Numerical Ability

PART B

Quantitative Aptitude

UNIT 1

Chapter 1 Simple Equations

Exercises

Practice Problems 1

Directions for questions 1 to 25: Select the correct alternative from the given choices.

1. P, Q, and R are successive even natural numbers in ascending order. Five times R is eight more than seven times P. Find Q.
(A) 6 (B) 8 (C) 12 (D) 14

2. Divide 1 kg weight into two parts such that the sum of the parts is $5/4^{th}$ the difference.
(A) 550 g, 450 g
(B) 200 g, 800 g
(C) 900 g, 100 g
(D) 400 g, 600 g

3. A is greater than B by $1/3^{rd}$ the sum of A and B. If B is increased by 40, it becomes greater than twice A by 10. Find A, B.
(A) 30, 20 (B) 60, 30
(C) 20, 10 (D) 20, 40

4. Ajay was asked to find $(2/9)^{th}$ of a number. He instead multiplied the number by (9/2) and obtained an answer which was 4235 more than the correct answer. Find the number.
(A) 900 (B) 945
(C) 990 (D) 810

5. An amount of ₹5,600 is divided among A, B, and C. The sum of the shares of B and C is equal to thrice the share of A. The sum of the shares of A and C is equal to nine-fifths the share of B. What is the share of C?
(A) ₹1,400 (B) ₹2,400
(C) ₹2,200 (D) ₹2,000

6. Four times the sum of the digits of a two-digit number is 18 less than the number and is also 9 less than the number formed by reversing its digits. Find the product of its digits.
(A) 12 (B) 20
(C) 30 (D) 42

7. Six years ago, Ram's age was four times Shyam's age. Six years hence, Ram's age will be thrice Shyam's age. After how many years from now will their combined age be 150 years?
(A) 21 (B) 9 (C) 36 (D) 18

8. The sum of the ages of Bharat and Sharat is twice the sum of their ages seven years ago. What is the product of their present ages, if the sum of the squares of their ages is 400?
(A) 192 (B) 180 (C) 200 (D) 164

9. Ashok has a total of 30 notes in denominations of ₹20 and ₹5. The total value of the notes with him is ₹300. Find the number of ₹20 notes with him.
(A) 5 (B) 10
(C) 8 (D) 6

10. A fraction is such that the numerator is five less than the denominator. Also four times, the numerator is one more than the denominator. Find the fraction.
(A) 4/9 (B) 3/8 (C) 2/7 (D) 7/12

11. The digits of a two digit number differ by 3. Find the difference of the number and the number formed by reversing its digits.
(A) 18 (B) 27 (C) 36 (D) 45

12. Two chocolates, three milk shakes and four cakes cost ₹190. Four chocolates and eight cakes cost ₹320. Find the cost of a milkshake (in ₹).
(A) 10 (B) 20
(C) 30 (D) Cannot be determined

13. Three consecutive even integers are such that one-third of the second number is equal to one-fourth of the third number. Find the three numbers.
(A) 4, 6, 8 (B) 8, 10, 12
(C) 12, 14, 16 (D) 2, 4, 6

14. Amar, Bhavan, Chetan, and Dinesh have a total of ₹150 with them. Amar has one-fourth of the total amount with the others. Find the amount with Amar (in ₹).
(A) 20 (B) 25 (C) 30 (D) 37.5

15. Ramesh is thrice as old as Suresh. Two years hence, Ramesh will be twice as old as Suresh. Find Ramesh's present age (in years).
(A) 2 (B) 3 (C) 4 (D) 6

16. Nalini has an amount of ₹20 in coins of denominations of 50 paise and ₹1. If she has a total of 30 coins with her, how many ₹1 coins does she have?
(A) 20 (B) 10 (C) 15 (D) 30

17. A two-digit number is one more than six times the sum of its digits and also five more than forty six times the difference of its digits. Find the number.
(A) 79 (B) 97 (C) 49 (D) 94

18. Find the value of k if the equations $3x + (k/3 + 2)y = 1$ and $kx + 2ky = 4$ have infinite solutions.
(A) 9 (B) 6 (C) 18 (D) 12

19. Cost of two pens, five pencils, and seven erasers is ₹37. Cost of seven pens, one eraser, and two pencils is ₹49. What is the cost of nine pencils and fortyseven pens?
(A) ₹184 (B) ₹276
(C) ₹284 (D) None of these

20. The sum of two numbers is 250. The difference of their squares is 12,500. Find the larger number.
(A) 130 (B) 140 (C) 150 (D) 160

21. Five three-digit numbers including N, were to be added. While adding, the reverse of N was added by mistake instead of N. Hence, the sum increased by 11 times the sum of the digits of N. Eight times the difference of N's units and hundreds digits is 6 more than twice its hundreds digit. Find its tens digit.

(A) 4 (B) 6
(C) 8 (D) 2

22. The cost of two pens, one eraser, and three sharpeners, is ₹23. The cost of six pens, three erasers, and one sharpener is ₹45. The cost of fourteen pens, seven erasers, and twenty one sharpeners is ₹161. Find the cost of each pen (in ₹).
(A) 3
(B) 4
(C) 5
(D) Cannot be determined

23. A child went to a shop to buy a pen, a pencil and a ruler where costs are integral values (in ₹) and are in decreasing order. Each item costs at least ₹4. The total cost is ₹15 and the cost of a pencil is ₹5. How many pencils can he purchase with the amount required to purchase ten rulers?
(A) 10 (B) 12
(C) 8 (D) 9

24. Nitya and Satya have some marbles with them. Nitya says to Satya, 'If you give one marble to me, we will have equal number of marbles'. Satya says to Nitya, 'If you give me one marble, I will have twice the number of marbles you have'. How many marbles do Nitya and Satya have respectively?
(A) 4, 6 (B) 5, 7
(C) 6, 4 (D) 7, 5

25. John covers 10 km per hour more than Peter while driving. On doubling his speed, Peter covers 15 km per hour more than John who is driving at his normal speed. What is John's speed?
(A) 40 km/hr (B) 25 km/hr
(C) 45 km/hr (D) 35 km/hr

Practice Problems 2

Directions for questions 1 to 5: Select the correct alternative from the given choices.

1. In a four-digit number, the sum of the middle two digits is twice the units digit. The sum of the hundreds digit and six times the thousands digit is twice the sum of the other two digits. The sum of the units digit and five times the thousands digit is twice the hundreds digit. How many values can the four-digit number assume?
(A) 1 (B) 2 (C) 3 (D) 4

2. The sum of two numbers is 4 and the reciprocal of one exceeds the reciprocal of the other by twice the product of their reciprocals. What is the product of the reciprocals of the two numbers?
(A) 3 (B) 2 (C) 1/3 (D) 1/2

3. Ashok went to a casino to play a card game. In each round, he happened to double the amount with himself and in each round he gave ₹x to his friend. At the end of three rounds, he was left with no money. If the amount he gave to his friend in each round was ₹20 more than the amount he started with, find the amount (in ₹) that he started with.
(A) 110 (B) 120 (C) 130 (D) 140

4. The cost of printing the first fifteen hundred copies of a book, is ₹1500. It costs y rupees to print each subsequent copy. The cost of printing the first 7500 copies of the book is ₹7080. Find y.
(A) 0.91 (B) 0.89 (C) 0.93 (D) 0.87

5. Mohan and Sohan went to a bakery shop. Mohan bought six puffs, four burgers, and three cakes and used up all the money he had. Sohan bought three puffs, eight burgers, and six cakes and spent 20% more than Mohan. What percent of his money did Mohan spend on puffs?
(A) $53\frac{1}{3}\%$ (B) $58\frac{3}{4}\%$
(C) $59\frac{4}{9}\%$ (D) $57\frac{1}{7}\%$

Directions for questions 6 and 7: Answer these questions based on the information below.

Rakesh went to a stationery shop to purchase a total of 38 pens, erasers, and sharpeners. He purchased at least 11 items of each. He purchased more sharpeners than erasers and more erasers than pens.

6. How many pens did he purchase?
(A) 11 (B) 12 (C) 13 (D) 14

7. If each pen cost ₹2, each eraser cost ₹3, and each sharpener cost ₹4, find the minimum expenditure be could have incurred on the items (in ₹).
(A) 116 (B) 118 (C) 117 (D) 119

Directions for questions 8 to 15: Select the correct alternative from the given choices.

8. A test had 200 questions. Each correct answer carried 2 marks. Each wrong answer carried –1/2 mark and unanswered questions carried no mark. Ajay attempted all the questions in the test and scored 360 marks. What would his marks be, if for each correct answer he got only 1/2 mark and for each wrong answer he lost 2 marks?
(A) 80 (B) 60 (C) 40 (D) 100

9. Anand bought a total of 30 white and brown pens for a total of ₹32. The cost of each white pen is 70 paise more than the cost of each brown pen. Which of the following represents a possible value of the cost of each brown pen (in paise)?

(A) 40 (B) 35 (C) 45 (D) 60

10. Runs scored by Bhangar in a match are 10 more than the balls faced by Karthik. The number of balls faced by Bhangar is 5 more than the number of runs scored by Karthik. Together they have scored 50 runs and Bhangar has faced 15 balls less than Karthik. What is the number of runs scored by Bhangar?
(A) 30 (B) 20 (C) 40 (D) 50

11. An amount of ₹9000 is divided among four people A, B, C, and D. The sum of the shares of A, C, and D is four times the share of B. The sum of the shares of B and D is equal to four-fifths the sum of shares of A and C. Find the share of D
(A) ₹1,800
(B) ₹2,400
(C) ₹2,200
(D) Cannot be determined

12. Gopi, Murthy, and Hari had some amount of money. Gopi gives half his amount to Murthy, who then gives half of what he now has to Hari. Hari gives half of what he now has to Gopi, who, now has exactly what he started with. If the sum of Murthy's initial amount and twice Hari's initial amount is ₹45, what was the amount (in rupees) Gopi started with?
(A) 10 (B) 15 (C) 30 (D) 40

13. A housewife, with a given amount, can buy either 10 apples or 15 oranges or 2 watermelons. Find the maximum number of oranges which she can buy with six times the initial amount such that she gets each of the three varieties of fruits.
(A) 75 (B) 81 (C) 60 (D) 72

14. A bag has balls of four colours—red, blue, pink, and green. Half the total number of balls are pink. One-fourth of the number of green balls equals one-third of the number of blue balls. The number of pink balls is 4 less than twice the total number of green and blue balls. The number of green balls is 22 less than the total number of blue and pink balls. Find the total number of balls in the bag.
(A) 48 (B) 36 (C) 40 (D) 54

15. The cost of four apples, six bananas, and eight oranges is p. The cost of five apples, eight bananas, and eleven oranges is q. The cost of eight apples, fourteen bananas, and twenty oranges is r. Which of the following is always true?
(A) $3p + 4q = \frac{3r}{4}$ (B) $-3p + 4q = \frac{r}{2}$
(C) $-4q + 3p = \frac{r}{2}$ (D) $-3p + 4q = r$

Directions for questions 16 and 17: Answer these questions based on the information below.

A shopkeeper sold a certain number of toys. The number of toys as well as the price of each toy (in ₹) was a two digit number. By mistake, he reversed the digits of both the number of toys he sold and the price of each toy. As a result, he found that his stock account at the end of the day showed 81 items more than it actually was.

16. Find the actual number of toys sold.
(A) 92 (B) 81
(C) 90 (D) 29

17. If the faulty calculations show a total sale of ₹882, find the actual selling price of each toy (in ₹).
(A) 89 (B) 98
(C) 97 (D) 79

Directions for questions 18 to 25: Select the correct alternative from the given choices.

18. A three-digit number is eleven times the two-digit number formed by using the hundreds and the units digit of the three-digit number, respectively, in the tens and units place of the two-digit number. If the difference between the digit in tens place and the digit in hundreds place is 1, then what is the digit in the units place?
(A) 2 (B) 3 (C) 4 (D) 1

19. A person on an exercise regime decides to lose 500 calories in a day. For every 30 minutes of exercise, he loses 80 calories, but gains 25 calories per cake that he eats. In the morning, he exercises for a certain time and he eats a certain number of cakes. In the afternoon, he exercises for double the time but eats thrice the number of cakes. In the evening, he exercises for 1.5 times the time he spends in the morning session and eats 4 times the number of cakes he eats in the morning. Had he not eaten any cake, he would have exceeded his target by 220 calories. If he actually falls short of his target by 180 calories, then, what is the number of cakes he has eaten in the evening?
(A) 12 (B) 8 (C) 16 (D) 10

20. A test consists of 120 questions. Each correct answer, each wrong answer, and each unanswered question in the test carry 1 mark, –½ mark, and –¼ mark, respectively. Find the maximum number of questions that the candidate could have answered wrongly in the test, if he scores 50 marks in it.
(A) 40 (B) 42 (C) 45 (D) 46

21. If $3x + y - 3z = 11$ and $2x - 2z + 5y = 29$ what is the value of $x + y - z$?
(A) 7 (B) 14 (C) 9 (D) 8

22. Classes A and B have 35 students each. If seven girls shift from class A to class B, then the number of girls

in the classes would interchange. If four girls shift from class B to class A, then the number of girls in class A would be twice the original number of girls in class B. What is the number of boys in class A and in class B?

(A) 18 and 11 (B) 24 and 17
(C) 18 and 27 (D) 17 and 24

23. Ten years ago, the age of a man was 35 years more than twice his son's age. After how many years from now will the man be

(i) twice his son's age?
(A) 15
(B) 20
(C) 25
(D) Cannot be determined

(ii) thrice his son's age?
(A) 10
(B) 15
(C) 20
(D) Cannot be determined

24. A grocer uses a weighing balance in which one pan weighs 0.5 kg and the other 0.75 kg. He puts a certain quantity of food grains in 0.5 kg pan and finds the weight (in kg) as a two-digit number. However, as the customer insists, he puts it in 0.75 kg pan. Now the indicated weight is 9.5 kg more than the weight which is obtained by reversing the digits of previous weight. Which of the following cannot be the actual weight (in kg) of the food grains?

(A) 43.25
(B) 36.25
(C) 41.5
(D) More than one of the above

25. There are ten children (aged 1 to 10 years) who have equal amounts of money. In the first transaction, the eldest child gives one rupee to every child younger to him. In the second transaction, the second eldest child gives one rupee to every child younger to him. This type of distribution continues for the next two transactions. In the end, the total sum with the children who have given money to other children is half the sum of money with the children who did not give any money. What was the original amount with each child?

(A) ₹36 (B) ₹48 (C) ₹50 (D) ₹32

HINTS/SOLUTIONS

Practice Problems I

1. Given that P, Q and R are successive even natural numbers in ascending order. As we need Q, we express P and R in terms of Q.

$P = Q - 2$ and $R = Q + 2$

$\therefore \quad 5(Q + 2) = 7(Q - 2) + 8 \Rightarrow Q = 8$

Hence, the correct option is (B).

2. Let the two parts be x grams and y grams

$x + y = 1000$

$x + y = \frac{5}{4}(x - y)$

$x - y = 800 \Rightarrow x = 900$

$y = 100$

Hence, the correct option is (C).

3. $\frac{A+B}{3} = A - B$

$3A - 3B = A + B$

$2A - 4B = 0$

$A - 2B = 0$ (32)

$B + 40 = 2A + 10$

$2A - B = 30$ (33)

$\Rightarrow \quad 4A - 2B = 60$

$3A = 60$

$\Rightarrow \quad A = 20$ and $B = 10$

Hence, the correct option is (C).

4. Let the number be x.

$\frac{9}{2}x - \frac{2}{9}x = 4235 \Rightarrow x = 990$

Hence, the correct option is (C).

5. Let the shares of A, B and C be A, B and C respectively.

then, $A + B + C = 5600$ (34)

$B + C = 3A$ (35)

$A + C = 9/5B$ (36)

Solving (34) and (35)

$4A = 5600 \Rightarrow A = ₹1400$

Solving (34) and (36)

$\frac{14B}{5} = 5600 \Rightarrow B = ₹2000$

Share of $C = 5600 - 1400 - 2000 = ₹2200$

Hence, the correct option is (C).

6. Let the number be 'ab'.

$4(a + b) = 10a + b - 18 \Rightarrow b + 6 = 2a$

Also, $10a + b - 18 = 10b + a - 9 \Rightarrow a - b = 1$

$\therefore \quad b + 6 = 2(b + 1)$

$b = 4$ and $a = 5$

$\therefore$ Product of digits = 20

Hence, the correct option is (B).

7. Let the present ages of Ram and Shyam be r years and s years respectively.

$r - 6 = 4(s - 6) \Rightarrow r = 4s - 18$

And $\quad r + 6 = 3(s + 6) \Rightarrow r = 3s + 12$

$\therefore \quad 4s - 18 = 3s + 12 \Rightarrow s = 30, r = 102$

$\therefore$ their present combined age is 132 years. It has to increase by 18 years to become 150 years. This will happen if the age of each increases by 9 years.

Hence, the correct option is (B).

8. Let the age of Bharat be A and Sharat be B.

$A + B = 2(A + B - 14) \Rightarrow A + B = 28$

$A^2 + B^2 = 400$

$(A + B)^2 - 2AB = 400$

$\therefore \quad AB = 192$

Hence, the correct option is (A).

9. Let the number of ₹20 notes with Alok be x. Number of ₹5 notes with him = $30 - x$. Given that,

$20x + 5(30 - x) = 300 \Rightarrow x = 10$

Hence, the correct option is (B).

10. Let the fraction be $\frac{x}{y}$

$x = y - 5$ (37)

and $\quad 4x = y + 1$ (38)

Solving (37) and (38)

$x = 2, y = 7$

$\therefore$ the fraction is $\frac{2}{7}$

Hence, the correct option is (C).

11. The difference between any two digit number and the number formed by reversing its digits is equal to 9 times the difference of its digits

$\therefore$ the required difference = 9 (3) = 27

Hence, the correct option is (B).

12. Let the costs of each chocolate, each milk shake and each cake be ₹ch ₹m and ₹c respectively.

$2ch + 3m + 4c = 190$

$4ch + 8c = 320$

The coefficients of ch and c are proportional, we can determine m, but not ch or c.

$2ch + 4c = 160$

$\therefore \quad 3m = 190 - (2ch + 4c) = 30$

$\therefore \quad m = 10.$

Hence, the correct option is (A).

13. Let the three consecutive numbers be $2x$, $2(x + 1)$, $2(x + 2)$

then $\frac{1}{3} \times 3\,(x + 1) = \frac{1}{4} \times 2(x + 2) \Rightarrow x = 2$

$\therefore$ the three numbers are 4, 6, 8

Hence, the correct option is (A).

14. Let the amount with Amar be ₹a

Total amount with the others = ₹$(150 - a)$

$a = \frac{1}{4}(150 - a) \Rightarrow a = 30$

Hence, the correct option is (C).

15. Let the present ages of Ramesh and Suresh be r years and S years respectively

$r = 3s$

$r + 2 = 2\,(s + 2)$

$\Rightarrow r = 2s + 2 = 3s$

$s = 2 \qquad r = 6$

$\therefore$ Ramesh's present age is 6 years.

Hence, the correct option is (D).

16. Let the number of 50 np coins with Nalini be x.

Number of ₹1 coins with her will be $(30 - x)$

Amount $= x \times 50 + (30 - x)\ 100 = 2000$

$\Rightarrow \quad x + 60 - 2x = 40 \ \Rightarrow x = 20$

$\therefore$ number of ₹1 coins with her is $(30 - 20) = 10$

Hence, the correct option is (B).

17. Let the number be 'ab' i.e. or $10a + b$.

Given that $10a + b = 6(a + b) + 1 \Rightarrow 4a = 5b + 1$

$a = b + \frac{b+1}{4}$

As a is a natural number, $b + 1$ must be divisible by 4.

$0 \le b \le 9$.

$\therefore$ b can be only 3 or 7 and correspondingly $a = 4$ or 9

$\therefore$ $ab = 43$ or 97

The second condition is satisfied only if 'ab' = 97.

$\therefore$ 'ab' = 97

Hence, the correct option is (B).

18. As the system has infinite solutions, the coefficients are proportional

i.e., $\frac{3}{k} = \frac{\frac{k}{3}+2}{2k}$; $k\left(\frac{k}{3} - 4\right) = 0$

$k = 0$ or 12. If $k = 0$, the second equation becomes inconsistent.

$\therefore$ $k = 12$

Hence, the correct option is (D).

19. Cost of 1 pen = x

Cost of 1 pencil = y

Cost of 1 eraser = z

$2x + 5y + 7z = 37$ (39)

$7x + 2y + z = 49$ (40)

Multiplying equation (40) by 7 and subtracting equation (39) gives the required value.

$\Rightarrow 49x + 14y + 7z = 343$

$2x + 5y + 7z = 37$

$47x + 9y = 306$

Hence, the correct option is (D).

20. Let the larger number be a.

Smaller number = $250 - a$

$a^2 - (250 - a)^2 = 12500$

$(a + 250 - a)\ (a - (250 - a)) = 12500$

$\Rightarrow \quad 2a - 250 = 50;\ \therefore\ a = 150$

Hence, the correct option is (C).

21. Let the number N be 'abc'.

Let the sum of the remaining numbers be R.

$R + cba = R + abc + 11(a + b + c)$

$\Rightarrow \quad 99(c - a) = 11(a + b + c)$

$\Rightarrow \quad 8c - 10a = b$ (41)

As $b \ge 0$ and $c \ge \frac{5}{4}a$, $c > a$

$\therefore$ 8 (Difference of a and c) $= 8(c - a) = 6 + 2a$

$8c - 10a = 6$ (42)

From (41) and (42), $b = 6$

Hence, the correct option is (B).

22. Let the cost of each pen, each eraser and each sharpener be ₹p, ₹e and ₹s respectively.

$2p + e + 3s = 23$ (43)

$6p + 3e + s = 45$ (44)

$14p + 7e + 21s = 161$ (45)

But (43), (45) are equivalent.

$\therefore$ We effectively have only 2 equations (43) and (44) or (44) and (45).

In order to find the cost of each pen, the ratio of the coefficients of erasers and sharpeners must be the same. As this is not the case, Choice (D) follows.

Hence, the correct option is (D).

23. If cost of a pencil is ₹5 then the only feasible values for cost of a pen and cost of a ruler are 6 and 4 respectively.

∴ Cost of 10 rulers = 40

∴ Number of pencils he purchased with ₹40 is

$= \frac{40}{5} = 8$

Hence, the correct option is (C).

24. Let the number of marbles with Nitya and Satya be n and s respectively.

$n + 1 = s - 1$

$\Rightarrow \quad n - s + 2 = 0 \qquad (46)$

$s + 1 = 2(n - 1)$

$\Rightarrow \quad 2n - s - 3 = 0 \qquad (47)$

Solving (46) and (47)

i.e. $2n - 2s + 4 = 0$

$2n - s - 3 = 0$

$\Rightarrow \quad s = 7, n = 5$

Hence, the correct option is (B).

25. Let the speeds of John and Peter be x and y respectively (in kilometres per hour)

$\Rightarrow \quad x = y + 10 \qquad (48)$

$x + 15 = 2y \qquad (49)$

$y = 25; x = 35$

∴ John's speed is 35 km/hr

Hence, the correct option is (D).

Practice Problems 2

Solutions for questions 1 to 5:

1. Let the number be '*abcd*'

Given that,

$b + c = 2d \qquad (50)$

$b + 6a = 2(c + d) \qquad (51)$

$d + 5a = 2b \qquad (52)$

Let us that the equations (50), (51) and (52) as the linear equations in a, b and c and express the values of a, b and c in terms of b.

By (51) – (50), we get $3c = 6a \Rightarrow a = \frac{c}{2}$

By substituting $c = 2a$ in (50), it becomes

$2d - 2a = b \qquad (53)$

Subtracting (53) from 2 × (52), we get

$12a = 3b \Rightarrow a = \frac{b}{4}$

As $c = 2a$, $c = \frac{b}{2}$

By substituting $a = \frac{b}{4}$ in (53), we get $2d =$

$\Rightarrow \quad d = \frac{3b}{4}$

∴ $a : b : c : d = 1 : 4 : 2 : 3$

∴ '*abcd*' can be 1423 or 2846.

Hence, the correct option is (B).

2. Let the two numbers be $\frac{1}{x}$ and $\frac{1}{y}$

$\frac{1}{x} + \frac{1}{y} = 4 \qquad (54)$

and $\quad x - y = 2xy$

$\Rightarrow \quad \frac{1}{y} - \frac{1}{x} = 2 \qquad (55)$

(54) + (55) gives $\frac{2}{y} = 6 \Rightarrow y = \frac{1}{3}$

(54) – (55) gives

$\frac{2}{x} = 2 \Rightarrow x = 1$

∴ Product of the reciprocals = (x) (y) = (1) (1/3) = (1/3)

Hence, the correct option is (C).

3. Suppose Alok started with ₹A. He gave ₹x to his friend in each round. The amounts with him at the end of the first, second and third rounds are (in ₹)

$2A - x$, $4A - 3x$ and $8A - 7x$ respectively.

Given that, $8A - 7x = 0$

$\Rightarrow \quad x = \frac{8}{7}A$

As $x = A + 20$, $\frac{8}{7}A = A + 20$

$\Rightarrow \quad A = 140$

Hence, the correct option is (D).

4. Number of sequence copies made = 7500 – 1500 = 6000.

Cost of making these copies (in ₹) = 7080 – 1500 = 5580.

$y = \frac{5580}{6000} = 0.93$

Hence, the correct option is (C).

5. Let the costs of each puff, each burger and each cake be ₹p, ₹b and ₹c respectively. Let the amount spent by Mohan be ₹x.

$6p + 4b + 3c = x \qquad (56)$

$3p + 8b + 6c = \frac{6x}{5} \qquad (57)$

Multiplying (56) by 2 and subtracting (57) from it,

$9p = \frac{4x}{5}$

$\Rightarrow \quad p = \frac{4x}{45}$

$\therefore \quad 6p = \frac{8}{15}x$

$\therefore$ Mohan spent $53\frac{1}{3}\%$ of his money on puffs.

Hence, the correct option is (A).

Solutions for questions 6 to 7:

Let the number of pens, erasers and sharpeners that Rakesh purchased be p, e and s respectively.

$p + e + s = 38$

$p < e < s$

$\therefore$ (p, e, s) can be only (11, 12, 15) or (11, 13, 14).

6. In either of the above combinations $p = 11$

Hence, the correct option is (A).

7. (p, e, s) = (11, 13, 14) results in the minimum expenditure.

Minimum expenditure = ₹ 117.

Hence, the correct option is (C).

Solutions for questions 8 to 15:

8. Let the number of questions correctly answered by Ajay be x. Number of questions wrongly answered by him = $200 - x$.

His mark = $2x - \frac{1}{2}(200 - x) = \frac{5}{2}x - 100 = 360$

$\Rightarrow \quad x = 184$

Upon interchanging of the marks, his marks would be

$= 184(1/2) - 16(2) = 60$

Hence, the correct option is (B).

9. Let the cost of each brown pen be b paise, cost of each white pen = $(b + 70)$ paise. Suppose Anand bought B brown pens. He must have bought $(30 - B)$ white pens.

Total cost = $bB + (30 - B)(b + 70)$

$30b - 70B + 2100 = 3200$

$B = \frac{3b - 110}{7}$. Among the given choices, B has a feasible value only when $b = 60$.

Hence, the correct option is (D).

10. Data is tabulated below

Bhangar	Karthin
Runs	$x + 10y$
Balls	$y + 5x$

$x + y + 10 = 50$

$x + y = 40$ (58)

$y + 5 = x - 15$

$x - y = 20$ (59)

$\Rightarrow \quad x = \left(\frac{40 + 20}{2}\right) = 30$

Runs scored by Bhangar = 30 + 10 = 40

Hence, the correct option is (C).

11. $A + B + C + D = 90000$ (60)

$A + C + D = 4B$ (61)

also, $B + D = 4/5\ (A + C)$ (62)

From (60) and (61)

$4B + B = 9000$

$\Rightarrow \quad B = ₹1800$

From (61) and (62)

$B + D = 4/5\ (4B - D)$

$5B + 5D = 16B - 4D$

$\Rightarrow \quad 11B = 9D$

$\Rightarrow \quad D = \frac{11}{9} \times 1800 = ₹2200$

Hence, the correct option is (C).

12. Let the initial amount with Gopi, Murthy and Hari in rupees be x, y and z respectively.

Gopi gives $\frac{x}{2}$ to Murthy

They now have $\frac{x}{2}$, $\frac{x}{2} + y$ and z

Murthy gives half of his amount to Hari

They now have

$\frac{x}{2}; \frac{x}{4} + \frac{y}{2}; \frac{x}{4} + \frac{y}{2} + z$

Hari gives half his amount to Gopi.

Gopi now has $\left(\frac{x}{2} + \frac{x}{8} + \frac{y}{4} + \frac{x}{4} + \frac{z}{2}\right)$ and this is equal to x.

$\Rightarrow \quad x = \frac{x}{2} + \frac{x}{8} + \frac{y}{4} + \frac{z}{2}$

$\Rightarrow \quad \frac{3x}{8} = \frac{y}{4} + \frac{z}{2}$

$\Rightarrow 3x = 2y + 4z = 2\ x\ 45 = 90$, as $(y + 2z)$ is given equal to 45.

$\Rightarrow \quad x = 30$

Hence, the correct option is (C).

13. Let the initial amount = A

Cost of 1 apple = $(A/10)$

Cost of 1 orange = $(A/15)$

Cost of 1 watermelon = $(A/2)$

As all three types of fruits are bought, the minimum shall be one.

As oranges are to be maximum, others are one each.

Amount spent = $6A$ (as per data).

Hence, the number of oranges

$= [6A - \{(A/10) + (A/2)\}] / (A/15)$

$= [6A - (6A/10)] / (A/15) = (54A/10) / (A/15) = 81$

Hence, the correct option is (B).

14. Let the number of balls of he colours red, blue, pink, green be r, b, p, g respectively.

Half the total number of balls in the box are pink

$$\therefore \quad p = b + r + g \tag{63}$$

$$\frac{g}{4} = \frac{b}{3} \Rightarrow \therefore g = \frac{4b}{3} \tag{64}$$

$$p = 2(g + b) - 4 \tag{65}$$

$$g = b + p - 22 \tag{66}$$

$\therefore \quad g = b + 2(g + b) - 4 - 22$

$\Rightarrow \quad 26 = 3b + g = 3b + \frac{4b}{3} = \frac{13b}{3}$

$b = 6$

$\therefore \quad g = 8, p = 24, r = 10$

Total number of balls $= b + g + p + r = 2p = 48$

Hence, the correct option is (A).

15. Let the costs of an apple, a banana and an orange be ₹a, ₹b and ₹c respectively.

$4a + 6b + 8c = p$

$5a + 8b + 11c = q$

$8a + 16b + 20c = r$

Going back from the options, only Choice (D) satisfy the given equation.

Hence, the correct option is (D).

Solutions for questions 16 and 17:

Let the number of toys sold by the shopkeeper be 'ab'. Let the selling price of each toy be ₹ 'cd'. Let his initial stock be s.

Actual stock left = s – 'ab'.

Stock as shown on record = s – 'ba'

$s - ba = s - ab + 81$

Given that,

$10a + b - (10b + a) = 81$

$a - b = 9$

$\therefore \quad a = 9$ and $b = 0$

16. $ab = 90$

Hence, the correct option is (C).

17. Mistaken sale price $= \frac{882}{'ba'} = \frac{882}{09} = 98 = ₹98$

Actual sale price = ₹89

Hence, the correct option is (A).

Solutions for questions 18 to 25:

18. Let the hundreds, tens and units digits be x, y, z respectively.

$(100x + 10y + z)$ is the 3-digit numbers $= 11(10x + z)$

$= (100x + z) + (10x + 10z)$

$\therefore \quad y = x + z$

Given $y - x = 1$

$\therefore \quad z = 1$

Note: If $(x - y)$ is taken as 1, $z = -1$, which is inadmissible.

Hence, the correct option is (D).

19. Calories lost by him on the particular day = 500 – 180 = 320

If he exercises for x hours, he loses $(2x) \times (80)$ calories $= 160x$.

If he eats y cakes he gains $= 25y$ calories

$\therefore \quad$ In the morning he loses $= 160x - 25y$ calories

In the afternoon he loses $= (320x - 75y)$ calories

In the evening he loses $= (240x - 100y)$ calories

$\therefore$ Total calories lost $= 720x - 200y = 320$ calories (1)

The second relation, as per the data is,

$720x = 500 + 220$

$x = 1, y = 2$

$\therefore \quad$ Cakes eaten in the evening $= 2 \times 4 = 8$

Hence, the correct option is (B).

20. Let the number of questions correctly answered, wrongly answered and unanswered by the candidate be C, W and U respectively. Given that

$$C + W + U = 120 \tag{67}$$

$$C - \frac{1}{2}W - \frac{1}{4}U = 50 \tag{68}$$

Subtracting (68) from (67),

$$\frac{3}{2}W + \frac{5}{4}U = 70$$

$$U = \frac{280 - 6W}{5} = 56 - \frac{6}{5}W \tag{69}$$

As $U \geq 0$, $W \leq 46\frac{2}{3}$.

To satisfy (69), W must be divisible by 5.

$\therefore \quad W = 45.$

Hence, the correct option is (C).

21. $3x + y - 3z = 11$

$\Rightarrow 3(x - z) + y = 11$ (70)

$2(x - z) + 5y = 29$ (71)

Let $x - z = a$

$\Rightarrow \quad 3a + y = 11$ (72)

$2a + 5y = 29$ (73)

(72) × 5 − (73)

$\Rightarrow \quad 13a = 26$

$\Rightarrow \quad a = 2$

$y = 5$

$a = x - z$

$\Rightarrow \quad y + x - z = 5 + 2 = 7$

Hence, the correct option is (A).

22. Let the number of girls in class A and class B be x and y respectively

$x = y + 7$

$\Rightarrow \quad x - y = 7$ (74)

$x + 4 = 2y$

$x - 2y = -4$ (75)

Solving (74) and (75), we get

$y = 11, x = 18$

$\therefore$ Number of boys in $A = 35 - 18 = 17$

Number of boys in $B = 35 - 11 = 24$

Hence, the correct option is (D).

23. Let the present ages of the man and the son be m years and s years respectively.

$m - 10 = 2(s - 10) + 35$

$m = 2s + 25$

(i) let us say after x years the father will be twice his son's age.

$m + x = 2(s + x)$

$2s + 25 + x = 2s + 2x$

$25 = x$

Hence, the correct option is (C).

(ii) Let us say after x years the father will be thrice his son's age

$2s + 25 + x = 3(s + x)$

$25 = s + 2x$

We have only 1 equation but 2 unknowns. So x cannot be found.

Hence, the correct option is (D).

Solutions for question 24:

Let the actual weight of food grains be w.

Case I: If it is kept in the pan of weight 0.5 kg

Total weight (including that of food grains) = $w + 0.5$

On the other side of the balance let an indicated weight 'xy' be kept,

$w + 0.5 = 10x + y + 0.75$ (76)

Case II: If food grains are kept in the other pan of the balance

$w + 0.75 = 10y + x + 9.5 + 0.5$ (77)

(77) − (76) gives

$0.25 = 9(y - x) + 9.25$

$9(x - y) = 9$

$x - y = 1$ (78)

24. $w = 10x + y + 0.25$

So, the actual weight $w = 10x + y + 0.25$

So, in the actual weight after the decimal part 0.25 must appear, and in the integral part, the ten's digit should be more than between the unit's digit. Both these conditions are satisfied for (A), not for choices (B) and (C).

Hence, the correct option is (D).

25. Let the children be $C_{10}, C_9, \ldots, C_1$ in decreasing order of their ages.

Let each child have ₹x at the beginning. The following table gives the amounts with each of them at the end of each transaction.

$1 \rightarrow (x - 9), (x + 1)(x + 1), \ldots, (x + 1)$

$2 \rightarrow (x - 9), (x - 7)(x + 2), (x + 2), \ldots, (x + 2)$

$3 \rightarrow (x - 9), (x - 7), (x - 5), (x + 3), (x + 3), \ldots, (x + 3)$

$4 \rightarrow (x - 9), (x - 8), (x - 5), (x - 3), (x + 4), (x + 4), \ldots, (x + 4)$

At the end, the total sum with children who have given some money

$= (x - 9) + (x - 7) + (x - 5) + (x - 3) = 4x - 24$

The total sum with those children who have not given any money $= 6 - (x + 4) = 6x + 24$

$\therefore \quad 6x + 24 = 2(4x - 24)$

$\Rightarrow \quad 2x = 72$

$x = 36$

Hence, the correct option is (A).

CHAPTER 2 RATIO-PROPORTION–VARIATION

EXERCISES

Practice Problems 1

Directions for questions 1 to 18: Select the correct alternative from the given choices.

1. If $p:q = 5:4$ and $p = a + b$ and $q = a - b$, find $a:b$.
(A) 1:9 (B) 9:1
(C) 5:4 (D) 4:5

2. The number of marbles with A and B are in the ratio of 10:11. Which of the following cannot be a possible number of marbles with A and B together?
(A) 189 (B) 210 (C) 231 (D) 153

3. The ratio of the number of students in three classes A, B, and C is 3:7:8. If ten students, are transferred from C to B, B will have 80 students. Find the total number of students in the three classes.
(A) 150 (B) 160 (C) 180 (D) 210

4. Three positive numbers p, q, and r satisfy $\frac{q+r}{p}=\frac{p+r}{q}=\frac{p+q}{r} = K$. $K =$ _____.
(A) 3/2 (B) 5/2 (C) 3 (D) 2

5. What must be subtracted from p and added to q so that the ratio of the resultants becomes 1:3?
(A) $\frac{p+q}{3}$ (B) $\frac{3p-q}{4}$
(C) $\frac{p-q}{p+q}$ (D) $\frac{q-3p}{4}$

6. Vipin's present age is twice the age of Kishore one year ago. What is the sum of their present ages (in years), if the ratio of the sum of their present ages to the difference of their present ages is 19:5?
(A) 21 (B) 19 (C) 24 (D) 34

7. Three different types of balls priced at ₹5, ₹8, and ₹13 per piece are displayed in three different boxes by a trader. Mr. Paul bought from this shop all three types of balls spending a total sum of ₹768. The numbers of the balls he bought, taken in the order in which the prices are mentioned above, are in the ratio 5:4:3. How many balls of the costliest variety did he buy?
(A) 104 (B) 64 (C) 48 (D) 24

8. If $a:b = 2:3$ $b:c = 4:3$ and $c:d = 2:3$, then find $a:b:c:d$.
(A) 8:12:9:27
(B) 16:24:18:27
(C) 18:27:36:8
(D) 12:18:15:20

9. The weights of Bimal and Basu are in the ratio 2:3 and the weights of Basu and Bali are in the ratio 4:3. What is Basu's weight (in kg) if the sum of the weights of Bimal, Basu, and Bali is 203 kg?
(A) 84 (B) 76 (C) 49 (D) 65

10. If $3x - 4y + 2z = 0$ and $4x - 2y - z = 0$, find $x:z:y$.
(A) 8:10:11 (B) 8:11:40
(C) 11:40:8 (D) 8:40:11

11. If $a + b - c:b + c - a:a + c - b = 5:6:7$, then find $a:b:c$.
(A) 12:13:11 (B) 12:11:13
(C) 13:12:11 (D) 13:11:12

12. Which of the following represents a possible value of $p : q$ satisfying $\frac{20p^2-40pq}{pq+4q^2} = 20$?
(A) 3:1 (B) 1:4 (C) 4:1 (D) 5:1

13. Ninety three is divided into two parts such that thrice the first part and twice the second part are in the ratio 25:4. Find the first part.
(A) 60 (B) 75 (C) 50 (D) 70

14. If three is subtracted from the numerator and five is added to the denominator of a fraction, the new fraction formed is 1/2. If two is added to the numerator of the initial fraction, the ratio of the new numerator to the denominator becomes 1:1. Find the original fraction.
(A) 11/13 (B) 18/23
(C) 13/15 (D) 13/11

15. The ratio of the number of students in classes A, B, and C is 3:7:8. If 10 students leave C and join B, the ratio of the number of students in B and C would be reversed. Find the total number of students in the classes A, B, and C.
(A) 144 (B) 162 (C) 180 (D) 198

16. A person has with him a certain number of weighing stones of 100 g, 500 g, and 1 kg in the ratio of 3 : 5 : 1. If a maximum of 5 kg can be measured using weighing stones of 500 g alone, then what is the number of 100 g stones he has?
(A) 6 (B) 3 (C) 9 (D) 5

17. The ratio of the prices of tea last year and this year is 5:6. The ratio of the prices of coffee last year and this year is 7:8. The sum of prices of a kg of tea and a kg of coffee this year is ₹48. Find the price of tea (in ₹) last year if it was $\frac{20}{21}$ of the price of coffee last year.
(A) 15 (B) 20 (C) 25 (D) 10

18. Ajay and Vijay wrote a test. The sum of Ajay's score and twice Vijay's score is 310. The sum of Vijay's score and twice Ajay's score is 290. Find the ratio of the scores of Ajay and Vijay.
(A) 9:11 (B) 13:17
(C) 11:19 (D) 7:13

Directions for questions 19 and 20: These questions are based on the data given below.

A test of 60 minutes contains questions on Mathematics and English only. The time taken to solve a Mathematics question is twice the time taken to answer an English question and the ratio of time taken to solve all Mathematics questions to time taken to answer all English questions is 8/7.

19. What is the ratio of the number of English questions to that of Mathematics?
(A) 11/7 (B) 7/4
(C) 9/4 (D) 7/5

20. If the total number of questions is 22, how many English questions can be answered in 18 minutes?
(A) 8 (B) 10
(C) 11 (D) 9

Directions for questions 21 to 25: Select the correct alternative from the given choices.

21. The pressure of a gas varies directly with the temperature when the volume is constant and varies inversely with the volume when temperature is constant. If the present temperature is 100 K, what will be the increase in temperature if the pressure triples and the volume doubles?
(A) 200 K (B) 600 K
(C) 500 K (D) 100 K

22. For a body starting from rest, the distance travelled (d) is directly proportional to the square of the time elapsed from the start (t). When $t = 4$ s, $d = 48$ m. What is the value of d (in metres) at $t = 7$ s?
(A) 128 (B) 150 (C) 115 (D) 147

23. 'A' varies directly as the sum of two quantities 'B' and 'C'. 'B' in turn varies directly as 'x' and 'C' varies inversely as 'x'. When $x = 1$ or 2, $A = 3$. Find the value of A when $x = 4$.
(A) 5 (B) 4.5
(C) 5.5 (D) 6

24. The extension of a spring from its rest position is directly proportional to the force acting on the spring. An additional force applied on the already stretched spring produces a further extension, which is twice that of the initial extension. What is the ratio of the additional force to the initial force?
(A) 3:1 (B) 2:1
(C) 4:1 (D) 1:3

25. The kinetic energy of a body is directly proportional to the square of its speed when the mass is kept constant and is directly proportional to mass when its speed is kept constant. A body with a mass of 2 kg and a speed of 10 m/s has a kinetic energy of 100 joules. What is the kinetic energy of a body whose mass is 20 kg and speed is 1 m/s?
(A) 100 joules (B) 1000 joules
(C) 10 joules (D) 20 joules

Practice Problems 2

Directions for questions 1 to 8: Select the correct alternative from the given choices.

1. The ratio of the ages of four members of a family is 9 : 8:3:2. The average age of the family is 22 years. What is the age of the eldest person in the family?
(A) 36 years (B) 32 years
(C) 12 years (D) 6 years

2. The strength of a class is 70. Which of the following cannot be the ratio of the number of boys and girls in the class?
(A) 2:5 (B) 3:2 (C) 11:3 (D) 9:2

3. The ratio of the number of chocolates with A and B is 3:4. If A gives four chocolates to B, the ratio of the number of chocolates with them becomes 5:9. How many chocolates did A have initially?
(A) 32 (B) 24 (C) 20 (D) 36

4. The ratio of the monthly incomes of A and B is 3:4. The ratio of the monthly expenditures of A and B is 4:5. Which of the following represents a possible value of the ratio of their savings?
(A) 9:10 (B) 3:4
(C) 13:20 (D) 4:5

5. The ratio of the present ages of a husband and a wife is 5:4.
(i) Which of the following can be a possible ratio of their ages 20 years ago?
(A) 5:4 (B) 6:5
(C) 23:20 (D) 13:10
(ii) Which of the following can be a possible ratio of their ages 20 years hence?
(A) 5:4 (B) 6:5
(C) 7:5 (D) 13:10

6. There are 2 two-digit numbers. Their product equals the product of the numbers formed by reversing their digits. Which of the following holds true if the numbers are denoted by ab and cd?
(A) a, b, d, and c are in proportion.
(B) a, b, c, and d are in proportion.
(C) a, d, c, and b are in proportion.
(D) a, c, b, d are in proportion.

7. A certain number is added to each of a pair of numbers which are in the ratio 4:7. The sum of the resulting numbers is 75 and their ratio (taken in the same order as mentioned above) is 8:17. What is the number added?

(A) –12 (B) 9
(C) –13 (D) 8

8. Ajay went to a market to buy a total by 90 apples, oranges and bananas. He bought an equal number of oranges and bananas. The ratio of the number of apples and oranges he bought is 5:2. If the price of each orange was equal to that of each apple, he could have skipped the purchase of bananas and instead purchased the same number of apples and oranges as he actually bought for the same total amount. If the prices (in ₹) are all integers, find the minimum possible total expenditure he could have incurred (in ₹).
(A) 120 (B) 130 (C) 140 (D) 150

Directions for questions 9 and 10: These questions are based on the data given below.

The amount used to purchase one litre of petrol can be used to purchase three litres of diesel or five litres of kerosene. Out of a certain amount, ₹510 is spent on diesel.

9. How much is spent on kerosene if equal volumes of the three liquids are purchased with the total amount?
(A) ₹300 (B) ₹306 (C) ₹382 (D) ₹354

10. What will be the amount spent on petrol if the total amount referred in the above question is instead spent to purchase equal volumes of petrol and kerosene only?
(A) ₹1250 (B) ₹1275 (C) ₹1955 (D) ₹1360

Directions for questions 11 to 25: Select the correct alternative from the given choices

11. If $k=\frac{a+c}{b+d}=\frac{c+e}{d+f}=\frac{a+e}{b+f}$, when all quantities are positive, then which of the following must be true?
(A) $k=\frac{e}{f}$ (B) $k=\frac{a}{b}$
(C) $k=\frac{c}{d}$ (D) All of the above

12. P and Q are distinct two-digit numbers. P_s and Q_s denote the sums of the digits in P and Q, respectively. If $\frac{P}{P_s}=\frac{Q}{Q_s}$, then find the minimum possible value of $P_s + Q_s$.
(A) 8 (B) 9 (C) 6 (D) 3

13. If $a:b = 3:4$ and $c:d = 2:3$, then find $\frac{a^3c^2+b^3d^2}{ab^2d^2+a^2bcd}$.
(A) $\frac{19}{3}$ (B) $\frac{19}{4}$ (C) $\frac{19}{9}$ (D) $\frac{19}{18}$

14. Amar, Bhavan, and Chetan have some coins with them. The total number of coins with Bhavan and Chetan and those with Amar and Chetan are in the ratio 4 : 5. The total number of coins with Bhavan and Chetan and those with Amar and Bhavan are in the ratio 4 : 3. Which of the following is the ratio of the number of coins with Bhavan and Chetan?
(A) 7:4 (B) 3:2 (C) 7:5 (D) 1:3

15. Manoj and Shiva, who are colleagues in an office, have their monthly savings in the ratio of 2:3. Manoj spends two-thirds of his income every month. If the ratio of their monthly incomes is 3:4, what is the ratio of their expenditures?
(A) 3:2 (B) 4:5 (C) 4:3 (D) 5:3

16. In a three digit number, the units digit is the sum of the other two digits, and it exceeds the tens digit by as much as the latter exceeds the hundreds digit. How many such three digit numbers exist?
(A) 1 (B) 2 (C) 3 (D) 4

17. Mr. Ram has five sons – Ajay, Balu, Charan, Eswar and Ganesh. He had some chocolates with him which he distributed in the following manner. Twice the number of chocolates received by Ajay, thrice the number of chocolates received by Charan and four times the number of chocolates received by Eswar are equal. Five times the number of chocolates received by Balu, six times the number of chocolates received by Eswar and eight times the number of chocolates received by Ganesh are equal. Find the minimum number of chocolates that could have been distributed by Mr. Ram.
(A) 377 (B) 661 (C) 441 (D) 551

18. If $\frac{a^2+c^2}{a+c}=\frac{b^2+c^2}{b+c}=k$ and $a \neq b$, which of the following is equal to k?
(A) $a+c$ (B) $b+c$ (C) $a+b$ (D) $a-c$

19. The mean proportional between two numbers is 9. The third proportional of the same numbers is 6561. Find the greater of the 2 numbers.
(A) 18 (B) 27 (C) 36 (D) 81

20. In a three digit number, the tens digit is the average of the other two digits. The ratio of the number formed by its first two digits and their sum equals the ratio of the number formed by its last two digits and their sum. How many three digit numbers satisfy these conditions?
(A) 7 (B) 8 (C) 9 (D) 6

21. Two diaphragms partition a cylinder into three chambers, whose volumes are in the ratio of 1:2:3. Pressure of a gas, which is in joint variation with its mass and volume, is directly proportional to its mass and inversely proportional to its volume. If the masses of the gas in the three chambers are same, find the ratio of the pressures of the gases in the three chambers taking them in the same order as the volumes have been taken. (Assume no factors other than those mentioned play a role.)

(A) 3:2:1 (B) 1:2:3
(C) 6:3:2 (D) 1:1:1

22. The volume of a cone varies as the square of the radius of its base when its height is fixed and also varies as its height when radius of its base is fixed. If a radius of 3 cm and height of 7 cm give it a volume of 66 cubic centimetre, then what will be the radius, (in centimetres), if the cone has a height of 6 cm and volume of 308 cubic centimetres?
(A) 9 (B) 15 (C) 7 (D) 11

23. The amount collected per month from a consumer of electrical power consists of two parts—a fixed charge for providing the service and a variable charge, which is directly proportional to the number of units of power consumed. An amount of ₹700 is collected from the consumer when he consumed 50 units in a month. It is also noticed that when the consumption increases from 100 units per month to 200 units per month, the bill amount increases to 5/4 times that of the former. How much is the fixed charge per month?
(A) ₹600 (B) ₹350 (C) ₹500 (D) ₹400

24. The distance (in metres) to which a boy can throw a stone is inversely proportional to its weight (in kg). He breaks the stone into 3 pieces whose weights (in kg) are in the ratio 1:3:2. He then throws the stones one by one. The sum of the distances they cover is 22 metres. To what distance can he throw the unbroken stone?
(A) 3 m (B) 4 m (C) 6 m (D) 2 m

25. A garrison of 900 soldiers had food-stock sufficient for 30 days when the rate of consumption is 2.5 kg/day/soldier. After some days of consumption at that rate, 300 soldiers were transferred to another garrison and the balance food lasted for 25 days for the remaining soldiers. If the rate of consumption of the remaining soldiers was 3.0 kg/day/soldier, after how many days from the start, were the soldiers transferred?
(A) 12 (B) 10 (C) 8 (D) 15

HINTS/SOLUTIONS

Practice Problems I

1. Let $p = 5x$ $\quad q = 4x$

$5x = a + b$ (3)

$4x = a - b$ (4)

From (3) and (4),

$2a = 9x$ and $2b = x$

$\therefore a : b = 2a : 2b = 9 : 1$

Hence, the correct option is (B).

2. Let the number of marbles with A and B be $10x$ and $11x$ respectively

Total number of marbles = $21x$

$\therefore$ the total number of marbles must be divisible by 21. Only Choice (D) violates this condition.

Hence, the correct option is (D).

3. Let the numbers of students in A, B and C be $3x$, $7x$ and $8x$ respectively.

If 10 students leave C and join B, C and B would have $8x - 10$ students and $7x + 10$ students respectively.

$7x + 10 = 80$; $x = 10$

Total number of students = $18x = 180$

Hence, the correct option is (C).

4. Given that $\frac{q+r}{p} = \frac{p+r}{q} = \frac{p+q}{r} = k$

$q + r = pk$, $p + r = qk$ and $p + q = rk$

$q + r + p + r + p + q = k(p + q + r)$

$(p + q + r)(2 - k) = 0$

$\therefore p + q + r \neq 0$,

$\therefore 2 - k = 0 \Rightarrow k = 2$

Hence, the correct option is (D).

5. $\frac{p-x}{q+x} = \frac{1}{3}$

$x = \frac{3p-q}{4}$

Hence, the correct option is (B).

6. Let the present age of Kishore be x years,

Vipin's present age is $2(x - 1)$ years

Given the ratio of the sum of their present ages to the difference of their present age is 19 : 5

i.e. $\frac{3x-2}{x-2} = \frac{19}{5}$ $\therefore x = 7$

Sum of present ages = $(k + v)$ = 19 years

Hence, the correct option is (B).

7. Ratio of prices = 5 : 8 : 13

Ratio of number of balls = 5 : 4 : 3

Ratio of amounts spent = $(5 \times 5) : (8 \times 4) : (13 \times 3)$

$= 25 : 32 : 39$

Total number of parts of the ratio = 25 + 32 + 39 = 96

Total amount spent, as per data = ₹768

Value of each part of the amounts' ratio = 768/96 = 8

Amount spent on costliest variety

= Value of 39 parts = 39×8

Number of costliest variety balls Paul purchased is

$= (39 \times 8)/13 = 24$

Hence, the correct option is (D).

8. $a : b = 2 : 3 = (2 \times 4) : (3 \times 4) = 8 : 12$

$b : c = 4 : 3 = (4 \times 3) : (3 \times 3) = 12 : 9$

$c : d = 2 : 3 = \left(2 \times \frac{9}{2}\right) : \left(3 \times \frac{9}{2}\right) = 9 : \left(\frac{27}{2}\right)$

$a : b : c : d = 8 : 12 : 9 : \frac{27}{2} = 16 : 24 : 18 : 27$

Hence, the correct option is (B).

9. Let a, b and c be the weights of the Bimal, Basu and Bali respectively.

$a/b = 2/3 = 8/12$; $c/b = 3/4 = 9/12$; $a : b : c = 8 : 12 : 9$

Given the sum of the weights (8 + 12 + 9 = 29 parts) is 203 kg i.e. one part is 7 kg. So Basu's weight i.e., 12 parts is $12 \times 7 = 84$ kg

Hence, the correct option is (A).

10. $3x - 4y + 2z = 0$ (5)

$4x - 2y - z = 0$ (6)

From (6)

$z = 4x - 2y$

Substituting $z = 4x - 2y$ in (5), we get

$3x - 4y + 8x - 4y = 0$

$11x = 8y$

Again, $z = 4x - 2y$

multiply by 4 on both sides, we get

$4z = 16x - 8y$; substituting $8y = 11x$

$4z = 16x - 11x = 5x$

$x : z : y = x : \frac{5}{4}x : \frac{11}{8}x = 8 : 10 : 11$

Hence, the correct option is (A).

11. Let $a + b - c = 5x$

$b + c - a = 6x$

$a + c - b = 7x$

$(a + b - c) + (b + c - a) = 11x$

$\therefore 2b = 11x$

$(a + b - c) + (a + c - b) = 12x$

$\therefore 2a = 12x$

$(b + c - a) + (a + c - b) = 13x$

$\therefore 2c = 13x$

$\therefore a : b : c = 2a : 2b : 2c = 12 : 11 : 13$

Hence, the correct option is (B).

12. Given, $\frac{20p^2 - 40pq}{pq + 4q^2} = 20 \Rightarrow 20p^2 - 60pq - 80q^2 = 0$

$\Rightarrow p^2 - 3pq - 4q^2 = 0$

$\Rightarrow (p - 4q)(p + q) = 0$

$\therefore \frac{p}{q} = 4 \text{ or } -1$ $\therefore$ Choice (C) is possible.

Hence, the correct option is (C).

13. Let the first and second parts be a and b.

Given, $\Rightarrow \frac{a}{b} = \frac{25}{6}$

$a = \frac{25}{31}(93) = 75$

Hence, the correct option is (B).

14. Let the fraction be x/y

$\frac{x-3}{y+5} = \frac{1}{2} \quad \Rightarrow 2x - y = 11 \qquad (7)$

$\frac{x+2}{y} = 1; \quad x - y = -2 \qquad (8)$

Subtracting (8) from (7), we get $x = 13$; $y = 15$

$\therefore$ Fraction = 13/15

Hence, the correct option is (C).

15. Let the number of students in three classes A, B and C be $3x$, $7x$ and $8x$ respectively.

Given, $\frac{7x+10}{8x-10} = \frac{8}{7} \Rightarrow x = 10$

Total = $18x = 180$

Hence, the correct option is (C).

16. Number of weighing stones of 500 gms = 5000/500 = 10

$\therefore$ Number of 100 gm weights = $(3/5) \times 10 = 6$

Hence, the correct option is (A).

17. Let the price of tea last year be ₹$5x$ per kg. Let the price of coffee last year be ₹$7y$ per kg.

$\frac{6}{5}(5x) + \frac{8}{7}(7y) = 48$

$5x = \frac{20}{21}(7y) \Rightarrow y = \frac{3}{4}x \Rightarrow 6x = \left(\frac{3}{4}x\right) = 48$

$\Rightarrow x = 4 \Rightarrow 5x = 20$

Hence, the correct option is (B).

18. Let the scores of Ajay be a and v respectively.

$a + 2v = 310 \qquad (9)$

$v + 2a = 290 \qquad (10)$

Solving (9) and (10)

$a = 90$ and $v = 110$

$\therefore \quad a : v = 9 : 11$

Hence, the correct option is (A).

Solutions for questions 19 and 20:

19.

	Number of questions	Time per question	Total time
Mathematics	a	2x	8y
English	b	X	7y

Given $15y = 60$

$y = 4 \quad \therefore 8y = 32$ min $\quad 7y = 28$ min

$\frac{(a)(2x)}{(b)(x)} = \frac{8}{7} \qquad [a/b = 16/28 = 4/7]$

$\therefore$ Ratio of the number of English questions to the number of Maths = $b/a = 7/4$

Hence, the correct option is (B).

20. Total number of questions is 22.

Number of English questions = $(7 \times 22)/11 = 14$

Time taken for English questions = $(60 \times 7)/15 = 28$ minutes

Number of questions that can be answered in 18 minutes = $18/(28/14) = 9$

Hence, the correct option is (D).

Solutions for questions 21 to 25:

21. $\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}; \quad \frac{xy}{100} = \frac{(3x)(2y)}{T_2}, \; T_2 = 600 \text{ K}$

$\therefore T_2 - T_1 = 500$ K

Hence, the correct option is (C).

22. $\frac{d_1}{t_1^2} = \frac{d_2}{t_2^2}; \frac{48}{16} = \frac{d_2}{49} \Rightarrow d_2 = 147$

Hence, the correct option is (D).

23. $A \; \alpha (B + C) \Rightarrow A = k_1 (B + C)$

$B \; \alpha x \Rightarrow B = k_2 x$

$C \; \alpha \; \frac{1}{x} \Rightarrow C = \frac{k_3}{x}$

$\therefore A = k_1 \left[k_2 x + \frac{k_3}{x}\right]$

As $A = 3$, when $x = 1$,

$3 = k_1 [k_2 + k_3] \qquad (11)$

As per data, $A = 3$, when $x = 2$

$\therefore 3 = k_1\left[2k_2 + \frac{k_3}{2}\right]$ (12)

(11) – (12) gives

$-k_1 k_2 + \frac{k_1 k_3}{2} = 0 \quad \Rightarrow k_1\left(\frac{k_3}{2} - k_2\right) = 0$

$\Rightarrow k_1 = 0$

or $k_2 = \frac{k_3}{2}$ are the possible cases

k_1 cannot be 0 (as A can't be 0).

Hence $k_2 = \frac{k_3}{2}$

From (11)

$3 = \frac{3k_1 k_3}{2}$

$k_1 k_3 = 2. \Rightarrow k_1 k_2 = 3 - 2 = 1.$

Hence $A = k_1 k_2 x + \frac{k_1 k_3}{x} = x + \frac{2}{x}$

When $x = 4$, $A = 4 + \frac{2}{4} = \frac{9}{2} = 4.5$

Hence, the correct option is (B).

24. $k \quad x \quad 2x$

0 1 2

'0' represents the unstretched position of the end of the spring. By the application of force F_1, the spring is stretched from position '0' to '1' and due to additional force F_2, the spring is additionally stretched from position '1' to '2'.

$F_1 = kx$

$F_1 + F_2 = k(x + 2x)$

$F_1 + F_2 = (k)(3x) \Rightarrow F_2 = 2kx$

F_2 as *a* fraction of $F_1 = \frac{(k)(2x)}{(k)(x)} = 2:1$

Hence, the correct option is (B).

25. Let k, M and S be the kinetic energy, mass and speed of *a* body respectively.

Given: $k \propto S^2$ (when M is kept constant) and $k \propto M$ (when S is kept constant)

$\Rightarrow k \propto MS^2 \Rightarrow k = CMS^2$

where C is the constant of proportionality.

Given that when $M = 2$ kg, $S = 10$ m/s, $k = 100$ joules

$\Rightarrow 100 = C \times 2 \times 10^2 \Rightarrow C = 1/2 \Rightarrow k = 1/2\ MS^2$

When $M = 20$ kg and $S = 1$ m/s

$k = 1/2 \times 20 \times 1^2 = 10$

$\therefore$ A body of mass 20 kg moving with a speed of 1 m/s has a kinetic energy of 10 joules.

Hence, the correct option is (C).

Practice Problems 2

Solutions for questions 1 to 8:

1. Let the ages of the four members be $9k$, $8k$, $3k$ and $2k$ years respectively.

Average age $= \frac{9k + 8k + 3k + 2k}{4} = 22 \Rightarrow k = 4$

$\therefore$ The age of the eldest family member $= 9 \times 4 = 36$

Hence, the correct option is (A).

2. Let the ratio of the number of boys and girls in the class be $a : b$. Number of boys and girls will be ak and bk.

Given $ak + bk = 70$

$$K = \frac{70}{a+b}.$$

Since k is an integer $a + b$ must be a factor of 70; from the options

The sum of the ratio in options (A), (B), (C) are factors of 70. But the sum of ratio 9 : 2 is not a factor of 70.

$\therefore$ 9 : 2 is not a possible ratio.

Hence, the correct option is (D).

3. Let the number of chocolates A and B be $3k$, $4k$

$\frac{3k-4}{4k+4} = \frac{5}{9}$

$\therefore A$ had $3k = 24$ chocolates initially.

Hence, the correct option is (B).

4. If $\frac{s_1}{s_2} < \frac{e_1}{e_2}$ then $\frac{s_1}{s_2} < \frac{s_1 + e_1}{s_2 + e_2} < \frac{e_1}{e_2}$

In the given problem, $\frac{e_1 + s_1}{e_2 + s_2} = \frac{3}{4}$ and $\frac{e_1}{e_2} = \frac{4}{5}$

$\therefore \frac{s_1}{s_2} < \frac{3}{4}.$

Only Choice (C), i.e. $\frac{s_1}{s_2} = \frac{13}{20}$ satisfies this condition.

Hence, the correct option is (C).

5. Let the present ages of the husband and the wife be $5x$ years and $4x$ years respectively.

(i) Ratio of their ages 20 years ago

$= \frac{5x-20}{4x-20} = \frac{5(x-5)+5}{4(x-5)} > \frac{5}{4}$ (some positive quantity)

$\therefore$ the ratio must exceed $\frac{5}{4}$. Only choice (D) satisfies this condition

Hence, the correct option is (D).

(ii) Ratio of their ages 20 years hence

$$= \frac{5x+20}{4x+20} = \frac{5(x+5)-5}{4(x+5)} < \frac{5}{4}$$

$\therefore$ the ratio must be less than $\frac{5}{4}$

Only Choice (B) satisfies this condition.

Hence, the correct option is (B).

6. $(10a + b)(10c + d) = (10b + a)(10d + c)$

$100ac + 10bc + 10ad + bd = 100bd + 10ad + 10bc + ac$

$ac = bd$

$\frac{a}{b} = \frac{d}{c}$ or $\frac{a}{d} = \frac{b}{c}$

Hence, the correct option is (A).

7. Two numbers are in the ratio 4 : 7

Let the numbers be $4k$ and $7k$

Let the number added to each be L.

Given that, $(4k + L) + (7k + L) = 75$

$\Rightarrow 11k + 2L = 75$ (13)

It is also given that, $\frac{4k + L}{7k + L} = \frac{8}{17}$

$\Rightarrow 12k + 9L = 0$ (14)

$\Rightarrow k = \frac{-9L}{12} = \frac{-3}{4}L$

Substituting the value of k in (13) we get $L = -12$

Alternate method:

Ratio of the resulting numbers = 8 : 17

Sum of the resulting numbers = 75

Hence, the numbers obtained after addition are

$(8/25) \times 75 = 24$ and $(17/25) \times 75 = 51$

i.e., $4k + L = 24$ and $7k + L = 51$

On subtraction,

$7k - 4k = 51 - 24, \Rightarrow 3k = 27, k = 9$ and $L = -12$

Hence, the correct option is (A).

8. The data is tabulated below

	Apples	Oranges	Banana	Total
Number	50	20	20	90
Price	a	r		b

Actual amount spent = $50a + 20r + 20b$

If $r = a$, amount spent = $70a$

Given $50a + 20r + 20b = 70a \Rightarrow r + b = a$

Amount spent = $70a$. Minimum value of a is 2, corresponding to $b = 1, r = 1$.

$\therefore$ Minimum possible amount = ₹140

Hence, the correct option is (C).

Solutions for questions 9 and 10:

9. Quantity of diesel purchased = x lts.

$\therefore$ Cost per liter of diesel = $510/x$

Cost per liter of kerosene = $\frac{510 \times 3}{x \times 5} = 306/x$

Since equal volumes are purchased, amount spent on kerosene = $\frac{306}{x} \times x =$ ₹306

Hence, the correct option is (B).

10. Amount spent on kerosene = x

Amount spent on petrol = $5x$

$\therefore$ Cost involved = $6x$ (15)

Ratio of prices per liter of Petrol, diesel and kerosene is 15 : 5 : 3

When equal volumes of all are purchased, the amounts are also in the ratio 15 : 5 : 3.

Amount spent on diesel, in this case = 510

Total amount spent = $(23 \times 510)5 = 23 \times 102$ (16)

From (15) and (16), $6x = 2346$

$x = 391$

$\therefore$ Amount spent on petrol = $391 \times 5 =$ ₹1955

Hence, the correct option is (C).

Solutions for questions 11 to 25:

11. $\frac{a+c}{b+d} = \frac{c+e}{d+f} = \frac{a+e}{b+f} = k$

Using the relationship each ratio =

$$k = \frac{\text{sum of numerators}}{\text{sum of denominator}}$$

$$k = \frac{(a+c)+(c+e)+(a+e)}{(b+d)+(d+f)+(b+f)} = \frac{2(a+c+e)}{2(b+d+f)} = \frac{a+c+e}{b+d+f}$$

Now $\frac{a+c}{b+d} = \frac{a+c+e}{b+d+f} = k = \frac{(a+c+e)-(a+c)}{(b+d+f)-(b+d)} = \frac{e}{f} = k$

Similarly $\frac{c+e}{d+f} = \frac{a+c+e}{b+d+f} = k$

$k = \frac{a+c+e)-(c+e)}{(b+d+f)-(d+f)} = \frac{a}{b}$ and lastly,

$k = \frac{(a+c+e)-(a+e)}{(b+d+f)-(b+f)} = \frac{c}{d}$, hence all the choices (A), (B) and (C) are true.

Hence, the correct option is (D).

12. Let $P = ab$ and $Q = cd$ $P_s = a + b$ and $Q_s = c + d$

$$\frac{10a+b}{a+b}=\frac{10c+d}{c+d}$$

$(10a + b)(c + d) = (10c + d)(a + b)$

$10ac + bc + 10ad + bd = 10ac + ad + 10bc + bd$

$ad = bc$ (17)

Let $a = 1$, $b = 0$, $c = 2$ $d = 0$

This would correspond to the minimum value of $P_s + Q_s$, which is 3.

Hence, the correct option is (D).

13. We will find it convenient to change the notation slightly.

Let $A : B = 3 : 4$ and $c : d = 2 : 3$

Let $A = 3x$ $\therefore$ $B = 4x$

Let $c = 2y$ $\therefore$ $d = 3y$

To find $E = \dfrac{A^3c^2 + B^3d^2}{AB^2d^2 + A^2Bcd}$

As each term has 3 upper case letters and 2 lower case, E is homogeneous and we can evaluate it.

$$E=\frac{3^3(2^2)+4^3(32)}{3(4^2)(3^2)+3^2(4)(2)(3)}\,\frac{x^3y^2}{x^3y^2}$$

$$=\frac{3+4^2}{3(4)+6}=\frac{19}{18}$$

Hence, the correct option is (D).

14. Let the number of coins with Amar, Bhavan and Chetan be A, B and C respectively.

The data is tabulated below

A	B	C	AB	AC	BC	$2(ABC)$
			3	5	4	12

$\therefore$ $A = 6 - 4 = 2$, $B = 6 - 5 = 1$ and $C = 6 - 3 = 3$

$\therefore$ $B : C = 1 : 3$

Hence, the correct option is (D).

15. Data can be tabulated as follows:

		Manoj		Shiva
1.	Income Ratio	3	:	4
2.	Savings Ratio	2	:	3
3.	Spending (Expenditure)	–		–

Manoj's expenditure is 2/3rds of his income. Let Manoj's monthly income be ₹36. (an arbitrary number is chosen in such a way that no fractions are encountered in the calculation)

Hence, Manoj's expenditure is $36 \times (2/3) = 24$ (18)

Hence, Manoj's savings is $36 - 24 = 12$

Ratio of savings of Manoj and Shiva = 2 : 3

Hence, Shiva's savings = $3 \times (12/2) = 18$ (19)

Income Ratio = 3 : 4

As Manoj's income = ₹36, Shiva's shall be

$4 \times (36/3) =$ ₹48 (20)

Shiva's expenditure = Income – Savings

$= 48 - 18 = 30$ (21)

From (18) and (21); Ratio of expenditures of Manoj and Shiva is 24 : 30 = 4 : 5

Hence, the correct option is (B).

16. Let the numbers satisfying the condition be denoted by xyz.

$z = x + y$, $z - y = y - x$, $z = 2y - x = x + y$

$y = 2x$

$z = 3x$

$\therefore$ $x : y : z = 1 : 2 : 3$

$\therefore$ (x, y, z) can be (1, 2, 3), (2, 4, 6) or (3, 6, 9)

Hence, the correct option is (C).

17. Let the number of chocolates distributed to Ajay, Balu, Charan, Eswar and Ganesh be a, b, c, e and g.

$2a = 3c = 4e$ (22)

$5b = 6e = 8g$ (23)

Multiplying (22) by 3 and (23) by 2, we get

$6a = 10b = 9c = 12e = 16g$

Let each of these equal k

$$a+b+c+e+g=\frac{k}{6}+\frac{k}{10}+\frac{k}{9}+\frac{k}{12}+\frac{k}{16}=\frac{377k}{720}$$

This has a minimum value of 377 when $k = 720$

Hence, the correct option is (A).

18. When two ratios like $\frac{m}{n}$ and $\frac{p}{q}$ are equal, each of them is equal to $\frac{m-p}{n-q}$, provided $n \neq q$.

$$\therefore \quad k=\frac{(a^2+c^2)-\left(b^2+c^2\right)}{(a+c)-\left(b+c\right)}=a+b$$

Hence, the correct option is (C).

19. Let the numbers be a and b

$a : 9 = 9 : b \Rightarrow ab$ (24)

$a : b = b : 6561 \Rightarrow b^2 = 6561a$ (25)

Solving (24) and (25)

$a = 1$ and $b = 81$

Hence, the correct option is (D).

20. Let the three-digit numbers satisfying the given conditions be denoted by abc

$$b=\frac{a+c}{2} \qquad (26)$$

$$\frac{ab}{a+b}=\frac{bc}{b+c}$$

$$\frac{10a+b}{a+b}=\frac{10b+c}{b+c}$$

$(10a + b)(b + c) = (10b + c)(a + b)$

$10ab + b^2 + 10ac + bc = 10ab + ac + 10b^2 + bc$

$ac = b^2$

From (26), $ac=\left(\frac{a+c}{2}\right)^2$

$(a - c)^2 = 0$

$\therefore a = c$

From (26), $b = c$

$\therefore a = b = c$

$\therefore$ Nine numbers satisfy these conditions.

Hence, the correct option is (C).

21. P $\alpha \frac{Q}{V}$, other factors being constant where Q and V are mass and volume of the gas.

Let the volumes of three chambers be V_1, V_2, V_3 respectively.

V_1	V_2	V_3

Let pressures in 3 chambers be P_1, P_2 and P_3

$P_1V_1 = P_2V_2 = P_3V_3$ (27)

[Since $PV = KQ$ and as Q is same for all chambers, KQ is constant]

$V_1 : V_2 : V_3 = 1 : 2 : 3$

So, $P_1 = 2P_2 = 3P_3$

$\therefore P_1 P_2 : P_3 = \frac{6}{1} : \frac{6}{2} : \frac{6}{3}$

$P_1 : P_2 : P_3 = 6 : 3 : 2$

Hence, the correct option is (C).

22. $V \alpha r^2h \Rightarrow V = k.r^2h \Rightarrow V_1/(r_1^2h_1)$

$= V_2 / (r_2^2h_2) = k$

Hence, $66/(9 \times 7) = 308/(6r^2)$;

$\Rightarrow r = 7$

Hence, the correct option is (C).

23. Fixed charge = x, unit rate = y, x and y are in Rupees.

$5/4(x + 100y) = x + 200y$

$5x + 500y = 4x + 800y$ (28)

$x + 50y = 700$ (data) (29)

$y = 2$, $x = 600$

Hence, the correct option is (A).

24. Distance = d weight = w

Given $d \alpha (1/w)$

$dw = k$ (constant)

Let weights of 3 pieces be x, $3x$, $2x$

$\therefore$ Sum of distances $= \frac{k}{x}+\frac{k}{3x}+\frac{k}{2x}=\frac{11}{6}\frac{k}{x} = 22$

$k = 12x$

Weight of unbroken stone is

$w = 6x = k/2$ as $d.\ w = k$

$d \times k/2 = k \Rightarrow d = 2\ m$

Hence, the correct option is (D).

25. The quantity of the balance of food after the transfer is such that

(900 − 300) = 600 soldiers, consumed at the rate of 3 kg/day/soldier, for 25 days (30)

If the soldiers were not transferred, 900 soldiers would have consumed it at the rate of 2.5 kg/day/soldier, the same food. (31)

The data can be tabulated as:

Soldiers	Consumption rate	Number of days
600	3.0	25
900	2.5	How many?

Number of soldiers and the number of days for which food lasts are inversely proportional. The number of soldiers increased; hence, number of days decreases. Hence multiplication factor is (600/900).

Consumption rate and number of days are also inversely proportional. Hence, multiplication factor is 3.0/2.5

Applying the above rates of variation, the number of days = 25 × (600/900) × (3.0/2.5) = 20 days

The initial stock was to last for 30 days.

⇒ Soldiers were transferred after 30 − 20 = 10 days

Hence, the correct option is (B).

Chapter 3 Numbers

Exercises

Practice Problems I

Directions for questions 1 to 50: Select the correct alternative from the given choices.

1. If x and y are irrational numbers, then $x + y - xy$ is _____.
(A) a real number (B) a complex number
(C) a rational number (D) an irrational number

2. Which of the following is a prime number?
(A) 851 (B) 589 (C) 429 (D) 307

3. Which of the following pairs of numbers are not twin primes?
(A) 131 and 133
(B) 191 and 193
(C) 157 and 159
(D) More than one of above

4. Which of the following is divisible by 11?
(A) 8787878
(B) 7777777
(C) 1234567
(D) More than one of the above

5. What is the least natural number that should be added to 52341693 so that the sum is a multiple of 8?
(A) 3 (B) 9 (C) 5 (D) 7

6. The product of 7 consecutive natural numbers is always divisible by
(A) 5040 (B) 10080
(C) 3430 (D) 6860

7. How many odd natural numbers have the same parity as their factorials?
(A) 1 (B) 2 (C) 0 (D) 3

8. N is a perfect number. What is the ratio of the sum of the factors of N and N?
(A) 1 (B) 2 (C) 3 (D) 4

9. $0.\overline{2\,55} =$
(A) $\frac{23}{90}$ (B) $\frac{23}{99}$
(C) $\frac{253}{990}$ (D) $\frac{253}{900}$

10. $0.3\overline{21} =$
(A) $\frac{53}{165}$ (B) $\frac{106}{333}$
(C) $\frac{10}{11}$ (D) None of these

11. $0.32\bar{1} =$
(A) $\frac{289}{900}$ (B) $\frac{289}{990}$
(C) $\frac{32}{99}$ (D) $\frac{16}{45}$

12. The least natural number that must be added to 599 so that the sum is a perfect cube is
(A) 120 (B) 125 (C) 130 (D) 135

13. There are 15 consecutive odd numbers. The sum of the first ten of those odd numbers is 200. What is the sum of the last five odd numbers?
(A) 125 (B) 175 (C) 150 (D) 200

14. Find the number of prime factors of 19019.
(A) 1 (B) 2 (C) 3 (D) 4

15. If $N = 2^a \times 3^b \times 5^c$, how many numbers (in terms of N) are less than N and are co-prime to it?
(A) $\frac{2}{15}N$ (B) $\frac{4}{15}N$
(C) $\frac{8}{15}N$ (D) $\frac{2}{5}N$

16. Which of the following numbers is divisible by 40 and 72?
(A) 7560 (B) 3840 (C) 5670 (D) 3780

17. What is the least whole number that should be added to 723111 to make the resultant is a multiple of 11?
(A) 4 (B) 8 (C) 7 (D) 3

18. (a) Prime factorize: 9000
(A) $2^2 \times 3^2 \times 5^2$ (B) $2^4 \times 3 \times 5^2$
(C) $2^3 \times 3^2 \times 5^3$ (D) $2^3 \times 3 \times 5^4$

(b) Prime factorize: 1936
(A) $2^2 \times 3 \times 11^3$ (B) $2^3 \times 11^3$
(C) $2^4 \times 11^2$ (D) $2^2 \times 3^2 \times 11^2$

(c) Write 3969 as a product of prime factors.
(A) $3^5 \times 7$ (B) $3^3 \times 7^3$
(C) $3^4 \times 7^2$ (D) $3^2 \times 7^4$

(d) Write 14553 as a product of prime numbers
(A) $3 \times 7^3 \times 11$ (B) $3^2 \times 7 \times 11^3$
(C) $3^3 \times 7^2 \times 11$ (D) $3 \times 7^2 \times 11^2$

19. Simplify the following:

(a) $248 \times 555 + 148 \times 445$
(A) 203500 (B) 302500
(C) 205300 (D) 305200

(b) $4\frac{1}{2} + 3\frac{1}{5} - 2\frac{1}{10} - 4\frac{1}{20}$
(A) $1\frac{1}{10}$ (B) $1\frac{11}{20}$
(C) $1\frac{1}{5}$ (D) $1\frac{11}{40}$

(c) $\dfrac{(3.37)^3 + 10.11(6.63)^2 + 19.89(3.37)^2 + (6.63)^3}{(3.37)^2 + 2 \times (6.63)(3.37) + (6.63)^2}$
(A) 3.26 (B) 6.74 (C) 10 (D) 8

20. Find the square root of 17689
(A) 143 (B) 137 (C) 133 (D) 147

21. The number of positive integers which are co-prime to 349247 is _____.
(A) 4 (B) 5
(C) 3 (D) infinite

22. The sum of the first N natural numbers is equal to x^2 where x is an integer less than 100. What are the values that N can take?
(A) 1, 9, 27 (B) 1, 7, 26
(C) 1, 8, 48 (D) 1, 8, 49

23. What is the unit's place of $(5^n + 4^{2n} + 7^{4n})^{4n}$?
(A) 4 (B) 8 (C) 2 (D) 6

24. What is the highest power of 5 in 240!?
(A) 58 (B) 17 (C) 116 (D) 39

25. The least possible number which when successively divided by 10, 7, and 6 leaves remainders of 8, 4, and 5 respectively is
(A) 256 (B) 148 (C) 398 (D) 198

26. The LCM and HCF of a pair of numbers is 1232 and 14, respectively. How many such pairs are possible?
(A) 3 (B) 2
(C) 1 (D) None

27. Find the square root of 12345654321.
(A) 1111 (B) 11111
(C) 111111 (D) 1111111

28. There are four prime numbers written in ascending order. The product of the first three prime numbers is 2431 and that of the last three is 4199. Find the greatest of them.
(A) 17 (B) 19 (C) 23 (D) 13

29. Find the minimum number of coins required to pay three persons 69 paise, 105 paise, and 85 paise, respectively, using coins in the denominations of 2 paise, 5 paise, 10 paise, 25 paise, and 50 paise.
(A) 9 (B) 10 (C) 14 (D) 11

30. If a, b, and c are prime numbers satisfying $a = b - 2 = c - 4$. How many possible combinations exist for a, b, and c?
(A) 4 (B) 3 (C) 2 (D) 1

31. Let p, q, and r be distinct positive integers that are odd. Which of the following statements cannot always be true?
(A) $pq^2 r^3$ is odd.
(B) $(p + q)^2 r^3$ is even
(C) $(p - q + r)^2 (q + r)$ is even.
(D) If p, q, and r are consecutive odd integers, the remainder of their product when divided by 4 is 3.

32. If *abcde* is a five-digit number the difference of *abcde* and *acdbe* would always be divisible by which of the following for all values of a, b, c, d and e?
(A) 9
(B) 18
(C) 99
(D) Both (A) and (B)

33. Find the value of the expression below

$$\frac{(0.68)^3 + (0.67)^3 - (0.5)^3 + (0.68)(0.67)(1.5)}{(0.68)^2 + (0.67)^2 + (0.5)^2 - (0.68)(0.67) + (0.67)(0.5) + (0.68)(0.5)}$$

(A) 1.85 (B) 0.51 (C) 0.49 (D) 0.85

34. Find the sum of all possible distinct remainders which are obtained when squares of a prime numbers are divided by 6.
(A) 7 (B) 8 (C) 9 (D) 10

35. The least number, which when successively divided by 2, 3, and 7 leaves respective remainders of 1, 2, and 3, is
(A) 56 (B) 130
(C) 68 (D) 23

36. Find the GCD of the numbers p and q where $p = 2^3 . 3^2 . 7^2 . 11^6$ and $q = 2^2 . 3^1 . 5^4 . 11^2 . 13^2$.
(A) 776 (B) 1452
(C) 1164 (D) 2028

37. Which of the following sets of numbers are relative primes?
(a) 57,61
(b) 396,455
(c) 693,132
(d) 6561,1024
(e) 384,352
(A) (c), (e) (B) (a), (b), (d)
(C) (a), (c), (d) (D) (b), (e)

38. (a) Find the units digit of 8^{173}.
(A) 2 (B) 4 (C) 8 (D) 6
(b) What is the last digit of $518^{163} + 142^{157}$?
(A) 2 (B) 4 (C) 6 (D) 8
(c) Find the last digit of $1567^{143} \times 1239^{197} \times 2566^{1027}$
(A) 2 (B) 3 (C) 4 (D) 6

39. If n is a positive integer, then $43^{5n} - 21^{5n}$ is always divisible by
(A) 11 (B) 18 (C) 25 (D) 64

40. Find the greatest number which when divides 6850 and 2575 leaving respective remainders of 50 and 25.
(A) 425 (B) 850 (C) 1700 (D) 1275

41. Find the least number which when divided by 12, 18, and 33 leaves a remainder of 5 in each case.
(A) 394 (B) 396 (C) 391 (D) 401

42. Find the smallest number that must be added to 1994 such that a remainder of 28 is left when the number is divided by 38 and 57.
(A) 66 (B) 68 (C) 86 (D) 98

43. Find the greatest number which divides 3300 and 3640 leaving respective remainders of 23 and 24.
(A) 13 (B) 113
(C) 339 (D) 226

44. Find the greatest number which divides 68, 140, and 248 leaving the same remainder in each case.
(A) 36 (B) 18 (C) 72 (D) 108

45. Five bells toll at intervals of 5, 6, 10, 12, and 15 seconds respectively. If they toll together at the same time, after how many seconds will they toll together again, for the first time?
(A) 300 (B) 120
(C) 60 (D) 30

46. If three numbers are in the ratio 3 : 4 : 5, and their LCM is 480, then find the sum of the three numbers.
(A) 96 (B) 72 (C) 84 (D) 108

47. If $(121)_8 = (x)_2$, then $x =$
(A) 101001 (B) 1010011
(C) 1010001 (D) 1011001

48. If $(ACD)_{16} = (x)_{10}$, then $x =$
(A) 2765 (B) 6725 (C) 5672 (D) 7625

49. Find the digit in the unit's place, in the product of $(25)^7 \times (37)^{12} \times (123)^9$.
(A) 1 (B) 5 (C) 3 (D) 9

50. What is the remainder when 3^{86} is divided by 6?
(A) 2 (B) 3
(C) 4 (D) 0

Practice Problems 2

Directions for questions 1 to 50: Select the correct alternative from the given choices.

1. If N = 1223334444 … and is a 100-digit number, find the remainder when N is divided by 16.
(A) 15 (B) 13 (C) 11 (D) 9

2. What should be subtracted from 546789 so that it becomes a multiple of 7?
(A) 5 (B) 6 (C) 7 (D) 8

3. A, B, and C are digits and 64A3B6C is divisible by 360. How many values can (A, B) take?
(A) 8 (B) 10 (C) 9 (D) 5

4. If n is a positive integer greater than 1, $n^5 - n$ is always divisible by
(A) 5 (B) 7 (C) 11 (D) 13

5. The difference of any 40-digit number and its reverse is always divisible by _____.
(A) 9 but not always by 11
(B) 11 but not always by 9
(C) 99
(D) 198

6. An organization has 99 employees, who were all assigned numerical codes from 2 to 100. A certain number of rounds of an emergency exit drill took place. In the first round, all the employees whose codes were divisible by 2, made an exit. In the second round, all the remaining employees whose codes were divisible by 3 made an exit and so on, until all the employees exit. In each round, at least one employee made an exit. How many rounds of the drill took place?
(A) 25 (B) 24 (C) 49 (D) 99

7. The product of seven integers, which are not necessarily distinct, between 5 and 19 both exclusive is 16081065. Find the sum of these integers.
(A) 81 (B) 83 (C) 85 (D) 79

8. The middle digit of a three-digit number is equal to the sum of the other two digits. Find the number of factors of the greatest such odd number.
(A) 10 (B) 12 (C) 8 (D) 14

9. The sum of all the factors of 11111111 is _____.
(A) 11599268 (B) 13549728
(C) 15479848 (D) 12499488

10. If $1 \le R \le 50$, how many values of R are such that $(R-1)!$ is not divisible by R?
(A) 20 (B) 16 (C) 15 (D) 22

11. Let S be the set of positive integers such that the following conditions are satisfied.
I. The elements of S range from 2000 to 2400.
II. Each element of S has only even digits.
How many elements of S are divisible by 6?
(A) 18 (B) 15 (C) 13 (D) 17

12. $\left(1-\frac{1}{4}\right)\left(1-\frac{1}{9}\right)\left(1-\frac{1}{16}\right)\left(1-\frac{1}{25}\right)\ldots\left(1-\frac{1}{900}\right) =$
(A) $\frac{29}{60}$ (B) $\frac{31}{60}$ (C) $\frac{29}{30}$ (D) $\frac{31}{30}$

13. What is the largest three-digit number which when divided by 9 leaves a remainder 6 and when divided by 7 leaves a remainder 5?
(A) 988 (B) 989
(C) 992 (D) 978

14. Find the sum of all the possible distinct remainders which are obtained when cubes of prime numbers are divided by 6.
(A) 9 (B) 11 (C) 13 (D) 15

15. From a book, in which the pages are numbered in the usual way with odd numbers appearing on the right pages and even numbers on the left pages, 31 consecutive leaves were torn off. Which of the following could be the sum of the 62 page numbers on these leaves?
(A) 1955 (B) 2201
(C) 2079 (D) None of these

16. $K(N)$ denotes the number of ways in which N can be expressed as a difference of two perfect squares. Which of the following is maximum?
(A) K(110) (B) K(105)
(C) K(216) (D) K(384)

17. N is a natural number obtained by adding 16 to the product of four consecutive even natural numbers. How many of the following statements are always true?
(1) N is divisible by 32
(2) N is divisible by 16
(3) N is divisible by 64
(4) N is a perfect square
(A) 0 (B) 1 (C) 2 (D) 3

18. In how many ways can 152100 be expressed as a product of two different factors?
(A) 24 (B) 36 (C) 39 (D) 40

19. If $a, b, c,$ and d are natural numbers such that $a^d + b^d = c^d$, which of the following is true?
(A) The minimum of $a, b,$ and c is at least d.
(B) The maximum of $a, b,$ and c is at most d.
(C) d lies between the minimum of $a, b,$ and c and the maximum of $a, b,$ and c.
(D) $d = 1$

20. Let $w, x, y,$ and z be four natural numbers such that their sum is $8m + 10$ where m is a natural number. Given m, which of the following is necessarily true?
(A) The maximum possible value of $w^2 + x^2 + y^2 + z^2$ is $6m^2 + 40m + 26$.
(B) The maximum possible value of $w^2 + x^2 + y^2 + z^2$ is $16m^2 + 40m + 28$.
(C) The minimum possible value of $w^2 + x^2 + y^2 + z^2$ is $16m^2 + 40m + 28$.
(D) The minimum possible value of $w^2 + x^2 + y^2 + z^2$ is $16m^2 + 40m + 26$.

21. N is a natural number greater than 6, find the remainder obtained when $N^7 - N$ is divided by 6.
(A) 0 (B) 1 (C) 2 (D) 3

22. Find the number of ways in which 24700 can be expressed as a product of two co-prime factors.
(A) 4 (B) 16 (C) 8 (D) 32

23. Find the value of $2+\cfrac{1}{2+\cfrac{2}{2+\cfrac{2}{2+\cfrac{2}{2+\cfrac{1}{2}}}}}$
(A) 103/52 (B) 113/52
(C) 123/52 (D) 133/52

24. When the square of a number and the cube of a smaller number are added the result is 593. If the square of the smaller number exceeds the bigger number by 55, find the difference of the two numbers.
(A) 1 (B) 2 (C) 3 (D) 4

25. Ramu was given a problem of adding a certain number of consecutive natural numbers starting from 1. By mistake, he added a number twice. He obtained the sum as 860. Find the number he added twice.
(A) 15 (B) 40 (C) 25 (D) 30

26. How many integers when squared would exceed a perfect square by 113?
(A) 5 (B) 4 (C) 3 (D) 2

27. If x, y are positive integers and $x^2 - y^2 = 255$, how many ordered pairs $(x - y, x + y)$ are there?
(A) 4 (B) 8 (C) 6 (D) 12

28. If $N = 2^4 \times 3^2 \times 7^3 \times k$ is a perfect square as well as perfect cube, find the total number of factors of the least value of k, given k is a natural number.
(A) 40 (B) 120 (C) 80 (D) 60

29. If the least positive integer divisible by $2^2 \cdot 3 \cdot 5$, $3 \cdot 5^2 \cdot 7$ and $5 \cdot 7 \cdot 11^2$ has x distinct prime factors, then find x.
(A) 3 (B) 5 (C) 6 (D) 7

30. Find the smallest four-digit number which when divided by 9 leaves the reminder 5 and when divided by 11 leaves the remainder 7.
(A) 1058 (B) 1041 (C) 1089 (D) 1085

31. Find the largest four-digit number which when divided by 7, 9, and 11 leaves a remainder of 5 in each case.
(A) 9236 (B) 9467 (C) 9707 (D) 9763

32. N_1 and N_2 are natural numbers not more than 100. $A \oplus B$ is defined as the remainder of A divided by B. $A \# B$ is defined as the product of A and B.
$(N_1 \oplus 8) \# (N_2 \oplus 7) = 21$. How many possible values does (N_1, N_2) have?
(A) 144 (B) 180 (C) 168 (D) 192

33. There are a certain number of soldiers in a field. If the soldiers are arranged in rows of 8 or 15 or 20, one soldier is left out. If the soldiers are arranged in rows of 9 or 13, four soldiers only are left out. Find the number of soldiers in the field.
(A) 109 (B) 113 (C) 117 (D) 121

34. Rohan has a certain number of (less than 10000) sweets with him. He would be left with 1 sweet, if he distributes them equally among 12 or 16 or 18 children. If he distributes them equally among 17 children, he would be left with no sweets. Find the number of possibilities for the sweets he has.
(A) 2 (B) 3 (C) 4 (D) 5

35. Four blocks of chocolates of weights $6\frac{1}{8}$ kg, $10\frac{1}{2}$ kg, $8\frac{3}{4}$ kg, and $3\frac{15}{16}$ kg, respectively, were bought for a birthday party. The blocks were divided into pieces such that all the pieces are of the same weight. What is the least number of pieces that can be obtained?
(A) 61 (B) 63 (C) 66 (D) 67

36. The LCM of two numbers is 196 and the HCF is 7. If the difference of the two numbers is 21, find the larger of the two numbers.
(A) 28 (B) 35 (C) 42 (D) 49

37. Find the units digit of the sum of the factorials of the first 100 natural numbers.
(A) 1 (B) 5 (C) 3 (D) 7

38. What is the remainder of $(25^3 + 27^3 + 29^3 + 31^3)$ divided by 112?
(A) 84 (B) 56 (C) 28 (D) 0

39. Find the remainder when 2^{168} is divided by 105.
(A) 7 (B) 15 (C) 3 (D) 1

40. $N = 10^{51} - 750$
Consider the following statements.
I. The remainder of N when divided by 11 is 8.
II. The remainder of N when divided by 7 is 5.
Which of the following can be concluded?
(A) Only I is true
(B) Only II is true
(C) Both I and II are true
(D) Neither I nor II is true

41. Find the remainder when the 300-digit number 112222333333 … is divided by 8.
(A) 1 (B) 3 (C) 5 (D) 6

42. $N = 161^3 - 77^3 - 84^3$, which of the following statements is not true?
(A) N is divisible by 4 and 23.
(B) N is divisible by 23 and 11.
(C) N is divisible by 4 and 7.
(D) N is divisible by 8 and 11.

43. What is the remainder when $98^{100} + 100^{100}$ is divided by 99?
(A) 2 (B) 0
(C) 1 (D) 98

44. What is the remainder when 2^{123} is divided by 61?
(A) 1 (B) 2
(C) 8 (D) 60

45. A three-digit number N leaves the same remainder upon dividing 68488 and 67516. How many possible values does N have?
(A) 8 (B) 6
(C) 5 (D) 4

46. If $a^3 + b^3 + c^3 = 3abc$, then
(A) $a + b + c = 0$
(B) $a = b = c$
(C) Both (A) and (B)
(D) At least one of (A) and (B)

47. $(111)_2 + (222)_3 + (333)_4 + (444)_5 + (555)_6 + (666)_7 =$
(A) $(999)_{10}$ (B) $(777)_8$
(C) $(777)_{10}$ (D) $(888)_9$

48. The square root of the hexa-decimal number 310 is
(A) $(3C)_{16}$ (B) $(2C)_{16}$
(C) $(1C)_{16}$ (D) $(C1)_{16}$

49. The numbers $(11)_7$, $(55)_7$, and $(404)_7$ are in
(A) arithmetic progression
(B) geometric progression
(C) harmonic progression
(D) arithmetic geometric progression

50. If the geometric mean of the numbers $(24)_6$ and $(34)_7$ is $(24)_n$, then $n =$
(A) 6 (B) 7
(C) 10 (D) 8

HINTS/SOLUTIONS

Practice Problems I

1. If $x = \sqrt{2}$ and $y = \sqrt{3}$, $x + y - xy = \sqrt{2} + \sqrt{3} - \sqrt{6}$

In this case, $x + y - xy$ is irrational.

If $x = \sqrt{2}$ and $y = -\sqrt{2}$,

$x + y - xy = \sqrt{2} + (-\sqrt{2}) - (\sqrt{2})(-\sqrt{2}) = 2$

In this case, $x + y - xy$ is rational.

$\therefore$ We can only conclude that $x + y - xy$ is real

($\because$ Any real number is one which is either rational or irrational)

Hence, the correct option is (A).

2. Choice (A)

$851 = 30^2 - 7^2 = (23)(37)$

$\therefore$ Choice (A) is not prime

Choice (B)

$589 = 25^2 - 6^2 = (19)(31)$

$\therefore$ Choice (B) is not prime.

Choice (C) is divisible by 3.

Choice (D) is prime.

Hence, the correct option is (D).

3. Twin primes are prime numbers, which differ by 2.

In Choice (A), 133 is divisible by 7 and hence it is not *a* prime

In Choice (B), the numbers are twin primes.

In Choice (C), 159 is divisible by 3 and hence it is not prime.

Hence, the correct option is (D).

4. Choice (A)

Sum of the digits in the odd places= 32

Sum of the digits in the even places = 21

(Sum of the digits in the odd places) – (Sum of the digits in the even places) is divisible by 11.

$\therefore$ Choice (A) is divisible by 11.

Choices (B) and (C) are not divisible by 11.

Hence, the correct option is (A).

5. The number formed by the last three digits of *a* number must be divisible by 8 for the number to be divisible by 8. The least natural number which should be added to the number formed by the last 3 digits of the given number to make it divisible by 8 is 3.

Hence, the correct option is (A).

6. The product of any N consecutive natural numbers is divisible by $N!$, any for all values of N.

$\therefore$ When $N = 7$, any such product is divisible by $7! = 5040$.

Hence, the correct option is (A).

7. If the odd natural number is more than or equal to 3 its factorial's parity would be even

$1! = 1$. $\therefore$ 1! is the only odd number satisfying the given condition.

Hence, the correct option is (A).

8. For any perfect number, the sum of its factors is twice the number.

Hence, the correct option is (B).

9. Let $x = 0.2\overline{55} = 0.2\bar{5}$

$10x = 2.\bar{5}$ (7)

$100x = 25.\bar{5}$ (8)

Subtracting (7) from (8)

$x = \frac{23}{90}$

Hence, the correct option is (A).

10. Let $x = 0.3\overline{21}$

$10x = 3.\overline{21}$ (9)

$1000x = 321.\overline{21}$ (10)

Subtracting (9) from (10), $x = \frac{318}{990} = \frac{53}{165}$

Hence, the correct option is (A).

11. Let $x = 0.32\bar{1}$

$100x = 32.\bar{1}$ (11)

$1000x = 321.\bar{1}$ (12)

Subtracting (11) from (12),

$x = \frac{289}{900}$

Hence, the correct option is (A).

12. Least natural number = (Least perfect cube greater than 599) – 599 = 729 – 599 = 130.

Hence, the correct option is (C).

13. Let the least odd number be x.

Given, $x + x + 2 + \ldots + x + 18 = 200 \Rightarrow x = 11$

Hence the sum of the last five terms $\Rightarrow 5x + 120$ i.e., 175.

Hence, the correct option is (B).

14. $19019 = 19(1001) = (19)(13)(11)(7)$

$\therefore$ 19019 has 4 prime factors.

Hence, the correct option is (D).

15. The number of numbers less than N and coprime to it

$= N\left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{5}\right) = \frac{4}{15}N$

Hence, the correct option is (B).

16. $40 = 5 \times 8$; $72 = 8 \times 9$ its enough to check out the divisibility of the number by 5, 8 and 9

$\therefore$ the number divisible is 7560.

Hence, the correct option is (A).

17. 723111

units sum = 4

tens sum = 11

difference = 7

units sum < tens sum, add difference i.e., 7

Hence, the correct option is (C).

18. (a) $9000 = 9 \times 1000$

$= 3^2 \times 10^3 = 2^3 \times 3^2 \times 5^3$

Hence, the correct option is (C)

(b) $1936 = 11 \times 176 = 11 \times 11 \times 16$

$= 11^2 \times 2^4$

Hence, the correct option is (C)

(c) $3969 = 9 \times 441$

$= 3^2 \times 3^2 \times 7^2 = 3^4 \times 7^2$

Hence, the correct option is (C)

(d) $14553 = 11 \times 1323$

$= 11 \times 3 \times 21^2 = 11 \times 3^3 \times 7^2$

Hence, the correct option is (C).

19. (a) $248 \times 555 + 148 \times 445$

$= (100 + 148)\ 555 + 148 \times 445$

$= 100 \times 555 + 148\ (555 + 445)$

$= 203500$

Hence, the correct option is (A)

(b) $4\ 1/2 + 3\ 1/5 - 2\ 1/10 - 4\ 1/20 = 1^{11}/_{20}$

Hence, the correct option is (B)

(c) $$\frac{(3.37)^3 + 3 \times 3.37(6.63)^2 + 3 \times 6.63(3.37)^2 + (6.63)^3}{(3.37)^2 + 2 \times 3.37 \times 6.63 + (6.63)^2}$$

$$= \frac{(3.37 + 6.63)^3}{(3.37 + 6.63)^2} = 3.37 + 6.63 = 10$$

Hence, the correct option is (C).

20.

1	1 76 89	133
	1	

23	76	
	69	

263	789	
	789	

	0	

$\therefore \sqrt{17689} = 133$.

Hence, the correct option is (C).

21. 349247 is odd,

$\therefore$ All the powers of 2 are co-prime to it. There are an infinite number of powers of 2.

An infinite number of positive integers are co-prime to it.

Hence, the correct option is (D).

22. Sum of the first N natural numbers $= \frac{N(N+1)}{2}$

$= \frac{N(N+1)}{2} = x^2 \Rightarrow \frac{N(N+1)}{2}$ is a perfect square.

Let us go by the choices.

Choice (A): When $N = 1$, $\frac{N(N+1)}{2} = 1$ which is a perfect square

When $N = 9$, $\frac{N(N+1)}{2} = 45$ which is not a perfect square.

$\therefore$ Choice (A) is ruled out.

Choice (B): When $N = 1$, $\frac{N(N+1)}{2}$ is a perfect square.

When $N = 7$, $\frac{N(N+1)}{2} = 28$ which is not a perfect square.

$\therefore$ Choice (B) is ruled out.

Choice (C): When $N = 1$, $\frac{N(N+1)}{2}$ which is a perfect square.

When $N = 8$, $\frac{N(N+1)}{2} = \frac{8(9)}{2} = 36$ which is a perfect square

When $N = 48$, $\frac{N(N+1)}{2} = (24)\ (49)$ which is not a perfect square

$\therefore$ Choice (C) is ruled out.

Choice (D): When $N = 1$ or 8, $\frac{N(N+1)}{2}$ is a perfect square. (proved above)

When $N = 49$, $\frac{N(N+1)}{2} = (49), (25)$ which is a perfect square

Choice (D) follows.

Hence, the correct option is (D).

23. For any value of n, unit's place of 5^n is 5.

As $2n$ is always even, unit's digit of 4^{2n} is 6.

As $4n$ is a multiple of 4, 7^{4n} ends with the digit 1.

$5^n + 4^{2n} + 7^{4n}$ ends with (5 + 6 + 1) i.e., 12

Hence 2^{4n} ends with 6.

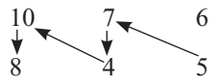
Hence, the correct option is (D).

24. Highest power of 5 in 240! is given by

$$\frac{240}{5}+\frac{240}{5^2}+\frac{240}{5^3} \text{ i.e., } 48 + 9 + 1 = 58$$

Hence, the correct option is (A).

25.



i.e, $[(5 \times 7) + 4]\ 10 + 8 = 398$ is the least number satisfying the given condition.

Hence, the correct option is (C).

26. As the HCF is 14, the numbers are of the form $14a$, $14b$, where a, b are co-primes.

We know that the product of two numbers will be always product of their LCM and HCF.

$14a \times 14b = 1232 \times 14 \Rightarrow ab = 88$

The possible pairs of a and b are: (1, 88), (2, 44), (4, 22) and (8, 11)

But the pairs which are co primes are (1, 88) and (8, 11).

Hence two pairs are possible.

Hence, the correct option is (B).

27. For number of this type (i.e., 1234 . . . n^{n-1} … 1), the number of digits in the square root of the number will be equal to the middle digit of the number.

Hence, the correct option is (C).

28. Let the four prime numbers be a, b, c and d.

Given $a \times b \times c = 2431$ and $b \times c \times d = 4199$

$$\therefore \frac{a\times b\times c}{b\times c\times d} = \frac{2431}{4199} \Rightarrow \frac{a}{d} = \frac{11}{19}$$

$\therefore d = 19$

Hence, the correct option is (B).

29. A minimum of 5 coins are required to pay 69 paise. (1 50 p, 1 10 p, 1 5 p and 2 2 p).

A minimum 3 coins are required to pay ₹1.05 (2 50 p and 1 5 p)

A minimum of 3 coins are required to pay 85p (1 50 p, 1 25 p and 1 10 p)

$\therefore$ Minimum number of coins required in total = 11

Hence, the correct option is (D).

30. $b = a + 2$ and $c = a + 4$

If a is even, b and c are also even. As a, b and c are prime, this is not possible.

$\therefore$ a, b and c are odd.

Least possible value of a is 3.

If $a = 3$, $b = 5$ and $c = 7$

If $a > 3$, it must be in the form $6k \pm 1$ where k is a natural number. If a is of the form $6k - 1$, c is not prime.

$(a, b, c) = (3, 5, 7)$ is the only possibility.

Hence, the correct option is (D).

31. Choice (A)

As q and r are odd, q^2 and r^3 are odd.

$\therefore$ pq^2r^3 is odd

$\therefore$ Choice (A) is always true.

Choice (B)

$(p + q)^2$ is even.

$\therefore$ $(p + q)^2 r^3$ is even.

$\therefore$ Choice (B) is always true.

Choice (C)

$q + r$ is even.

$\therefore$ $(p - q + r)^2 (q + r)$ is even.

$\therefore$ Choice (C) is always true.

Choice (D)

If $p = 1$, $q = 3$ and $r = 5$, pqr leaves a remainder of 3 when divided by 4.

If $p = 3$, q = 5 and $r = 7$, pqr leaves a remainder of 1 when divided by 4.

$\therefore$ Choice (D) is not always true.

Hence, the correct option is (D).

32. $abcde = 10000a + 1000b + 100c + 10d + e$

$acdbe = 10000a + 1000c + 100d + 10b + e$

The difference of $abcde$ and $acdbe$

$= (10000a + 1000b + 100c + 10d + e) - (10000a + 1000c + 100d + 10b + e) = (990b - 900c - 90d)$

$18\ (55b - 50c - 5d)$ which is always divisible by 9 and 18

Hence, the correct option is (D).

33. The given expression is of the form

$$= \frac{a^3+b^3-c^3+3abc}{a^2+b^2+c^2-ab+bc+ca}$$

where $a = 0.68$, $b = 0.67$ and $c = 0.5$

$$\frac{a^3+b^3+c^3-3abc}{a^2+b^2+c^2-ab-bc-ca} = a+b-c$$

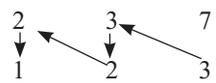
$$= 0.68 + 0.67 - 0.5 = 0.85$$

Hence, the correct option is (D).

34. 2^2 and 3^2 when divided by 6 leave remainders of 4 and 3 respectively. All other primes are of the farm $(6k \pm 1)^2$ when divided by 6 leave a remainder of 1. Hence the sum of the distinct possible remainders is 8.

Hence, the correct option is (B).

35.



The least possible number is $(3 \times 3 + 2) \times 2 + 1 = 23$

Hence, the correct option is (D).

36. $p = 2^3 . 3^2. 7^2. 11^6$

$q = 2^2. 3^1. 5^4. 11^2. 13^2$

The common prime factors of p and q are 2, 3 and 11.

$\therefore$ GCD (p, q) must have only these prime factors.

$\therefore$ GCD $(p, q) = 2^{\min(3, 2)}. 3^{\min(2, 1)}. 11^{\min(6, 2)}$

$= 2^2. 3^1. 11^2 = (4)\ (3)\ (121) = 1452$

Hence, the correct option is (B).

37. Two numbers having no common factor except one are called as relative primes. Among the options the pairs which are relative primes are 57,61; 396,455; and 6561, 1024

Hence, the correct option is (B).

38. (a) The units digit of 8 repeats after every four powers. Expressing 173 in terms of 4, we have $8^{173} = 8^{4 \times 43 + 1}$ and hence the last digit of 8^{173} and 8^1 should be the same. Hence 8^{173} has the units digit of 8.

Hence, the correct option is (C).

(b) Last digit of 518^{163} is the same as the last digit of 8^{163}; $8^{163} = 8^{(4 \times 40) + 3}$. Since the last digit of the power of 8 is 3, 8^{163} will have the same units digit as 8^3 whose last digit is 2, 142 will have same units digit as 2^{157} $2^{157} = 2^{(4 \times 39 + 1)}$ cycle for the last digit of power of 2 is also 4.

Hence 2^{157} will have the same units digit as 2^1 whose units digit is 2. Hence $518^{163} + 142^{157}$ will have the last digit of $2 + 2 = 4$

Hence, the correct option is (B).

(c) 1567^{143} has the same last digit as 7^{143}

$7^{143} = 7^{(4 \times 35) + 3}$

Since the last digit of the power of 7 has a cycle of 4, 7^{143} will have the same last digit as 7^3 i.e., 3.

1239^{197} has the same last digit as 9^{197}.

For 9^{197}, since power of 9 is odd its last digit is 9.

Hence, 1239^{197} has last digit of 9.

2566^{1027} has the same last digit as 6^{1027}, a i.e., 6, since 6 raised to any power will always have a last digit of 6.

Hence, last digit of $1567^{143} \times 1239^{197} \times 2566^{1027}$ will be the last digit of $3 \times 9 \times 6 = 162$

i.e., 2

Hence, the correct option is (A).

39. This is of the form $(43)a - (21)^a$, where $a = 5n$. This is always divisible by $43 - 21 = 22$.

Hence it is also divisible by 11.

Hence, the correct option is (A).

40. Two different dividends and two remainders,

$\therefore$ take HCF of difference of dividends and remainders

$6850 - 50 = 6800$

$2575 - 25 = 2550$

$6800 = 17 \times 2^4 \times 5^2$

$2550 = 17 \times 150 = 17 \times 2 \times 3 \times 5^2$

HCF $= 17 \times 2 \times 5^2 = 850$

$\therefore$ HCF = 850 is the greatest number

Hence, the correct option is (B).

41. It is a case of three divisors and the same remainder

Least number = LCM of (12,18 and 33) + 5

$= 396 + 5 = 401$

Hence, the correct option is (D).

42. LCM of 38 and 57 = 114

Remainder when 1994 is divided by 114 is 56. Number to be added to 1994 to make it a multiple of 114 is 58. in order to leave a remainder of 28, the number to be added is

$58 + 28 = 86$

Hence, the correct option is (C).

43. Take HCF of difference of dividers and remainders.

HCF of $3300 - 23 = 3277$ and $3640 - 24 = 3616$

$3616 = 2^5 \times 113$

Checking divisibility of 3277 by 113

$3277 = 29 \times 113$

HCF = 113

Hence, the correct option is (B).

44. Take difference of two pairs of dividends and find their HCF

$140 - 68 = 72$

$248 - 140 = 108$

HCF of 72, 108

$36 \times 2 = 72$

$36 \times 3 = 108$

HCF = 36.

Hence, the correct option is (A).

45. Time to toll together again = LCM of 5, 6, 10, 12, 15

= 60 seconds.

Hence, the correct option is (C).

46. Let the numbers be $3x$, $4x$ and $5x$.

LCM $(3x, 4x, 5x) = x$ LCM $(3, 4, 5) = 60x$

Given $60x = 480$

$x = 8$

sum $3x + 4x + 5x = 12x = 96$

Hence, the correct option is (A).

47. $(121)_8 = (1 \times 8^2 + 2 \times 8 + 1 \times 8^0)_{10} = (81)_{10}$

2	81
2	40 – 1
2	20 – 0
2	10 – 0
2	5 – 0
2	2 – 1
2	1 – 0

$\therefore (121)_8 = (81)_{10} = (1010001)_2$

Hence, the correct option is (C).

48. $(ACD)_{16} = (A \times 16^2 + C \times 16 + D \times 16^0)$

$= (10 \times 256 + 12 \times 16 + 13 \times 1)_{10}$

$= (2765)_{10}$

Hence, the correct option is (A).

49. $5^1 = 5$

$5^2 = 25$

$5^3 = 125$

We observe that for five, every time the unit's digit is repeated.

Similarly for 7, it is for every 4 times and for 3, it is for every 4 times.

$\therefore 5^7 \times 7^{12} \times 3^9 = 5 \times 7^4 x^3 \times 3^4 x^{2+1}$ which has unit's digit

$= 5 \times 1 \times 3 = 5$

$\therefore$ Unit's digit is five.

Hence, the correct option is (B).

50. When, $3^1 \div 6$, remainder = 3

When, $3^2 \div 6$, remainder = 3

When, $3^3 \div 6$, remainder = 3

We observe that, 3 power any number when divided by 6, leaves a remainder of 3.

$\therefore$ When 3^{86} is divided by 6, the remainder 3.

Hence, the correct option is (B).

Practice Problems 2

Solutions for questions 1 to 50:

1. The number n, the number of times it occur in N and the number of digits it contributes and the total number of digits are tabulated below.

n	No. of occurrences	No. of digits	Total number of digits
1	1	1	1
2	2	2	3
3	3	3	6
–	–	–	–
9	9	9	45
10	10	20	65
11	11	22	87
12	6	12	99
1 (of 12)	Half	1	100

$\therefore$ The last 4 digits of N are 2121

The first 2 comes from the 5^{th} 12

The 12 comes from the 6^{th} 12

The 1 is part of the 7^{th} 12.

$\therefore \text{Rem}\frac{N}{16} = \text{Rem}\frac{2121}{16} = 9$

Hence, the correct option is (D).

2. When 546789 is divided by 7, the remainder is 5. Hence 5 should be subtracted from 546789 so that it becomes a multiple of 7.

Hence, the correct option is (A).

3. The number is divisible by 10.

$\therefore \quad c = 0$

The number is divisible by 8

$\therefore \quad B = 1, 3, 5, 7$ or 9.

The number is divisible by 9. The sum of all the known digits of the number leaves a remainder of 1.

$\therefore A + B = 8$ or 17.

$\therefore (A, B)$ can be (7, 1) (5, 3) (3, 5) (7, 1) or (8, 9).

It can take 5 values.

Hence, the correct option is (D).

4. If $n = 2$, $2^5 - 2 = 32 - 2 = 30$

If $n = 3$, $3^5 - 3 = 243 - 3 = 240$

If $n = 4$, $4^5 - 4 = 1024 - 4 = 1020$

In all the cases $n^5 - n$ is divisible by both 3 and 5.

Only 5 is there in the options.

Hence, the correct option is (A).

General proof: Any value of n where n is a positive integer can be expressed in the form $5k$ or $5k - 1$ or $5k - 2$ or $5k - 3$ or $5k - 4$ where k is an integer.

If $n = 5k$, n is always divisible by 5

If $n = 5k - 1$, $n + 1 = 5k$ is always divisible by 5

If $n = 5k - 2$, $n^2 + 1 = 5k'$ is always divisible by 5

If $n = 5k - 3$, $n^2 + 1 = 5k'$ is always divisible by 5

If $n = 5k - 4$, $n - 1 = 5k - 4 - 1 = 5k - 5 = 5(k - 1)$ and is always divisible by 5.

Hence in general, for any n,

$n(n - 1)(n + 1)(n^2 + 1) = n^5 - n$ is always divisible by 5.

5. Consider an n-digit number.

Consider $n = 3$ and 4

Let $P = abc = 100a + 10b + c$

The reverse $Q = cba = 100c + 10b + a$

$\therefore P - Q = 99(a - c)$

Let $P = abcd = 1000a + 100b + 10c + d$

The reverse $Q = dcba = 1000d + 100c + 10b + a$

$\therefore P - Q = 999(a - d) + 90(b - c)$

For $n = 3$, $P - Q$ is divisible by both 9 and 11

For $n = 4$, $P - Q$ is divisible by 9 but not necessarily by 11. In general when n is odd, the difference is divisible by 9 and 11. When n is even, the difference is divisible by 9 but need not be divisible by 11. In the given problem, N is 40, an even number.

$\therefore$ The difference must be divisible by 9 but not necessarily by 11.

Hence, the correct option is (A).

6. In the first round, employees 2, 4, …, 180 made an exit. In the second round, 3, 9, 15 made an exit.

In the third round, 5, 25, 35, 55, 65, 85, 95 made an exit.

The number of round is equal to the number of primes less than 100, which is 25.

Hence, the correct option is (A).

7. $16081065 = 5\ (3216213) = 5(9)\ 357357$

$= 5(9)(357)(1001)$

$= 5(9)3(7)(17)7(11)(13)$

$= 7(7)(9)15(11)(13)(17)$

This is the only way to express the number as the product of 7 numbers between 5 and 19. The sum of these 7 factors is 79.

Hence, the correct option is (D).

8. Let the three-digit number be abc.

$b = a + c$

The greatest odd number must have the greatest possible value of a and an odd value of c. (Also $b \le 9$)

$\therefore a = 8$, $c = 1$ and $b = 9$

or $abc = 891 = 11^1(3^4)$

Number of factors of 891 is $(1 + 1)\ (4 + 1) = 10$

Note: $b = a + c$ means the number is divisible by 11.

Hence, the correct option is (A).

9. $11111111 = 11(1010101) = 11(101)\ (10001)$

$= 11(101)\ (11025 - 1024) = 11(101)\ (105^2 - 32^2)$

$= 11(101)\ (137)\ (73)$

Sum of all the factors $= \frac{11^2-1}{11-1} \cdot \frac{101^2-1}{101-1} \cdot \frac{137^2-1}{137-1} \cdot \frac{73^2-1}{73-1} = 12\ (102)\ (138)\ (74) = 12499488$

Hence, the correct option is (D).

10. If $(R - 1)!$ is not divisible by R, R is a prime number. Since $1 \le R \le 50$, the objective is to find the number of prime numbers between 1 and 50. On calculating we find there are 15 primes between 1 and 50. Hence there are 15 values of R such that $(R - 1)!$ is not divisible by R. Also for $R = 4$, $(R - 1)!$ i.e., 3! is not divisible by 4. Hence, the total number of values is $(15 + 1) = 16$

Hence, the correct option is (B).

11. The numbers between 2000 and 2400 (both inclusive), which have only even digits and which are multiplies of 3 are listed below. (The sum of the digits has to be a multiple of 6)

2004	2202	2400
2022	2208	
2028	2220	
2040	2226	
2046	2244	
2064	2262	
2082	2268	
2088	2280	
2286		

There are 18 such numbers.

Hence, the correct option is (A).

12. $\left(1-\frac{1}{4}\right)\left(1-\frac{1}{9}\right)\left(1-\frac{1}{16}\right)\left(1-\frac{1}{25}\right)\ldots\ldots\left(1-\frac{1}{900}\right)$

$= \left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\left(1-\frac{1}{4^2}\right)\left(1-\frac{1}{5^2}\right)\ldots\ldots\left(1-\frac{1}{30^2}\right)$

$= \left(1-\frac{1}{2}\right)\left[\left(1+\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\right]\left[\left(1+\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\right]$

$\left(1+\frac{1}{4}\right)\ldots\left(1-\frac{1}{30}\right)\left(1+\frac{1}{30}\right)$

$= \frac{1}{2} \times 1 \times 1 \times \ldots\ldots 1 \times \frac{31}{30} = \frac{31}{60}.$

Hence, the correct option is (B).

13. The number is of the form $9k + 6$

This number when divided by 7 leaves a remainder 5.

So, $9k + 6 - 5$ is divisible by 7

Substituting the smallest value 3 for k the multiple of 7 is obtained.

$\therefore$ The smallest number $= 9k + 6 = 9\ x\ 3 + 6 = 33$

So, the general form of the number satisfying the condition is $33 + 63k$

Dividing 999, the largest 3-digit number that is possible, by 63, the quotient is 15. Hence, $k = 15$ and $63k + 33 = 978$; which is the largest 3-digit number

Hence, the correct option is (D).

14. Prime numbers less than 5 are 2 and 3.

2^3 and 3^3 leave respective remainders of 2 and 3 when divided by 6.

Prime numbers greater than or equal to 5 are of the form $6k \pm 1$ where k is a natural number.

$(6k + 1)^3$ leaves a remainder of 1 when divided by 6.

$(6k - 1)^3$ leaves a remainder of 5 when divided by 6.

$\therefore$ Sum of all the distinct possible remainders is 11.

Hence, the correct option is (B).

15. Let S be the sum of the 62 page numbers. The sum of the first 62 numbers is 31(63) = 1953.(i.e., in this case $S = 1953$). If first leaf is left intact and instead leaf 32 (comprising pages 63, 64) is torn off, S would be 1953 – (1 + 2) + (63 + 64) or 1953 + 124. If the leaves torn off are 3 to 33, S would be 1953 + 2(124). In general, S would be of the form 1953 + 124n, i.e, 1953, 2077, 2201, …. Among the choices, only 2201 is of this form.

Hence, the correct option is (B).

16. Let $N = a^2 - b^2 = (a - b)\ (a + b)$

If $a - b$ and $a + b$ are of opposite parity, a and b will not be natural numbers.

$\therefore$ For a and b to be natural numbers, $a - b$ and $a + b$ must be positive.

$\therefore$ $K(N)$ represents the number of ways of expressing N in the form $(a - b)\ (a + b)$, where $a - b$ and $a + b$ are positive.

The numbers in the 5 choices (N), their prime factors, and $k(N)$ are tabulated below.

N	Prime factors	$K(N)$
110	2(5) (11)	0
105	(3)(5)(7)	4
216	$2^3\ 3^3$	4
384	$2^7\ 3$	6
450	$2^1 3^2 5^2$	0

If there is only one 2 in N, $K(N) = 0$

If there is no. 2, $K(N)$ = number of ways of expressing N as a product of two factors.

If there is more than one 2, (say if these are $2m$ or $2m + 1$) 2's), the 2's can be split in m ways. If there are n of some other prime factors, those factors can be split it $(n + 1)$ ways.

$\therefore$ For $N = 216 = 2^3 3^3$, the 2's can be split as 2, 2^2, i.e., in 1 way. The 3 threes can be split as 0.3; 1.2; 2.1 or 3.0, i.e., in 4 ways.

$\therefore$ $K(N) = 4$.

For $N = 384 = 2^7(3)$, the 2's can be split as 2, 2^6; 2^2, 2^5 or 2^3, 2^4 . For each of these split, the one 3 can go with either part.

$\therefore$ $k(N) = 3\ (N) = 6$. This is the maximum.

Hence, the correct option is (D).

17. Let the even natural numbers be $2k$, $2k + 2$, $2k + 4$ and $2k + 6$.

$$N = 16 + (2k)\ (2k + 2)\ (2k + 4)\ (2k + 6)$$
$$= 16(1 + k(k + 1)\ (k + 2)\ (k + 3))$$
$$= 16(1 + k(k + 3)\ (k + 1)\ (k + 2))$$
$$= 16(1 + (k^2 + 3k)\ (k^2 + 3k + 2)$$
$$= 16(1 + (k^2 + 3k)^2 + 2(k^2 + 3k))$$
$$= 16(k^2 + 3k + 1)^2$$

$k^2 + 3k + 1$ is odd for any positive integral value of k.

$\Rightarrow (k^2 + 3k + 1)^2$ is also odd.

$\therefore$ 16 $(k^2 + 3k + 1)^2$ is a perfect square divisible by 16. Hence only (B) and (D) are true.

Hence, the correct option is (C).

18. Expressing 152100 as product of prime factors, we have $152100 = 2^2 \times 3^2 \times 5^2 \times 13^2$

Number of ways in which 152100 can be expressed as a product of two different factors

$$= 1/2\ [(2 + 1)(2 + 1)(2 + 1)(2 + 1) - 1]$$
$$= 1/2\ [81 - 1] = 1/2\ [80] = 40$$

Hence, the correct option is (D).

19. Suppose, $a = 3$, $b = 4$, $c = 5$ and $d = 2$

Then they satisfy $a^d + b^d = c^d$

In this case, the minimum of a, b and c is at least d.

Hence, the correct option is (A).

20. Given $w + x + y + z = 8\ m + 10$.

In $m = 1$, $x + y + z = 6\ m + 10 = 16$

When the sum of three natural numbers is constant, the sum of their squares is minimum when the numbers are as close as possible.

So the four numbers must be $2m + 2$, $2m + 2$, $2m + 3$ and $2m + 3$.

$\therefore$ the minimum value of $w^2 + x^2 + y^2 + z^2 = (2m + 2)^2 + (2m + 2)^2 + (2m + 3)^2 + (2m + 3)^2 = 16m^2 + 40m + 26$

Hence, the correct option is (D).

21. $N^7 - N = N\ (N^6 - 1) = N((N^3)^2 - 1) = N(N^3 - 1)\ (N^3 + 1)$
$N(N - 1)\ (N^2 + N + 1)\ (N + 1)\ (N^2 - N + 1)$

$$= N(N - 1)\ (N + 1)\ (N^2 + N + 1)\ (N^2 - N + 1)$$

$N - 1$, N and $N + 1$ are consecutive integers. The product of any three consecutive integers is divisible by 6.

$\therefore$ $N^7 - N$ is divisible by 6.

$\therefore$ the remainder is 0.

Hence, the correct option is (A).

22. $24700 = (247)\,(100) = (19)\,(13)\,(12^2)\,(5^2)$

Number of ways in which a number having n prime factors can be expressed as a product a two co-prime factors $= 2^{n-1}$

$\therefore$ required number of ways $= 2^{4-1} = 8$

Hence, the correct option is (C).

23. $$2+\cfrac{1}{2+\cfrac{2}{2+\cfrac{2}{2+\cfrac{2}{2+\cfrac{1}{2}}}}} = 2+\cfrac{1}{2+\cfrac{2}{2+\cfrac{2}{2+\cfrac{2}{\frac{5}{2}}}}}$$

$$= 2+\cfrac{1}{2+\cfrac{2}{2+\cfrac{2}{2+\cfrac{4}{5}}}} = 2+\cfrac{1}{2+\cfrac{2}{2+\cfrac{2}{\frac{14}{5}}}}$$

$$= 2+\cfrac{1}{2+\cfrac{2}{2+\frac{10}{14}}} = 2+\cfrac{1}{2+\cfrac{2}{\frac{28}{14}+\frac{10}{14}}} = 2+\cfrac{1}{2+\cfrac{2}{\frac{38}{14}}}$$

$$= 2+\cfrac{1}{2+\frac{28}{38}} = 2+\cfrac{1}{\frac{76}{38}+\frac{28}{38}} = 2+\cfrac{1}{\frac{104}{38}} = 2+\frac{38}{104}$$

$$= \frac{208}{104}+\frac{38}{104} = \frac{246}{104} = 123/52$$

Hence, the correct option is (C).

24. Let the smaller number be n and the larger number be N.

$N^2 + n^3 = 593$

$\Rightarrow N < 25$ and $N + 55 = n^2$, $\Rightarrow n^2 < 55 + 25 \Rightarrow n < 9$

for $n = 8$, $n^3 = 512$ and $N^2 = 593 - 512 = 81 \Rightarrow N = 9$

$\therefore N - n = 1$

Hence, the correct option is (A).

25. Let the number added twice be x.

The correct sum $= 860 - x$.

This must be in the form $= \dfrac{N(N+1)}{2}$

where N is a natural number.

When $N = 39$, $\dfrac{N(N+1)}{2} = 780$

When $N = 40$, $\dfrac{N(N+1)}{2} = 820$

When $N = 41$, $\dfrac{N(N+1)}{2} = 861$

$860 - x < 860$

$\therefore N \le 40$.

If $N = 40$, $x = 40$.

If $N < 40$, $x > N$. This is not possible.

Hence, the correct option is (B).

26. Let each integer satisfying the given condition be denoted by N.

N^2 exceeds a perfect square by 113.

Let us denote the perfect square by x^2

$N^2 - x^2 = 113$.

$(N - x)\,(N + x) = 113$.

113 is prime.

$\therefore (N - x, N + x) = (-1, -113)$ or $(1, 113)$ or $(113, 1)$ or $(-113, -1)$.

$\therefore (N, x) = (-57, -56)$ or $(57, 56)$ or $(57, -56)$ or $(-57, 56)$

N has two possible values, -57 or 57.

Hence, the correct option is (D).

27. $x^2 - y^2 = 255$

$(x - y)\,(x + y) = 255$.

Both $x - y$ and $x + y$ must be positive.

Also $x - y < x + y$.

$\therefore (x - y, x + y) = (1, 255)$; $(3, 85)$; $(5, 51)$ or $(15, 17)$

Hence, the correct option is (A).

28. $N = (2^4)\,(3^3)\,(7^3)$ k is a perfect square and a perfect cube.

Hence the index of each of its prime factors must be divisible by 6

$\therefore$ Least value of k is $2^2\,(3^4)\,(7^3)$

$\therefore$ Total number of factors of k is $(3)\,(5)\,(4) = 60$

Hence, the correct option is (D).

29. Least positive integer divisible by $2^2.3.5, 3.5^2.7$ and $5.7.11^2$ is their LCM. $= 2^2.\ 3.5^2.\ 7.\ 11^2$.

$\therefore$ Its distinct prime factors are 2, 3, 5, 7 and 11.

Number of distinct prime factor = 5.

Hence, the correct option is (B).

30. LCM of 9 and 11 is 99. When the smallest four-digit number 1000 is divided by 99, we have the remainder as 10. Hence, $1000 - 10 = 990$ is divisible by 99

Thus the smallest four-digit number which is divisible by 9 and 11 is $990 + 99 = 1089$

The smallest four-digit number which when divided by 9 leaves a remainder 5 and when divided by 11 leaves a remainder $7 = 1089 - C$, where C is the common difference between the divisor and the remainder in both the cases. $C = 9 - 5 = 4$ or $C = 11 - 7 = 4$

Hence the smallest four-digit number required

$= 1089 - 4 = 1085$

Hence, the correct option is (D).

31. LCM of 7, 9 and 11 = $7 \times 9 \times 11 = 63 \times 11 = 693$.

Dividing the largest 4-digit number 9999 by 693 we get a remainder of 297. Subtracting 297 form 9999, we have 9999 – 297 = 9702 which is exactly divisible by 693 and hence by 7, 9 and 11. The largest 4-digit number which when divided by 7, 9 and 11 leaves a remainder of 5 in each case = 9702 + 5 = 9707

Hence, the correct option is (C).

32. $(N_1 \oplus 8)\ \#\ (N_2 \oplus 7) = 21$. In other words, the product of the remainders of N_1 divided by 8 and N_2 divided by 7 is 21.

$\therefore$ The only possible remainders when N_1 and N_2 are divided by 8 and 7 are 7 and 3 respectively.

($\because$ Any remainder must be less than the divisor).

N_1 and N_2 are natural numbers not more than 100.

N_1 can be 7, 15, 23, …, 95

N_2 can be 3, 10, 17, …, 94

N_1 has 12 possible values and N_2 has 14 possible values.

$\therefore$ (N_1, N_2) has 168 possible values.

Hence, the correct option is (C).

33. If the soldiers are arranged in rows of 8 or 15 or 20, one soldier is left to stand alone in the last row. Hence if the total number of soldiers is divided by 8 or 15 or 20, the remainder will be 1. Similarly, if the total number of soldiers is divided by 9 or 13, the remainder will be 4. An option satisfying both these conditions is only.

Alternate method:

Number of soldiers on the field = LCM (8, 15, 20)c + 1

= (120c + 1), where c is *a* constant

Number of soldiers on the field = LCM (9, 13)k + 4

= 117k + 4 where k is *a* constant.

Hence 120c + 1

= 117k + 4.

The above equation is satisfied when $k = 1$ for $c = 1$

Thus number of soldiers in the field = 1(120) + 1 = 121

Hence, the correct option is (D).

34. Let the number of sweets with Rohan we N.

$N = k_1$ LCM(12, 16, 18) + 1 = 17 k_2, where k_1 and k_2 are natural numbers.

$144k_1 + 1 = 17k_2$

$$8k_1 + \frac{8k_1 + 1}{17} = k_2$$

The least value of k_2 (17) is realized when $k_1 = 2$.

$\therefore$ The least value of $N = 289$.

N must be of the form 289 + k LCM (144, 17)

= 2448k + 289, where k is a whole number.

$\therefore$ $2448k + 289 < 10000$

$\therefore$ k can be 0, 1, 2 or 3 i.e., it has 4 possible values.

Hence, the correct option is (C).

35. Weight of each piece (in kg)

$$= \text{HCF}\left(6\frac{1}{8}, 10\frac{1}{2}, 8\frac{3}{4}, 3\frac{15}{16}\right)$$

$$= \text{HCF}\left(\frac{49}{8}, \frac{21}{2}, \frac{35}{4}, \frac{63}{16}\right) = \frac{\text{HCF}\ (44, 21, 35, 63)}{\text{LCM}\ (8, 2, 4, 16)} = \frac{7}{16}$$

$$\text{Number of pieces obtained} = \frac{\frac{49}{8} + \frac{21}{2} + \frac{35}{4} + \frac{63}{16}}{\frac{7}{16}} = 67$$

Hence, the correct option is (D).

36. Since the HCF of the two numbers is 7, we have 7x and 7y as the two numbers where x and y are co-primes. $7x - 7y = 7(x - y) = 21$, $x - y = 21/7 = 3$, $x = y + 3$

The LCM of the two numbers is $7xy = 196$

$xy = 196/7 = 28$

$\Rightarrow$ $(y + 3)y = 28$

$\Rightarrow$ $y^2 + 3y = 28$

$y^2 + 3y - 28 = 0$

$\Rightarrow$ $(y + 7)(y - 4) = 0$

Since y can't be negative, $y = 4$

$x = 28/y = 28/4 = 7$

Hence the larger of the two numbers is $7x = 7 \times 7 = 49$

Alternate method:

Going by the options, option (A) says the larger number is 28. The smaller number would then be 28 – 21 = 7. LCM of 28 and 7 is 28. Option (B) says the larger number is 35. Since 196 is not a multiple of 35, option (B) is ruled out. Option (C) says the larger number is 42; smaller number would then be 42 – 21 = 21

LCM of 42 and 21 is 42. Hence not possible.

Option (D) says the larger number is 49. Smaller number would then be 49 – 21 = 28

LCM of 49 and 28 is 196

Hence, the correct option is (D).

37. Units digit of the factorial of any natural number which is 5 or more is 0. Required units digit = units digit of 1! + 2! + 3! + 4! = 1 + 2 + 6 + 4 = 3.

Hence, the correct option is (C).

38. $a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$. Also 25 + 31 = 27 + 29

= 56 and 112 = 56 (2)

$25^3 + 31^3 = 56\ [25^2 + 31^2 - 25 \times (31)] = 56$ (an odd number)

Similarly $27^3 + 29^3 = 56$ (an odd number)

$25^3 + 31^3 + 27^3 + 29^3 = 56$ (an even number)

= A number divisible by 112.

∴ Remainder is 0.

Hence, the correct option is (D).

39. $105 = 3\,(5)\,(7)$

$$\text{Rem}\left(\frac{2^{168}}{3}\right) = \text{Rem}\left(\frac{(3-1)^{168}}{3}\right)$$

$$= \text{Rem}\left(\frac{(-1)^{168}}{3}\right) = 1 \qquad (13)$$

$$\text{Rem}\left(\frac{2^{168}}{5}\right) = \text{Rem}\left(\frac{4^{84}}{5}\right) = \text{Rem}\left(\frac{(5-1)^{84}}{5}\right) = 1 \qquad (14)$$

Remainders of 2^N divided by 7 have *a* cycle of 3.

$$\therefore \text{Rem}\left(\frac{2^{168}}{7}\right) = \text{Rem}\left(\frac{2^{56\times 3}}{3}\right) = \text{Rem}\left(\frac{2^3}{7}\right) = 1 \qquad (15)$$

From (13), (14), (15), $2^{168} - 1$ is divisible by 3, 5, 7 and hence by their L.C.M i.e., 105.

$$\therefore \text{Rem}\left(\frac{2^{168}}{105}\right) = 1$$

Hence, the correct option is (D).

40. $N = 10^{51} - 750$

I. The remainder of 10^n, where n is any odd number, when divided by 11 is always 10.

A more general statement is that, if n is odd, the remainder of A^n divided by $A + 1$ is always A.

$$\text{Rem}\left(\frac{N}{11}\right)$$

$$= \text{Rem}\left(\frac{(11k+10) - 11(68)+2}{11}\right) = 8$$

I is true.

II. $10^{51} = (7 + 3)^{51} = (7 + 3)$ multiplied 51 times.

$(7 + 3)\,(7 + 3) = M(7)$, where $M(7)$ denotes an unspecified multiple of $7 + 3^2$

$(7 + 3)^3 = (7 + 3)\,M(7) + 3^2)$

= A multiple of $M(7) + 3^3$.

It follows in general that $(7 + 3)^N = M(7) + 3^N$

$10^{51} = M(7) + 3^{51}$.

$$\text{Rem}\left(\frac{10^{51}}{7}\right) = \text{Rem}\left(\frac{3^{51}}{7}\right) = \text{Rem}\left(\frac{(3^3)^{17}}{7}\right)$$

$$= \text{Rem}\left(\frac{27^{17}}{7}\right) = \text{Rem}\ \frac{(28-1)^{17}}{7}$$

$$= \text{Rem}\left(\frac{(-1)^{17}}{7}\right) = -1$$

∴ 27^{17} is 1 less than a multiple of 7, (or 6 more than a multiple of 7) while $750 = 7\,(107) + 1$

$$\therefore \text{Rem}\left(\frac{N}{7}\right) = \text{Rem}\left[\frac{(M(7)+6)-(7(107)+1)}{7}\right]$$

$$= \text{Rem}\left(\frac{5}{7}\right) = 5.$$

II is true.

Both I and II are true.

Hence, the correct option is (C).

Note: $10^{51} = 2^{51} \times 5^{51}$. Remainders of 2^N divided by 7 have a cycle of 3.

$$\therefore \text{Rem}\left(\frac{2^{51}}{7}\right) \text{can be found} \qquad (16)$$

$$\text{Rem}\left(\frac{5^{51}}{7}\right) = \text{Rem}\left(\frac{(5^3)^{17}}{7}\right) = \text{Rem}\left(\frac{(18(7)-1)^{17}}{7}\right)$$

$$= (-1)^{17} = -1 \qquad (17)$$

From (16) and (17), $\text{Rem}\left(\frac{N}{7}\right)$ can be found.

41. We can define the following concepts to describe the 300-digit number, say N.

Block	Number of units in the block	No.of digits in the block	Cummulative no of digits
11	2	2	
2222	4	4	6
⋮	⋮	⋮	⋮
9....9	18	18	90
1010...10	20	40	
1111...11	22	44	
1212...12	24	48	
1313...13	26	52	274
14.....14	13	26	300

∴ The last 3 digits of N are 414 and

$$\text{Rem}\ \frac{N}{8} = \text{Rem}\ \frac{414}{8} = 6$$

Hence, the correct option is (D).

42. As $(a + b)^3 - a^3 - b^3 = 3ab(a + b)$,

$N = 161^3 - 77^3 - 84^3 = 3(77)(84)(161)$

$= 3^2(7)^3(11)(4)(23)$

∴ N is divisible by 4, 23, 11, 7 but not by 8. Choice (D) is false.

Hence, the correct option is (D).

43. $$\text{Rem}\ \frac{98^{100}}{99} = \text{Rem}\ \frac{(99-1)^{100}}{99}$$

$$= \text{Rem}\ \frac{99k + (-1)^{100}}{99} = 1$$

Similarly, Rem $\frac{100^{100}}{99}$ = Rem $\frac{1^{100}}{99} = 1$

$\therefore$ Rem $\frac{98^{100}+100^{100}}{99} = 2$

Hence, the correct option is (A).

44. We need Rem $\frac{2^{123}}{61}$. As the index x is close to a multiple of the divisor which is prime, we think of Fermat's Little theorem

Rem $\frac{2^{60}}{61} = 1 \Rightarrow$ Rem $\frac{2^{120}}{61} = 1$

$\therefore$ Rem $\frac{2^{123}}{61}$ = Rem $\frac{2^3}{61} = 8.$

Hence, the correct option is (C).

45. Let the number be N.

$68488 = N.k_1 + R$ and

$67516 = Nk_2 + R$ where k_1 and k_2 are natural numbers and R is the remainder $68488 - 67515 = N(k_1 - k_2)$.

$972 = N(k_1 - k_2)$

N must be *a* factor of 972.

$972 = 1 \times 972 = 2 \times 480 = 3 \times 324$

$= 4 \times 243 = 6 \times 162 = 9 \times 108$

$\therefore$ N has 6 possibilities.

Hence, the correct option is (B).

46. $a^3 + b^3 + c^3 = 3abc$

$a^3 + b^3 + c^3 - 3abc = 0$

$(a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca) = 0$

$(a + b + c) \frac{\left((a-b)^2 + (b-c)^2 + (a-c)^2\right)}{2} = 0$

$a + b + c = 0$ or $a - b = b - c = c - a = 0$

i.e. $a = b = c$ or both.

Hence, the correct option is (D).

47. $(111)_2 + (222)_3 + \ldots + (666)_7$

$= (111)_2 = 2^3 - 1, (222)_3 = 3^3 - 1, \ldots$

$(666)_7 = 7^3 - 1$

$\therefore$ The given sum $= 2^3 - 1 + 3^3 - 1 + 4^3 - 1 + \ldots + 7^3 - 1$

$1^3 + 2^3 + 3^3 + \text{- - - -} 7^3 - 1 - 1 - 1 + \ldots - 1$ for 7 times

$\left(\frac{7(7+1)^2}{2}\right) - 7 = 784 - 7 = 777 = (777)_{10}$

Hence, the correct option is (C).

48. $(310)_{16} = 0 + 1 \times 16 + 3 \times 16^2$

$= 0 + 16 + 768 = 784$

The square root of 784 is 28.

$16 \overline{)\ 28}$

$1 - 12$

$28 = (1C)_{16}$

$\therefore$ The square root of $(310)_{16} = (1C)_{16}$

Hence, the correct option is (C).

49. $(11)_7 = 1 + 7 = 8$

$(55)_7 = 5 + 35 = 40$

$(404)_7 = 4 + 0 \times 7 + 4 \times 7^2 = 200$

$\therefore$ 8, 40 and 200 are in G.P

Hence, the correct option is (B).

50. Given

$(24)_6 = 4 + 2 \times 6 = 16$

$(34)_7 = 4 + 3 \times 7 = 25$

The geometric mean of 16 and 25 is $\sqrt{16 \times 25} = 20$

Given, geometric mean $= (24)_n$

$(20)_{10} = (24)_n$

$20 = 4 + 2n \Rightarrow 2n = 16 \Rightarrow n = 8$

Hence, the correct option is (D).

Chapter 4 Percentage, Profit and Loss

Exercises

Practice Problems I

Directions for questions 1 to 30: Select the correct alternative from the given choices.

1. Ganesh owns $83\frac{1}{3}\%$ of a property. Three fourths of the part of it he owns is worth ₹5 lakhs. Find the value of the property (in ₹ lakhs).
(A) 7.2 (B) 8
(C) 6.4 (D) 8.8

2. The salaries of two persons are equal. If the salary of one of them is increased by 20% and the salary of the other is decreased by 20%, find the percentage change in the total salary of the two persons.
(A) 4% increase B) 4% decrease
(C) 0% (D) None of these

3. The price of a TV is decreased by 20%. By what percent must it be increased to bring it back to the original price?
(A) 25% (B) 20%
(C) $16\frac{2}{3}\%$ (D) 15%

4. The ratio of two numbers is 5/6 : 2/3. By what percentage is the second number more/less than the first number?
(A) 20% less (B) 25% more
(C) 25% less (D) 20% more

5. In a test, Mohan's mark was 25% more than Sohan's mark. Mohan got the minimum mark required to pass the test. The pass mark was 35. Find Sohan's mark.
(A) 21 (B) 26
(C) 27 (D) 28

6. In 2004, the price of a shampoo bottle increases by 10% with respect to that in 2003. By what percentage is its price in 2003 less than that in 2004?
(A) 10% (B) $9\frac{1}{11}\%$
(C) 11% (D) $10\frac{1}{11}\%$

7. Due to inflation the total cost of monthly household items has gone up by 20%, but the salary of the family increased by only 10%. Initially, the family used to spend 20% of the salary on household items. What percentage of the present salary should the family spend to buy the same quantities of household items?
(A) 10% (B) $20\frac{2}{11}\%$
(C) $22\frac{2}{11}\%$ (D) $21\frac{9}{11}\%$

8. If the area of a rectangle is increased by 32% and its breadth is increased by 10%, what is the percentage increase in its perimeter?
(A) 10%
(B) 12%
(C) 20%
(D) Cannot be determined

9. *A*'s salary is 20% less than *B*'s salary. If *C*'s salary is ₹10000 and it is 25% more than *B*'s salary, then what is *A*'s salary?
(A) ₹6000 (B) ₹9600
(C) ₹8000 (D) ₹6400

10. School *A* has 30% more students than school *B*. If 120 more students join school *B*, the two schools will have the same number of students. What is the sum of the number of students in school *A* and school *B* initially?
(A) 600 (B) 400
(C) 800 (D) 920

11. The price of petrol increased by 2% in a certain week and increased by 4% in the next week. Find the net percentage increase in the price of petrol over these two weeks.
(A) 6.12% (B) 6.08%
(C) 6.16% (D) 6.20%

12. Rahul got 150 marks in a test. He scored 25% more marks than the pass mark in it. Rajesh got 165 marks in it. By what percent did his mark exceed the pass mark?
(A) 40% (B) 37.5%
(C) 45% (D) 32.5%

13. In a college of total strength 1000, 30% of the students are girls. There are 600 PGs and 120 more male UGs than female UGs. What percent of the males are the female UGs?
(A) 20% (B) 15%
(C) 25% (D) 10%

14. A machine costs ₹4,00,000. It depreciates by 18% in value in the first year, 16.5% in the second year, 15% in the third year and so on. Find the amount by which it depreciates in the seventh year (in ₹) (Assume all percentages apply to the original cost of the equipment).
(A) 32000 (B) 28000
(C) 36000 (D) 40000

15. The monthly income of Ram increased by 26%. His expenditure which is 70% of his monthly income increased by 20%. His savings must have increased by
(A) 40% (B) 30%
(C) 50% (D) 25%

16. A shopkeeper sells an item for ₹60 at a profit of 20%. At what price (in ₹) should he sell it to gain 30%?
(A) 63 (B) 65 (C) 68 (D) 70

17. The profit made on selling 5 m of a cloth equals the cost price of 2 m of that cloth. Find the profit percentage in selling each m of the cloth.

(A) $66\frac{2}{3}\%$ (B) 50%

(C) 40% (D) $28\frac{4}{7}\%$

18. A company manufactures a product for ₹50. It sold it to a dealer for ₹60. The dealer sold it to a shopkeeper for ₹75. The shopkeeper sold it to a customer for ₹100. Find the profit percentage of the company.

(A) $16\frac{2}{3}\%$ (B) 25%

(C) 20% (D) $33\frac{1}{3}\%$

19. The cost price of 80 articles is ₹12.50 per article. Twenty of them were sold for ₹18 each. At what price should each of the remaining articles be sold so as to get an overall profit of ₹4.50 per article?

(A) ₹15 (B) $₹16\frac{2}{3}$

(C) $₹17\frac{1}{3}$ (D) ₹18

20. Rohit marked his goods 40% above his cost price. He sold it after a discount at 12% profit. Find his discount percentage.

(A) 20% (B) 25% (C) 15% (D) 30%

21. Two successive discounts of 30% and 10% are equal to a single discount of

(A) 33% (B) 35% (C) 37% (D) 36%

22. *P* and *Q* started a business in which *P* invested ₹10000 and *Q* invested ₹20000. They received a profit of ₹9600 at the end of a year. Find *Q*'s share in profit (in ₹).

(A) 8000 (B) 6400 (C) 4800 (D) 3200

23. Ramesh and Suresh started a business. Ramesh invested ₹9000 for ten months and Suresh invested ₹6000 for a year. If the profit at the end of a year was ₹4500, find Suresh's share.

(A) ₹3600 (B) ₹2700 (C) ₹2500 (D) ₹2000

24. Kanchan has bought 50 articles. He sells 20% of the articles and makes a profit of ₹1200, which is also equal to the cost of 5 articles. If the selling price for all 50 articles is the same, what is the value of the remaining articles at the selling price?

(A) ₹14400 (B) ₹9600

(C) ₹18000 (D) ₹15000

25. If the discount and profit percentage are both 20% by what percent is the marked price above the cost price?

(A) 40% (B) 50% (C) 60% (D) 70%

26. A car dealer sold a car at a discount of ₹100000. Even after the discount, he made a profit of 15%. What is the marked price of the car, if the marked price is 25% more than the cost price?

(A) ₹1000000 (B) ₹1115000

(C) ₹2500000 (D) ₹1250000

27. Simon gets a discount of 25% on purchasing 100 VCD's from Samuel. He sells them and makes a profit equal to the undiscounted price of 25 VCD's. What is the gain percentage?

(A) 25% (B) 30%

(C) 66.66% (D) 33.33%

28. Ashwin bought an article at ₹200 and marked it at ₹300. He offered a discount and then sold it his profit/loss percentage and discount percentage are in the ratio 3 : 2. Find his profit/loss percentage.

(A) 29% profit (B) 25% profit

(C) 20% loss (D) 25% loss

29. Feroze marks up an article by 30% and sells it at a discount of 20% to Sohail. Sohail marks up the price of the article to a certain amount which happens to be 20% more than Feroze's cost price. What is the maximum discount Sohail can offer without going into loss?

(A) 30% (B) 20%

(C) $16\frac{2}{3}\%$ (D) $13\frac{1}{3}\%$

30. Gopal, Hari, and Karthik started a business with investments of ₹8000, ₹12000, and ₹16000 respectively. Hari and Karthik left the business after *x* months. Out of the annual profit share, Gopal got more than Hari but less than Karthik. If *x* is an integer, find the ratio of Gopal's, Hari's and Karthik's shares.

(A) 27 : 21 : 28 (B) 24 : 21 : 28

(C) 30 : 27 : 36 (D) 32 : 30 : 40

Practice Problems 2

Directions for questions 1 to 15: Select the correct alternative from the given choices.

1. Ravi's salary before he got an increment was 20% of the total income of his family. His increment was one-fourth of his salary after the increment. What percentage of the total income of his family is his new salary?

(A) $16\frac{2}{3}\%$ (B) 20%

(C) $33\frac{1}{3}\%$ (D) 25%

2. The price of fan *A* is twice that of another fan *B*. The price of *A* is increased by 10% and that of *B* is decreased by 20%. Find the percentage decrease in the sum of the prices of the fans.

(A) 10% (B) 0%

(C) 15% (D) 5%

3. The length of a rectangle is increased by 20% and its breadth is increased by 10%. Which of the following is a possible value of the percentage increase in its perimeter?

(A) 13% (B) 14%

(C) 12% (D) 16%

4. In an election among three contestants P, Q, and R, P gets 120% more votes than Q. P beats R by 3,50,000 votes. Q beats R by 5% of the total votes. Find the total number of votes polled (in lakhs).
(A) 12 (B) 10 (C) 9 (D) 11

5. The publisher of a novel, published the novel in six volumes numbered I, II, III, … VI. The number of pages in each volume is 25% more than that in the previous volume. If the number of pages in volume V of the book is 500 more than the number of pages in volume IV, how many pages were there in volume II of the book?
(A) 480 (B) 960 (C) 640 (D) 1280

6. In 2000, the market shares of the toilet soaps Margo, Palmolive, and Dove were 40%, 30%, and 30%, respectively. Starting from the next year, a new soap enters the market each year and gets 10% of total market share. The existing soaps share the remaining market share in the same ratio as they did in the previous year. What percent of the total market share will Margo have in 2002?
(A) 32% (B) 32.4% (C) 28.8% (D) 34%

7. In 2003, Brijesh paid a tax of 20% of his salary. In 2004, his salary increased by $93\frac{3}{4}\%$ and the tax scheme changed. Under the new tax scheme, he had to pay a fixed sum of ₹1000 and an additional 20% on the amount above ₹10000. His salary in 2004 was more than ₹10,000 and he paid a tax of ₹500 more than what he paid in 2003. What is his salary in 2004?
(A) ₹15500 (B) ₹17000
(C) ₹18200 (D) ₹20000

8. The production of rice in the year 2001 was 1000 tonnes, which was 25% of the total food grain production in that year. In the next year if the production of rice decreased by 4% and production of rice as a percentage of total food grain production increased by 5 percentage points, what is the total food grain production in 2002?
(A) 4020 tonnes (B) 3200 tonnes
(C) 3800 tonnes (D) 3540 tonnes

9. A salesman has to choose between two schemes of remuneration. The first scheme has a fixed salary of ₹3700 and a commission of 2% on sales above ₹50000. The second scheme has no salary but offers commission only. The commission, starting from 3% of sales for the first ₹50000 or part there of, increases at the rate of 1 percentage point for every increase of ₹50000 or part there of sales, upto a maximum of 20% of sales. What is the minimum value of the sales above which he can prefer the second scheme?
(A) ₹140000 (B) ₹90000
(C) ₹40000 (D) ₹240000

10. At the beginning of a year, the owner of a jewel shop raised the prices of all the jewels in his shop by $x\%$ and the lowered then by $x\%$. The price of one jewel after this up–down cycle reduced by ₹100. The owner carried out the same procedure after a month. After this second up–down cycle, the price of that jewel was ₹2304. Find the original price of that jewel (in ₹).
(A) 2600 (B) 2550 (C) 2650 (D) 2500

11. In a public sector unit (PSU), there are 45600 employees. When the PSU offered a voluntary retirement scheme (VRS), 40% of the employees applied for the VRS. After scrutinizing, the PSU has rejected 15% of the applications. But only 9120 employees took the retirement through the scheme. What percentage of the total number of employees did not take retirement even though their applications are not rejected?
(A) 25% (B) 14%
(C) 24% (D) 12.75%

12. One month Mrs. and Mr. Rai take home ₹20000 each. These amount represent an increase of 25% over Mrs. Rai's take home in the previous month and a decrease of 33.33% over Mr. Rai's take home in the previous month, respectively. What was their total income in the previous month?
(A) ₹40000 (B) ₹50000
(C) ₹46000 (D) ₹41666

13. Anand cut a rectangular piece of paper twice. With the first cut, he decreased its area by 40%. With the second cut, he decreased its area by 50%. Its final area was 30 sq cm less than its area after the first cut. Find its original area (in sq cm).
(A) 120 (B) 100 (C) 90 (D) 80

14. A new coach was appointed in the middle of a season for a football team. After he took over, the team won 80% of the 60 matches it played. But the overall performed of the team was only 60%. Find the minimum number of matches the team must have played that season before the new coach took over.
(A) 10 (B) 15 (C) 20 (D) 25

15. A motorist used 10% of his fuel to cover 20% of his total journey. He covered another 40% of his total journey under similar conditions. For the rest of journey, the conditions were different. Find the maximum percentage by which his fuel efficiency (distance covered per unit quantity of fuel) can drop, so that he can still cover the remaining journey without a refill.
(A) $71\frac{3}{7}\%$ (B) $72\frac{6}{7}\%$
(C) $27\frac{1}{7}\%$ (D) $74\frac{2}{7}\%$

Directions for questions 16 and 17: Answer the questions based on the following data:

In an exam, every question correctly answered fetches 2 marks. Every question wrongly answered loses 1 mark. Unanswered questions have no marks associated with them. Ram and Shyam wrote this exam. Ram attempted a certain

number of questions and 30% of them went wrong. Shyam attempted a certain number of questions and 40% of them went wrong. Ram got 40 marks more than the pass mark. Shyam got 25 marks more than the pass mark. The two of them together attempted a total of 100 questions.

16. Find the pass mark in the exam.
(A) 15 (B) 20 (C) 25 (D) 30

17. If there are 80 questions in the exam, find the percentage of marks secured by Ram.
(A) 40.625% (B) 45.75%
(C) 50.75% (D) 34.375%

Directions for questions 18 to 30: Select the correct alternative from the given choices.

18. The cost of production of a motorbike which is sold at 20% profit went up by 40%. What should be the percentage increase in the selling price to maintain the profit percentage the same even at the new cost of production?
(A) 40% (B) 20% (C) 68% (D) 60%

19. A man bought 100 mangoes at a certain price, with the intention of selling each at a profit of 25%. But 20 mangoes got spoilt. If he sold the rest at the intended price, what was his profit or loss percentage?
(A) 0% (B) 6.66% profit
(C) 6.25% loss (D) 12.5% profit

20. A trader sells an article at a profit of ₹25. If the cost price is reduced by ₹25 and consequently the selling price is reduced by 25% he would make a profit of 25%. What is his initial cost price?
(A) ₹100 (B) ₹50 (C) ₹75 (D) ₹40

21. A dealer bought 50 television sets at ₹10000 each. For every set purchased from him, he gave one set free. The loss made by him is equal to the selling price of 15 sets. What is the selling price of each set, that is bought?
(A) ₹10000 (B) ₹15000
(C) ₹12500 (D) ₹20000

22. A car manufacturing company sold a car to a show room at 25% profit. The show room sold it to a customer at the same profit as that at which the company sold it. The customer sold it to another customer at a loss percentage whose value equals that of the profit percentage made by the showroom. The first customer sold it for ₹288000. Find the cost of manufacturing the car (in ₹lakhs).
(A) 1 (B) 1.2 (C) 2.4 (D) 0.72

23. Mr. Singh is paid as per the number of hours he puts in per month. The rate of pay was increased by 20% per hour, but the number of hours put in by him went down by 20%. What is the percentage increase/decrease in his income?
(A) 2% decrease (B) 2% increase
(C) 4% decrease (D) 4% increase

24. In a certain year, the wholesale price index fluctuated as given below:

Period	Percentage of increase or decrease over the preceding period
(a) 1st April to 30th April	Increased by *x*%
(b) 1st May to 31st May	Decreased by *x*%
(c) 1st June to 30th June	Increased by *x*%
(d) 1st July to 31st July	Decreased by *x*%

If the decrease for the period 1st April to 31st May was 160 points and that for the period from 1st June to 31st July was 134.4 points, what was the price index on 1st April? (in points)
(A) 800 (B) 900 (C) 1000 (D) 1200

25. Ravi manufactures watches. Each day he manufactures as many watches as the cost price per watch (in ₹). Each day he sells all his watches at a profit of ₹60 per watch and at the end of the day his profit percentage is 10%. Find his daily profit (in ₹).
(A) 36000 (B) 20000
(C) 40000 (D) 50000

26. Ajay sold an article at 20% profit to Balu. Balu sold it at 30% profit to Chetan. Dinesh sold a similar article at 20% loss to David. David sold it at 30% loss to Edward. The sum of the price that Chetan and Edward paid for their respective articles is ₹28000 more than what Ajay paid. If Ajay and Dinesh bought the article for the same price, Find the sum of the prices paid by Ajay and Dinesh (in rupees) for their respective articles.
(A) 50000 (B) 40000
(C) 45000 (D) 52000

27. Mr. Londa imported 10000 hard disks from Korea at a discount of 10% on the marked price. Out of these, 20% of the hard disks were damaged in transit. If the selling price of an undamaged disk is 20% more than its cost price, what is the profit/loss percentage on total sales?
(A) 6.66% profit (B) 10% profit
(C) 6.66% loss (D) 4% loss

28. *P* and *Q* started a business. They made an annual profit of ₹50000. *Q* being a working partner received 20% of the annual profit as his salary. If the entire profits were divided in the ratio of their investments, *P* would have received ₹8000 more as his profit share than what he actually got. Find *P*'s actual profit share (in ₹).
(A) 33600 (B) 32400
(C) 32000 (D) 31600

29. *A*, *B*, *C*, and *D* started a business with investments in the ratio 3 : 4 : 5 : 6. As *B* and *C* were working partners they were paid equal salaries. The ratio of *B*'s and *C*'s total annual income is 9 : 10. If the total annual profit is ₹84000, find B's salary (in ₹).

(A) 9000 (B) 12000
(C) 15000 (D) 18000

30. Satish, Sanjay, and Sunil started a business with an investment of ₹20000 each. At the start of each month starting from the second, Satish adds ₹1000. At the start of each even month starting from the second, Sanjay adds ₹2000. At the start of the 3rd, 6th, 9th, and 12th months, Sunil adds ₹3000. Who gets the greatest share out of the annual profit?
(A) Satish
(B) Sanjay
(C) Both Satish and Sunil
(D) Sunil

HINTS/SOLUTIONS

Practice Problems I

1. Let the value of the property be ₹x

The value of the part that Ganesh owns (in ₹)

$$= \frac{83\frac{1}{3}}{100} x \times = \frac{5}{6}x$$

Three fourths of this part is worth ₹ $\frac{3}{4}\left(\frac{5}{6}x\right)$

i.e., ₹ $\frac{5}{8}x$

$$\frac{5}{8}x = 5, \quad x = 8$$

Hence, the correct option is (B).

2. As the salaries of the two persons are equal and since the percentage increase in the salary for one person equals the percentage decrease in the salary for the other, the increase in the salary and the decrease in the salary must be equal.

$\therefore$ the total salary of the 2 persons does not change.

Hence, the correct option is (C).

3. Let the original price be ₹x

Decrease in the price = ₹$0 \times 2x$

Price after the decrease = ₹$0 \times 8x$

$$\% \text{ increase} = \frac{0.2x}{0.8x}(100) = 25\%$$

Alternate method:

Any quantity which decreases by $x\%$ must be increased by $\frac{100x}{100-x}\%$ to become its original value

As $x = 20$, $\frac{100x}{100-x} = 25$

Hence, the correct option is (A).

4. Ratio = $\frac{5}{6} : \frac{2}{3}$

Let the first number be $5x$,

Second number is $4x$

$$\% \text{ less} = \frac{5x-4x}{5x} \times 100 = 20\%$$

Hence, the correct option is (A).

5. Let the marks of Mohan and Sohan be m and s respectively.

$$m = s\left(1+\frac{25}{100}\right) = \frac{5}{4}s.$$

m = minimum mark required to pass

i.e. pass mark = 35.

$$\frac{5}{4}s = 35$$

$$s = 28.$$

Hence, the correct option is (D).

6. Let the price in 2003 be 100.

Price in 2004 = 110

With respect to the price in 2004, its price is less by

$$\frac{110-100}{110} \times 100 = 9\frac{1}{11}\%.$$

Hence, the correct option is (B).

7. Let the initial cost of house hold items be x

$$\text{Present cost} = \frac{100+20}{100}x = 1.2x.$$

Let the initial salary of family be y.

$$\text{Present salary of family} = \frac{100+10}{100}y = 1 \times 1y$$

Initially $0 \times 2y = x$

$\Rightarrow 1 \times 2x = 0 \times 24y$

Percentage of present salary being spent on household items $= \frac{0 \cdot 24y}{1 \cdot 1y} \times 100 = \frac{240}{11} = 21\frac{9}{11}\%$.

Hence, the correct option is (D).

8. Let the length of the rectangle be 'l' and its breadth be 'b'

Area$a = lb$

New Area = $A^1 = 1 \times 32A = 1 \times 32lb$

New breadth = $1 \times 1b$

$$\Rightarrow \text{New length} = \frac{1 \cdot 32lb}{1 \cdot 1b} = 1 \times 2l.$$

Perimeter = $2(l + b)$

New perimeter = $2(1 \times 2l + 1 \times lb)$

l and b are unknown

Hence answer is cannot be determined.

Hence, the correct option is (D).

9. Let B's salary be x.

$$A\text{'s salary} = \frac{100-20}{100} \times x = 0 \times 8x$$

C's salary = 10000 and it is also 25% more than B's salary $\Rightarrow \frac{100+25}{100} \times x = 10000.$

$x =$ ₹8000.

A's salary = $0 \times 8x =$ ₹6400

Hence, the correct option is (D).

10. Let the numbers of students in school $B = x$

Number of students in school $A = 1 \times 3x$

Given $1.3\,x - x = 0 \times 3x = 120$

$\Rightarrow x = 400$

Total number of students in both the schools

$= (1 + 1 \times 3)x = (2 \times 3) \times 400 = 920.$

Hence, the correct option is (D).

11. Let the initial price be ₹100

$$\text{Final price} = \left(100 + \frac{2}{100}(100)\right) + \frac{4}{100}\left(100 + \frac{2}{100}(100)\right)$$

$= ₹106.08$

$\therefore$ Percentage increase $= 6.08\%$

Hence, the correct option is (B).

12. Let the pass mark be x.

$$\text{Rahul's mark} = x + \frac{25}{100}x = 150$$

$x = 120$

$$\text{Required percent} = \frac{165.120}{120}(100) = 37.5\%$$

Hence, the correct option is (B).

13. $$\text{Number of girls} = \frac{30}{100}(1000) = 300$$

Number of boys $= 1000 - 300 = 700$

Number of UGs $= 1000 - 600 = 400$

Let the number of female UGs be f. Number of male UGs $= 400 - f$

$\Rightarrow 400 - f - f = 120$

$\Rightarrow f = 140$

$$\text{Required percent} = \frac{140}{700}(100) = 20\%$$

Hence, the correct option is (A).

14. It can be seen that it depreciates by $(18 - 1.5(n - 1))$ in the n^{th} year.

$\therefore$ In the seventh year it depreciates by 9%.

$\therefore$ required amount $= 9\%\ (4{,}00{,}000) = 36{,}000$

Hence, the correct option is (C).

15. Let the monthly income of Ram be ₹100 initially.

Initial expenditure of Ram = ₹70.

Initial Saving of Ram = ₹30

Final monthly income of Ram = ₹126

Final expenditure of Ram = ₹84

Final savings of Ram = ₹42

Percentage increase his savings = 40%

Hence, the correct option is (A).

16. Let the cost price of the item be ₹x

Profit = ₹$0 \times 2x$

$x + 0 \times 2x = 60 \Rightarrow x = 50$

To gain 30% profit must be ₹$0 \times 3x$

$\therefore$ selling price must be ₹$1 \times 3x$ = ₹65

Hence, the correct option is (B).

17. Let the cost price of each in be ₹x

Profit made on selling 5 m = cost price of 2 m = ₹$2x$

Cost price of 5 m = ₹$5x$

$$\text{Profit percentage} = \frac{2}{5}(100)\% = 40\%$$

Hence, the correct option is (C).

18. $$\text{Required percentage} = \frac{60 - 50}{50}(100)\% = 20\%$$

Hence, the correct option is (C).

19. Total cost price (80) $(12 \times 50) = ₹1000$

Total profit = (80) $(4 \times 50) = ₹360$

Total selling price = ₹1360

Total selling price of 20 articles = ₹360

Selling price of each of the remaining articles

$$= \frac{1360 - 360}{60} = ₹16\frac{2}{3}$$

Hence, the correct option is (B).

20. Let his C.P. = ₹100

$$\text{M.P. (in ₹)} = 100\left(1 + \frac{40}{100}\right) = 140$$

$$\text{profit in (₹)} = \frac{12}{100}(100) = 12$$

S.P. = ₹112

discount = ₹28.

discount ($\because$ Discount = M.P. – S.P.)

$$= \frac{28}{140}(100)\ \% = 20\%$$

Hence, the correct option is (A).

21. Let the M.P. be ₹100

$$\text{First discount (in ₹)} = \frac{30}{100}(100) = 30$$

Price after this discount = ₹70

$$\text{Second discount (in ₹)} = \frac{10}{100}(70) = 7.$$

Price after this discount = ₹63

Total discount = ₹37

Equivalent single discount percentage = 37%

Hence, the correct option is (C).

22. Ratio of profits of P and Q = Ratio of the investments of P and $Q = 1 : 2$

Q's profit $= \frac{2}{3}(9600) = ₹6400$

Hence, the correct option is (B).

23. Ratio of profits of Ramesh and Suresh
$= (9000)(10) : (6000)(12) = 5 : 4$

Suresh's share $= \frac{4}{9}(4500) = ₹2000$

Hence, the correct option is (D).

24. Number of articles sold = 20% of 50 = 10
Cost of 5 articles = ₹1200
Cost of 1 article = ₹240
Cost of 10 articles = ₹2400
Selling price of 10 articles = 2400 + 1200 = ₹3600
Selling price of 1 article = ₹360
Number of articles remaining = 40.
Total value of the remaining articles at selling price
$= 40 \times 360 = 14400$.
Hence, the correct option is (A).

25. Let the marked price be ₹x.
Let the cost prices be ₹100
Profit = ₹20
Selling Price = ₹120
Discount = ₹$0.2x$
Given, $x - 0.2x = 120 \Rightarrow x = 150$
$\therefore$ The marked price is 50% alone the cost price.
Hence, the correct option is (B).

26. Let the cost of a car be $100x$.
After a discount of 100000, the profit = 15%
$\Rightarrow$ Selling price of the car = $115x$, and
Marked price of the car = $115x + 100000$
Given that the M.P of the car is 25% more than the C.P of the car $\Rightarrow 125x = 115x + 100000$
$\Rightarrow 10x = 100000,\ x = 10000$
Marked price of the car = $115x + 100000 = 1250000$.
Hence, the correct option is (D).

27. For Samuel
Let's assume the marked price of each VCD is ₹100
For Simon
Cost price of each VCD = $0 \times 75 \times 100 = 75$
Cost price of 100 VCDs = 7500
Now, undiscounted price of 25 VCDs = price marked by Samuel for 25 VCDs = 2500
So, profit on 100 VCDs = 2500
Percentage of profit $= \frac{2500}{7500} \times 100 = 33 \times 33\%$.
Hence, the correct option is (D).

28. Let his profit/loss percentage be $3x$%.
His discount percentage = $2x$%
His selling price $= 200\left(1+\frac{3x}{100}\right) = 300\left(1-\frac{2x}{100}\right)$
If we made a profit, $200\left(1+\frac{3x}{100}\right) = 300\left(1-\frac{2x}{100}\right)$
$x = \frac{25}{3}$
Profit percentage = 25
If he made a loss $200\left(1-\frac{3x}{100}\right) = 300\left(1-\frac{2x}{100}\right)$
$200 - 6x = 300 - 6x$ which is not possible.
$\therefore$ He made 25% profit.
Hence, the correct option is (B).

29. Let the cost price to Feroze be ₹100
Feroze marks up the price to ₹130
He sells it at 20% discount; at ₹104
Sohail marks up the price to 20% more than ₹100, equal to ₹120
The least sale price = 104
To sell it at ₹104, percentage discount to be offered
$= \frac{16}{120} \times 100 = \frac{40}{3} = 13\frac{1}{3}$ %.
Hence, the correct option is (D).

30. Ratio of the profit shares of Gopal, Hari and Karthik
$= (8000)(112) : (12000)(x) : (16000)(x)$
$= 96 : 12x : 16x$
$16x > 96 > 12x$
$\therefore x > 6$ and $x < 8$
As x is an integer, $x = 7$
$\therefore$ Required ratio = 24 : 21 : 28
Hence, the correct option is (B).

Practice Problems 2

Solutions for questions 1 to 15:

1. Let Ravi's salary after the increment be $4x$
$\therefore$ Before the increment it was $3x$ and the total family income was $15x$. After Ravi's increment, the family income is $16x$.
Required percentage
$= \frac{\text{Ravi's new salary}}{\text{Total family income}}(100\%)$
$= \frac{4x}{16x}(100\%) = 25\%$.
Hence, the correct option is (D).

2. Let the price of fan B initially be ₹x

Price of fan A initially = ₹$2x$

Increase in the price of $A = \frac{10}{100}(2x) = ₹0 \cdot 2x$

Decrease in the price of $B = \frac{20}{100}(x) = 0.2x$

As the increase equals the decrease, the sum of the prices does not change.

$\therefore$ required percentage change = 0%

Hence, the correct option is (B).

3. Let the length and the breadth be ℓ and b respectively. $\ell \geq$ b.

If ℓ = b, percentage increase in the perimeter is 15%.

Also if $\ell >$ b, percentage increase is more than 15%.

$\therefore$ Percentage increase can be 16%.

Hence, the correct option is (D).

4. Let the number of votes Q gets be x

Number of votes P gets = $2.2x$

Number of votes R gets = $2.2x - 3{,}50{,}000$

$$x - 2.2x - 3{,}50{,}000 = \frac{5}{100}(x + 2.2x + 2.2x - 3{,}50{,}000)$$

$x = 2{,}50{,}000$

Number of votes polled = $5.4x - 3{,}50{,}000 = 10{,}00{,}000$

Hence, the correct option is (B).

5. The volume number and the corresponding number of pages are tabulated below.

Volume I	II	III	IV	V	VI
Pages $4x$	$4^4(5)x$	$4^3(5^2)x$	$4^2(5^3)x$	$4(5^4)x$	$55x$

We see that the number of pages in V is $500x$ more than in VI $500x = 500 \Rightarrow x = 1$ and the number of pages in II is 1280.

Hence, the correct option is (D).

6. Ratio of market shares of Margo, Palmolive, Dove and the new soap in 2001 = 0.9(40) : 0.9(30) : 0.9(30) : 10

= 36 : 27 : 27 : 10

Market share of Margo in 2002 = $\frac{36}{100}(90) = 32.4\%$

Hence, the correct option is (B).

7. Let the salary in 2003 be x.

The tax paid by him in 2003 is $0 \times 2x$

Salary in 2004 = $x\left(1 + \frac{93 \cdot 75}{100}\right) = 1 \times 9375x$

This salary > 10000

Given tax difference = 500

$(1 \times 9375x - 10000)\ 0 \times 2 + 1000 - 0 \times 2x = 500$

$(1 \times 9375)\ (0 \times 2)x - 1000 - 0 \times 2x = 500$.

$0 \times 1875x = 1500$

$x = 8000$.

Salary in 2004 = $1 \times 9375x$ = ₹15500.

Hence, the correct option is (A).

8. Total food grain production in

$2001 = \frac{100}{25} \times 1000 = 4000$

Production of rice in 2002 = $\left(\frac{100 - 4}{100}\right) \times 1000 = 960$t

Rice production as a percentage of food grain production, in 2002 = 25 + 5 = 30%.

$\therefore$ Total food grain production in 2004

$= \frac{100}{30} \times 960 = 3200$.

Hence, the correct option is (B).

9. The commissions received under the two schemes, for various levels of sales amounts can be tabulated as follows:

	Sales Amount (₹)	Commission under scheme 1 (₹)	Commission under scheme 2 (₹)
1.	First 50000	3700	3% of 50000 = 1500
2.	Second 50000	2% of 50000 = 1000	4% of 50000 = 2000
3.	Fist 100000	3700 + 1000 = 4700	1500 + 2000 = 3500
4.	Third 50000	2% of 50000 = 1000	5% of 50000 = 2500
5.	First 150000	4700 + 1000 = 5700	3500 + 2500 = 6000

The above implies that for some value of sales amount between ₹100000 and ₹150000, the incomes from both the schemes is the same.

Let the commission for a sale amount of ₹ $(100000 + x)$ be equal.

$\Rightarrow$ commission on ₹$(100000 + x)$ under scheme 1

= commission on ₹$(100000 + x)$ under scheme 2

$\Rightarrow 4700 + 2\%$ of $x = 3500 + 5\%$ of x

$\Rightarrow 1200 = 3\%$ of x,

$\Rightarrow 1\%$ of $x = 400$ or $x = 40000$

Hence, $100000 + x = 140000$.

Hence, the correct option is (A).

10. Let the original price of that jewel be ₹P.

$$P\left(1 + \frac{x}{100}\right)\left(1 - \frac{x}{100}\right) = P - 100$$

$$\Rightarrow \left(1+\frac{x}{100}\right)\left(1-\frac{x}{100}\right)=\frac{P-100}{P}$$

$$\Rightarrow (P-100)\left(1+\frac{x}{100}\right)\left(1-\frac{x}{100}\right)=2304$$

$$\Rightarrow (P-100)\left(\frac{P-100}{p}\right)=2304$$

$\Rightarrow P^2 - 2504P + 10000 = 0$

$(P - 2500)\,(P - 4) = 0$

P must be greater than 2304 since each up down cycle reduces the price effectively.

$\therefore P = 2500$

Note: $(P-100)\left(1+\frac{x}{100}\right)\left(1-\frac{x}{100}\right)$

$$(P-100)\left(1+\frac{x}{100}\right)\left(1-\frac{x}{100}\right)-100\left(1+\frac{x}{100}\right)\left(1-\frac{x}{100}\right)$$

$= P - 100 -$ (some number which is less than 100)

= Some number which is more than $P - 200$)

$\therefore 2304 > P - 200$

$2504 > P$

Only Choice (D) satisfies this condition.

Hence, the correct option is (D).

11. From the given information, the percentage of employees who took retirement through the scheme

$$= \frac{9120}{45600}\times 100 = 20\%.$$

Now by taking 100 as the base, we can calculate the answer easily.

Out of 100 employees, 40 applied for VRS.

But PSU allowed only 85% of 40 i.e. 34 employees to take VRS.

$\therefore 34\% - 20\% = 14\%$

i.e., 14% of the total employees did not take retirement, although their applications are not rejected.

Hence, the correct option is (B).

12. Present salary of Mrs. Rai = 20000

This is a gain of 25% from last month's earnings.

$\Rightarrow$ Last month's salary of Mrs. Rai.

$$= \frac{100}{125}\times 20.000 = 16000$$

Mr. Rai's salary has come down to ₹20000. This represents *a* $33 \times 33\%$ drop.

$$\text{Mr. Rai's salary last month} = \frac{100}{66\cdot 66}\times 20000 = 30000$$

Total income of Mr. Rai and Mrs. Rai last month

$= 30000 + 16000 = 46000.$

Hence, the correct option is (C).

13. Let its original area be x sq cm.

$$\text{Area after the first cut} = x - \frac{40}{100}x = 0.6x$$

$$\text{Area after the second cut} = 0.6x - \frac{50}{100}x(0.6x) = 0.3x$$

Given, $0.6x - 0.3x = 30 \Rightarrow x = 100$

Hence, the correct option is (B).

14. Let the minimum number of matches be x. Let us say, of these x matches y matches were won. Number of matches won after the coach took over $= \frac{80}{100}(60) = 48$.

$$\text{Given, } \frac{48+y}{60+x}(100) = 60 \Rightarrow 48 + y = 0.6(60 + x)$$

$\Rightarrow x$ would be minimum when $y = 0$.

In this case, $x = 20$

Hence, the correct option is (C).

15. Let the total distance be 100 km. Let the total fuel be 10 litres. As 10% of the fuel was used to cover 20% of his total journey driving conditions, a litre is required for driving every 20 km (under given driving conditions).

$\therefore$ Total fuel required for conditions which were different = Fuel required for the 60% of the journey

$$= \frac{\frac{60}{100}(100)}{20} = 3 \text{ litres.}$$

$\therefore$ 7 litres is required for driving the balance 40 km.

$$\text{Required percent} = \frac{20-\frac{40}{7}}{20} = 71\frac{3}{7}\%$$

Hence, the correct option is (A).

Solutions for questions 16 and 17:

16. Let the number of questions attempted by Ram be x. Let the pass mark be p.

$$\text{Ram's mark} = 2\left(\frac{70x}{100}\right) - \frac{30}{100}x = 1.1x$$

$$\text{Shyam's mark} = 2\left(\frac{60}{100}\right)$$

$$(100 - x) - \frac{40}{100}(100 - x)$$

$\Rightarrow 0.8(100 - x)\ 1.1x = p + 40$ (3)

$\Rightarrow 0.8(100 - x) = p + 25$ (4)

Solving (3) and (4), $x = 50$ and $p = 15$

Hence, the correct option is (A).

17. Percentage secured by Ram

$= \frac{(1.1)(50)}{80(2)}(100)\% = \frac{1.1(50)}{80(2)}(100)\% = 34.375\%$

Hence, the correct option is (D).

Solutions for questions 18 to 30:

18. Let the initial cost be ₹100.

It went up by 40% i.e., new cost price = ₹140

Initial Selling Price = ₹120.

(as the profit is 20%)

Present Selling Price = $140(1 \times 2)$ = ₹168

Percentage increase in selling price

$= \frac{168-120}{120} \times 100$

$= \frac{48}{1 \cdot 2} = 40\%$

Hence, the correct option is (A).

19. Let the initial selling price of each mango be ₹x.

∴ New selling price = ₹$1.25x$.

Overall selling price now = $1.25x\ (80)$ = ₹$100x$.

∴ As there was 25% overall profit on selling,

Overall cost price = $\frac{(100x)(100)}{125}$ = ₹$80x$.

If the remaining 80 apples are sold at ₹x, then the selling price = ₹$80x$.

∴ Overall C.P. = Overall S.P., hence no profit or no loss.

Alternate method:

Let the cost price of 1 mango = ₹1

Cost Price of 100 mangoes = ₹100 (5)

Profit percentage expected = 25%

∴ Proposed sale price per mango = ₹1.25 (6)

20 mangoes are rotten; 80 mangoes are available for sale.

If these mangoes are sold at the original sale price of ₹1.25 per mango, the money received from the sale of 80 mangoes

= 80 (1.25) = ₹100 (7)

As (5) and (7) are equal, there is neither profit nor loss.

Hence, the correct option is (A).

Note: Information that sale price is increased by 25% is redundant because, no mangoes are sold of this price.

20. Let the initial cost price be ₹x.

Initial S.P. = $x + 25$

Now, the cost price is reduced by ₹25

New C.P. = $(x - 25)$

Selling price is reduced by 25%.

∴ New S.P. = $(x + 25) \times \frac{3}{4}$.

Profit = 25%

$\therefore (x - 25) \times \frac{5}{4} = (x + 25) \times \frac{3}{4}$

$5x - 125 = 3x + 75$

$2x = 200 \Rightarrow x = 100$.

Hence, the correct option is (A).

21. Let the selling price of one television be x.

Total money made = $\frac{1}{2}(50x) = 25x$

(since one TV is given free with each one bought by customer.)

Given total cost = $10{,}000 \times 50 = 500000$

Loss = $15(x) = 15x$

Selling price + Loss = Cost price

$\Rightarrow 25x + 15x = 500000$

$40x = 500000$

$x = 12500$

Selling price of each television = ₹12500.

Hence, the correct option is (C).

22. Let the cost of manufacturing be ₹x.

Profit made by the company = ₹$\frac{x}{4}$.

Profit made by the show room = ₹$\frac{x}{4}$.

Cost price to the show room = ₹$\frac{5x}{4}$.

Profit % to the show room = 20%

Loss% to the first customer = 20%

Cost price of the first customer = ₹$\frac{3x}{2}$

Selling price of the first customer $\frac{3x}{2} \times \frac{4}{5}$ i.e.,

= ₹$\frac{12x}{10} = 28800$. $\therefore x = 240000$

Hence, the correct option is (C).

23. Let Mr. Singh's initial income per hour be ₹100

Present income per hour = ₹120

Let the number of hours per month be 100, initially

Present number of hours = 80

Earlier monthly income = $100 \times 100 = 10000$

Present monthly income = $120 \times 80 = 9600$

Percentage decrease in his income

$= \frac{400}{10000} \times 100 = 4\%$.

Alternate method:

Mr. Singh's income (I) is the product of his hourly rate (R) and the number of hours he works (n).

i.e., $I = rn$

R increases by 20% to $\frac{6R}{5}$ and

n decreases by 20% to $\frac{4n}{5}$.

$\therefore$ The income changes to $= \frac{6R}{5} \times \frac{4n}{5}$

$= \frac{24}{25} I$, i.e., it decreases by 4%.

Hence, the correct option is (C).

24. Let the whole sale price index on 1st April be points. As per data,

$$p - p\left(1+\frac{x}{100}\right)\left(1-\frac{x}{100}\right) = 160$$

$$p - p\left(1-\frac{x^2}{10000}\right) = 160 \quad p\left(\frac{x^2}{10000}\right) = 160 \qquad (8)$$

The second equation:

$$p\left(1-\frac{x^2}{10,000}\right) - p\left(1-\frac{x^2}{10,000}\right)\left(1-\frac{x^2}{10,000}\right)$$

$$= 134 \times 4$$

$$p\left(1-\frac{x^2}{10000}\right).\frac{x^2}{10000} = 134 \times 4 \qquad (9)$$

from (1), $160\left(1-\frac{x^2}{10000}\right) = 134 \times 4$

$1-\frac{x^2}{10000} = 0 \times 84 \quad x = 40\%$

$\therefore$ from (8), $p = 1000$.

Hence, the correct option is (C).

25. Let his cost price be ₹x per watch

Given, $\frac{10}{100}x = 60 \Rightarrow x = 600$

$\Rightarrow$ Profit $= \frac{10}{100}[(600)(600)] = ₹36000$

Hence, the correct option is (A).

26. Let the cost prices for Ajay and Dinesh be ₹a and ₹d respectively.

Cost price of Balu = ₹1.560.

$\therefore$ Ajay's article's value increases by ₹0.56a.

Cost price of David = ₹0.8d.

Selling price of David = ₹0.56d

$0.56a + 0.56d = 28000$

$a + d = 50000$

Hence, the correct option is (A).

27. Let the cost of 10000 hard disks be ₹10000

After a discount of 10%, cost = ₹9000

20% of the hard disks are damaged.

$\Rightarrow$ Number of hard disks to be sold = 8000

Selling price = 20% more than the price at which he bought

Total sales = ₹$(8000 \times 1 \times 2 \times 0 \times 9)$ = ₹8640

Loss percentage $= \frac{9000-8640}{9000} \times 100$

$= \frac{360}{9000} \times 100 = 4\%$.

Alternate method:

Ratio of CP of undamaged disks and damaged disks $= 80\% : 20\% = 4 : 1$

Undamaged disks are sold at a price which is 20% more than the C.P.; $\Rightarrow$ profit percentage on these = (+)20%

Damaged disks are to be taken as unsaleable;

$\Rightarrow$ loss percentage on these = (–)100%

Overall percentage of profit/loss

$= [4 \times (+20) + 1(-100)]/(4 + 1) = (-20/5) = -4$

$\therefore$ 4% loss

Hence, the correct option is (D).

Note:

1. Information about number of disks is redundant.
2. The statement about the discount (10%) received at the time of purchase, is also redundant.

28. Let the investments of P and Q be ₹p and ₹q respectively.

Salary of Q = ₹10000

$$\frac{p}{p+q}(50000) = \frac{p}{p+q}(40000) + 8000$$

$p : q = 4 : 1$

Profit share of $p = \frac{4}{5}\ (40000) = ₹32000$

Hence, the correct option is (C).

29. Let the investments of A, B, C and D be ₹3x, ₹4x, ₹5x and ₹6x respectively. Let the salary of B be ₹S.

Given $\frac{4x+S}{5x+S} = \frac{9}{10}$

$\Rightarrow S = 5x$

Total amount profit = $18x + 2S$

$= 28x = 84000 \Rightarrow x = 3000$

$\therefore$ B's salary is 5x i.e., 15000

Hence, the correct option is (C).

30. The data is tabulated below. All the amounts are in thousands of rupees.

Month	Satish	Sanjay	Sunil
1st	20	20	20
2nd	21	22	20
3rd	22	22	23
4th	23	24	23
5th	24	24	23
6th	25	26	26
7th	26	26	26
8th	27	28	26
9th	28	28	29
10th	29	30	29
11th	30	30	29
12th	31	32	32

$\therefore$ the ratio of their profit shares is

$$\left(\frac{20+31}{2}\right)12:2\left(\frac{22+30}{2}\right)$$

$$(5)+20+32:20+20+32+3(3)(26)$$

$$= 51(6) : 52(6) : 72 + 9(26) = 153 : 156 : 153$$

We see that Sanjay gets the greatest share.

Hence, the correct option is (B).

Chapter 5 Simple Interest and Compound Interest

Exercises

Practice Problems I

Directions for questions 1 to 20: Select the correct alternative from the given choices.

1. Find the amount obtained by investing ₹24,000 at 18% p.a. simple interest for five years
(A) ₹21,600 (B) ₹44,000
(C) ₹45,600 (D) ₹48,000

2. The simple interest for the second year on a certain sum at a certain rate of interest is ₹1000. Find the sum of the interest accrued on it for the 6^{th}, 7^{th}, and 8^{th} years.
(A) ₹3200 (B) ₹3000
(C) ₹3300 (D) ₹3630

3. In how many years will a sum of money become sixteen times itself at 30% p.a. simple interest?
(A) 25 (B) 40 (C) 30 (D) 50

4. A sum of money becomes ten times itself at simple interest. If the time period (in years) is numerically equal to the rate of interest, find the annual rate of interest.
(A) 25% (B) 20% (C) 30% (D) 90%

5. An amount of ₹2400 is due after six years under simple interest at 10% p.a. Find its present value (in ₹).
(A) 2000 (B) 1600 (C) 1800 (D) 1500

6. If ₹3000 amounts to ₹3630 in two years under compound interest, interest being compounded annually, what is the annual rate of interest?
(A) 10% (B) 21% (C) 11% (D) 10.5%

7. ₹5000 is invested for two years under compound interest at 10% p.a., interest being compounded annually. Find the interest earned (in ₹).
(A) 500 (B) 1000 (C) 2100 (D) 1050

8. A sum under compound interest, interest being compounded annually amounts to ₹6000 in two years and ₹7200 in three years. Find the rate of interest.
(A) 10% p.a. (B) 20% p.a.
(C) 18% p.a (D) 15% p.a

9. The compound interest on a sum for the third year is ₹2420, interest being compounded annually. The interest on it for the fourth year is ₹2662. Find the rate of interest.
(A) 10% p.a. (B) 11% p.a.
(C) 12% p.a (C) 13% p.a

10. A sum of money becomes four times itself in eight years at compound interest. In how many years will the same sum become sixteen times itself?
(A) 64 (B) 32 (C) 44 (D) 16

11. A sum becomes 2.197 times of itself in three years at compound interest. Find the rate of interest.
(A) 30% (B) 13% (C) 39.9% (D) 235

12. Find the interest (in ₹) earned in the first year on ₹200 at 20% p.a. compound interest, interest compounded every six months.
(A) 40 (B) 42 (C) 44 (D) 48

13. Find the effective rate of interest if the rate of interest is 40% p.a., and the interest is compounded quarterly?
(A) 42% p.a (B) 40% p.a.
(C) 44% p.a (D) 46.41% p.a.

14. Ashok borrowed a total of ₹84000 from two banks at compound interest, interest being compounded annually. One of the banks charged interest at 10% p.a. while the other charged interest at 20% p.a. If Ashok paid ₹13200 as the total interest after a year, find the difference of the sums he borrowed (in ₹).
(A) 24000 (B) 48000 (C) 54000 (D) 12000

15. If the annual rate of simple interest at which a sum is lent for two years increases by 10 percentage points, the interest realized would be ₹4000 more. Find the sum (in ₹).
(A) 20000 (B) 10000 (C) 8000 (D) 16000

16. If a sum was ₹10000 more it would fetch ₹4000 extra as simple interest, if it was lent at a certain rate of interest for two years. Find the annual rate of interest.
(A) 5% (B) 10% (C) 20% (D) 25%

17. A sum was invested under compound interest, interest being compounded annually. It fetches ₹14400 as interest in the second year and ₹17280 as interest in the third year. Find the annual rate of interest.
(A) 10% (B) 15% (C) 20% (D) 25%

18. A sum takes T_1 years to double at R_1% p.a. simple interest. If it is lent at R_2% p.a. compound interest, interest being compounded annually, it would take the same time to double. Which of the following is always true if $T_1 > 1$?
(A) $R_1 > R_2$ (B) $0.5R_2 < R_1 < R_2$
(C) $R_1 = R_2$ (D) $R_2/3 < R_1 < R_2$

19. A sum takes two years to become 40% more under simple interest at a certain rate of interest. If it was lent at the same interest rate for the same time under compound interest, interest being compounded annually, it would amount to x% more than itself. Find x.
(A) 36 (B) 48 (C) 40 (D) 44

20. A sum was divided into two equal parts. One part was lent at 20% p.a. simple interest. The other part was lent at 20% p.a. compound interest, interest being compounded annually. The difference in the interests fetched by the parts in the second year is ₹400. Find the difference in the interests fetched by the parts in the fourth year (in ₹).
(A) 1414 (B) 1442
(C) 1456 (D) 1484

Practice Problems 2

Directions for questions 1 to 20: Select the correct alternative from the given choices.

1. What is the principal, if after five years at 11% p.a. simple interest, it amounts to ₹18600?
(A) ₹12000 (B) ₹8000
(C) ₹15000 (D) ₹11000

2. Given that a principal amounts to ₹10080 at 10% p.a simple interest after two years, what is the compound interest for two years on this principal at the same rate?
(A) ₹10164 (B) ₹1764
(C) ₹1640 (D) ₹3764

3. ₹10000 was lent at compound interest, interest being compounded annually for 3 years. The annual rates of interest for the first, second, and third years were 10%, 20%, and 30%, respectively. If it was instead, lent at 20% p.a. simple interest for the same time, how much more/less interest would be realized?
(A) ₹ 530 more (B) ₹530 less
(C) ₹1160 more (D) ₹1160 less

4. Krishna takes a loan of ₹8000 at simple interest. After four years, he takes an additional loan of ₹14440. From that point, compound interest at 10% per annum is calculated on the total amount repayable on the first loan as well as the second loan. He repays a total of ₹30250 after two more years to clear the entire loan amount. What is the annual rate of simple interest?
(A) 8% (B) 9%
(C) 7% (D) 6%

5. Prakash invested a certain amount in a six-year fixed deposit scheme, interest being compounded annually. The interests accrued on this deposit for the fourth and fifth years, respectively, are ₹1331 are ₹1464.10. If George deposited ₹12000 in the same scheme, how much interest would be accrued on this deposit for the first two years?
(A) ₹1320 (B) ₹1452
(C) ₹2520 (D) ₹1440

6. The difference between the amounts to be repaid by a man at the end of two years, at 20% per annum compounded annually and half yearly, is ₹1084.50. What is the principal?
(A) ₹45000 (B) ₹50000
(C) ₹51500 (D) ₹52500

7. Venkat lends a sum P at $r\%$ compound interest, compounded every moment for ten years. It becomes 'a' times P after ten years. What is the annual interest rate? (Assume that $a = e^2$)
(A) 100% (B) 20%
(C) 40% (D) 200%

8. Sourabh borrows ₹2500000 at 12% compound interest from a bank and invests in shares. The investment gives him a return of 20% per annum and he repays ₹500000 at the end of first year. How much does he make for himself after paying all the outstanding amount at the end of the second year?
(A) ₹424000 (B) ₹356241.50
(C) ₹525000 (D) ₹484241.80

9. What annual instalment will discharge a debt of ₹1450 due after five years at 8% p.a simple interest?
(A) ₹320 (B) ₹450
(C) ₹250 (D) ₹400

10. A man took a loan of ₹100000 at 8% per annum compound interest. He repays ₹10000 per annum. What is the amount due from him at the beginning of the third year?
(A) ₹91917 (B) ₹81917
(C) ₹93970 (D) ₹95840

11. Srikanth buys a car worth ₹525000. He pays ₹125000 as down payment and agrees to pay the remaining amount in instalments. What is the approximate yearly instalment amount to be paid by him, if at 12% p.a. compounded annually, he repays the remaining amount in three more years? Given that $(1/1.12)^3 = 0.71$.
(A) ₹160000 (B) ₹165000
(C) ₹180000 (D) ₹183000

12. A sum was lent for a year, another sum was lent for 2 years and another sum was lent for 3 years. Each sum was lent at 5% p.a compound interest. If each sum amounted to the same value, the ratio of the first, second and third sums is _____.
(A) 400 : 420 : 441 (B) 20 : 21 : 22
(C) 22 : 21 : 20 (D) 441 : 420 : 400

13. Suhaas borrowed a certain amount at 28% p.a. compound interest and repays it in one year. Bhanu borrows a certain amount at a certain interest rate under simple interest and returns it after four years. If the amounts repaid by Suhaas and Bhanu are the same and that is equal to ₹38400 and the sum of their principals borrowed is ₹54000, at what rate Bhanu paid the interest?
(A) 10 % p.a (B) 12% p.a
(C) 15% p.a (D) 18% p.a

14. Akshay deposits a sum P in a bank at $r\%$ compound interest. The amount becomes $27P$ after three years by compounding annually. Instead, if the bank had compounded half yearly, what is the additional amount Akshay would have received in terms of P?
(A) 54P (B) 37P (C) 18P (D) 5P

15. A sum was split into four parts (P_1, P_2, P_3, P_4) where $P_1 : P_2 : P_3 : P_4 = 1 : 4 : 5 : 2$. Each part was lent at simple interest. P_1 was lent at 10% p.a. for a year. P_2 was lent at 20% p.a. for 5 years. P_3 was lent at 2% p.a. for 4 years. P_4 was lent at 6% p.a. for 10 years. The greatest of the interests on the parts exceeds the least of the interests on the parts by ₹7800. Find the total interest fetched by the parts (in ₹).

(A) 5700 (B) 8550
(C) 11400 (D) 17100

16. A man invested ₹40000 in a bond which gives 10% p.a. interest, compounded half yearly. If the annual rate of interest is increased by 20 percentage points at the end of every half year, what will be the interest for the first one and half years?
(A) ₹15875 (B) ₹16750
(C) ₹20375 (D) ₹19875

17. A loan has to be repaid after three years under compound interest, interest being compounded annually at 30% p.a. Find the amount borrowed if the amount to be repaid is ₹87880 (in ₹).
(A) 36000 (B) 40000
(C) 48000 (D) 45000

18. Srikar saves ₹20000 at the beginning of each year in a savings bank account that pays 5% p.a, interest being compounded annually. If, at the beginning of the third year, instead of depositing ₹20000, he withdraws ₹10000, how much would be the total savings of the man at the end of three years?
(A) ₹28124.24 (B) ₹29324.20
(C) ₹31349.75 (D) ₹34702.50

19. Anwar borrowed ₹72000 at 20% p.a. compound interest, interest being compounded annually. He repaid ₹x at the end of the first year. He repaid ₹57600 at the end of the second year and thereby cleared the loan. Find x.
(A) 40000 (B) 36000
(C) 38400 (D) 32000

20. In the above question, if all the interests considered had been compound interest, interest being compounded annually, how much more/less interest would be realized?
(A) 1632 more (B) 1632 less
(C) 2014.40 more (D) None of these

HINTS/SOLUTIONS

Practice Problems I

Solutions for questions 1 to 20:

1. Simple Interest $= \frac{24000\times18\times5}{100} = ₹21{,}600$

Amount = Principal + Simple Interest

$= 24000 + 21600 = ₹45{,}600$

Hence, the correct option is (C).

2. The interest on a sum remains the same each year under simple interest.

$\therefore$ Total interest $= 3(1000) = ₹3000$

Hence, the correct option is (B).

3. Let the sum be ₹100.

Sixteen times of sum = ₹1,600

Interest = ₹1,500

$1500 = \frac{100\times30\times x}{100}$

$\Rightarrow x = 50$ i.e. 50 years.

Hence, the correct option is (D).

4. Let the sum be ₹x.

Then Amount = ₹$10x$.

Interest = ₹$9x$.

Let the time period be r years and rate of interest be r%.

$\Rightarrow 9x = \frac{x\times r\times r}{100}$

$\Rightarrow r = 30\%$

Hence, the correct option is (C).

5. Let its present value be ₹P.

$P\left(1 + \frac{6\ (10)}{100}\right) = 2400$

$P = 1500$

Hence, the correct option is (D).

6. Let the rate of interest be R% p.a.

Amount at the end of 2 years = ₹630

$3630 = 3000\left[\left(1 + \frac{R}{100}\right)^2\right]$

$(1.1)^2 = \left(1 + \frac{R}{100}\right)^2$

$1.1 = 1 + \frac{R}{100}\ \ 10 = R$

Hence, the correct option is (A).

7. Interest earned on a sum of ₹P invested for T years at R% p.a. under compound interest

$= ₹P\left[\left(1 + \frac{R}{100}\right)^T - 1\right]$

Interest earned $= 5000\left[\left(1 + \frac{10}{100}\right)^2 - 1\right] = ₹1050$

Hence, the correct option is (D).

8. Let the sum be ₹P and the rate of interest be R% p.a.

$P\left(1 + \frac{R}{100}\right)^2 = 6000$ (9)

$P\left(1 + \frac{R}{100}\right)^3 = 7200$ (10)

Dividing (10) by (9),

$1 + \frac{R}{100} = 1.2$

$R = 20$

Hence, the correct option is (B).

9. Let the sum be ₹P.

Let the rate of interest be R% p.a.

$P\left[\left(1 + \frac{R}{100}\right)^3 - \left(1 + \frac{R}{100}\right)^2\right] = 2420$

$P\left[\left(1 + \frac{R}{100}\right)^4 - \left(1 + \frac{R}{100}\right)^3\right] = 2662$

$P\ \frac{100t}{3}\ \frac{R}{100} = 2420$ (11)

$P\left(1 + \frac{R}{100}\right)^3\ \frac{R}{100} = 2662$ (12)

Dividing (12) by (11),

$1 + \frac{R}{100} = 1.1 \Rightarrow R = 10$

Alternate method:

(100) The interest on a sum for the k^{th} year under compound interest, interest being compounded annually is

$\frac{1+R}{100}$ (Interest for the $(k-1)^{th}$ year)

$2420\left(1 + \frac{R}{100}\right) = 2662$

$\therefore R = 10$

Hence, the correct option is (A).

10. Let the sum be ₹100

Amount after eight years = ₹400

Amount after sixteen years = ₹1,600

i.e., it takes 8 + 8 = 16 years for the sum to become sixteen times of itself.

$8 + 8 = 16$ years

Hence, the correct option is (D).

11. Let the sum be ₹x.

Then, the amount after three years = ₹$2.197 \times x$

$$\Rightarrow 2.197 \times x = x\left(\frac{100+r}{100}\right)^3$$

$$\Rightarrow \quad (1.3)^3 = \left(\frac{100+r}{100}\right)^3$$

$$\Rightarrow \quad 1.3 = \frac{100+r}{100}$$

$$\Rightarrow \quad 130 = 100 + r \Rightarrow r = 30\%$$

Hence, the correct option is (A).

12. Rate of interest = $\frac{20}{2}\% = 10\%$ per half year.

$$\text{Interest} = 200\left[\left(1+\frac{10}{100}\right)^2 - 1\right] = ₹42$$

Hence, the correct option is (B).

13. Rate of interest = 10% per quarter. A sum of ₹P under compound interest when invested at $R\%$ p.a. would become ₹$P\left(1+\frac{R}{100}\right)^N$

A sum of ₹100 would become

$$100\left[\left(1+\frac{10}{100}\right)\right] = 146.41$$

$\therefore$ This is the effective rate of interest is 46.41% p.a.

Hence, the correct option is (D).

14. Let the sum Ashok borrowed from the bank lending at 10% p.a. be ₹x

Sum he borrowed from the other bank = ₹$(84000 - x)$

$$\frac{10}{100}x + \frac{20}{100}(84000 - x) = 13200$$

$x = 36000$

difference = $84000 - 2x = 12000$

Hence, the correct option is (D).

15. Let the sum be ₹P

$$\text{Extra interest} = P\left(\frac{10}{100}\right)(2) = 4000$$

$P = 20000$

Hence, the correct option is (A).

16. Let the rate of interest be $R\%$ p.a.

$$\text{Extra interest} = (10000)\left(\frac{R}{100}\right)(2) = 4000$$

$R = 20$

Hence, the correct option is (C).

17. Let the sum be ₹P.

Let the rate of interest be $R\%$ p.a.

$$P\left[\left(1+\frac{R}{100}\right)^2 - \left(1+\frac{R}{100}\right)\right] = 14400 \qquad (13)$$

$$P\left[\left(1+\frac{R}{100}\right)^3 - \left(1+\frac{R}{100}\right)^2\right] = 17280 \qquad (14)$$

Dividing (14) by (13), $\frac{S}{x} \Rightarrow R = 20$

Hence, the correct option is (C).

18. If a sum is doubled in the same time at simple interest as well as compound interest, the rate of interest under simple interest is more than rate under compound interest.

Hence, the correct option is (A).

19. Let the sum be ₹P.

Let the rate of interest be $R\%$ p.a

$$0.4P = (P)\left(\frac{R}{100}\right)(2)$$

$$20 = R$$

$$\left(1+\frac{x}{100}\right)P = P\left(1+\frac{20}{100}\right)^2$$

$\Rightarrow x = 44$

Hence, the correct option is (D).

20. Let the sum be ₹$2P$.

₹P lent at simple interest fetches ₹$0.2P$ as interest each year. ₹P lent at compound interest fetches ₹$0.24P$ as interest in the second year and ₹$0.3456P$ as interest in the fourth year

$0.24P - 0.2P = 400$

$P = 10000$

Required difference = $0.3456P - 0.2P$

= ₹1456

Hence, the correct option is (C).

Practice Problems 2

Solutions for questions 1 to 20:

1. Given $P+\frac{Pnr}{100}=18600$

$P\left(1+\frac{nr}{100}\right)=18600\ n=5, r=11$

$\Rightarrow \quad P=\frac{18600}{1+\frac{5\times 11}{100}}=12000$

Alternate method:

$A=P\ (1+nr\%), \Rightarrow 18600=P(1+55\%)=1.55\times P$

$\Rightarrow 18600\times 100=155P, \Rightarrow P=12000$

Hence, the correct option is (A).

2. For Simple Interest $P\left(1+\frac{nr}{100}\right)=10080$

$n=2, r=10$

$P=10080/1.2=8400$

$n=2, r=10, P=8400$

$A=\ 8400\left(1+\frac{10}{100}\right)^2=10164$

$\therefore$ C.I. $=10164-8400=₹1764$

Alternate method:

$A=P(1+nr\%) \Rightarrow 10080 \Rightarrow P(1+20\%);$

Under C.I., $A=P(1+R\%)^n=\frac{10080}{1.2}(1.1)^2=10{,}164$

Hence, C.I. $=10{,}164-P=10{,}164-8400=1764.$

Hence, the correct option is (B).

3. Amount that ₹10000 becomes under compound interest

$=1000\left(1+\frac{10}{100}\right)\left(1+\frac{20}{100}\right)\left(1+\frac{30}{100}\right)=17160$

amount that ₹10000 becomes under simple interest

$=(10000)\ \left[1+3\left(\frac{20}{100}\right)\right]=16000$

interest realized is ₹1160 less

Hence, the correct option is (D).

4. Let the amount that ₹8000 becomes after 4 years be x. Then $(x+14440)\ (1.1)^2=30{,}250$

Or $x+14440=30250/1.21$

$x+14440=25000;\ 10560=x$

Now, let the rate of simple interest for the first 4 years be $r\%$ Then $8000(1+4r/100)=10{,}560;\ 320r=2560,$ $r=8$

Hence, the correct option is (A).

5. When compounded annually, the interest accrued for the n^{th} year is $r\%$ more than the interest accrued for the $(n-1)^{th}$ year, where r is the annual rate of interest.

$\therefore \quad I_5=\left(1+\frac{r}{100}\right)I_4$

$\Rightarrow 1464.1=\left(1+\frac{r}{100}\right)(1331) \Rightarrow 1+\frac{r}{100}=1.1$

$\Rightarrow r=10\%$

Interest accrued for the first two years when ₹12000 is invested in the same scheme

$=1200\left[\left(1+\frac{10}{100}\right)-1\right]=12000[(1.1)^2-1]$

$=12000\times 0.21=2520$

Hence, the correct option is (C).

6. If compounding is done annually $r=20\%, n=2,$

Let $P=100, A=100(1.2)^2=144$

If compounding is done half-yearly

$r=10\%, n=4, A=146.41$ Difference $=2.41$

If $P=100$ then, difference $=2.41$

If difference is 1084.5, $P=\frac{1084.5}{2.41}\times 100=₹45000$

Hence, the correct option is (A).

7. Sum $=P$, C.I $=r\%, n=10$ years

Given, $Pe^{\left(\frac{nr}{100}\right)}=a\,.\,P$

$e^{10x\frac{r}{100}}=a$

Given $e=a^2$ hence $e^{r/10}=e^2 \Rightarrow r=20$

Hence, the correct option is (B).

8. He borrows ₹2500000

At the end of the first year it becomes

$2500000\times(1+12/100)^1=₹2800000$

He repays ₹500000 (15)

₹2300000 is the principal for the 2nd year.

This becomes $230000\ (1+12/100)^1=₹2576000$

Which is repaid by the person, at the end of 2nd year.

In the first year, (on his earnings side), he earns 20% on ₹2500000 = ₹500000.

Out of this, (15) is repaid. Investment remains as ₹250000.

This earns 20% during the second year. So, it becomes ₹3000000. Out of this ₹2576000 is repaid. So, he makes, 3000000 – 2576000

= ₹424000

Hence, the correct option is (A).

9. Let the annual instalment be ₹x.

Let $(FV)_i$ denote the value of the i^{th} instalment at the end of the fifth year.

Sum of future values of all the instalments = ₹1450

$$(FV)_1 = x + \frac{(x)(8)(4)}{100}$$

$$(FV)_2 = x + \frac{(x)(8)(3)}{100}$$

$$(FV)_3 = x + \frac{(x)(8)(2)}{100}$$

$$(FV)_4 = x + \frac{(x)(8)(1)}{100}$$

$$(FV)_5 = x$$

$$\therefore 5x + \frac{x}{100}(8)(4 + 3 + 2 + 1) = 1450$$

$$\Rightarrow 5x + 0.8x = 1450$$

$$\Rightarrow x = \frac{1450}{5.8} = 250$$

Alternate method:

Each instalment must be less than ₹$(\frac{1}{5}^{th}$ of 1450) = ₹(290). From among the choices given, only Choice (C) satisfies the condition.

Hence, the correct option is (C).

10. Principal = ₹100000

At the end of first year it amounts to

$100000(1 + 8/100)^1 = 108000$

Out of this he repays ₹10000

So, amount due at the end of 1st year

= 108000 – 10000 = ₹98000.

At the end of the second year this amounts to

$98000\ [1 + 8/100]^1 = 105840$.

Out of this he repays ₹10000

∴ At the beginning of the third year amount due from him = 105840 – 10000 = ₹95840

Hence, the correct option is (D).

11. Cost of the car = ₹525000

Cash payment = ₹125000

∴ Principal of loan to be discharged in instalments = ₹400000

$$\text{Each instalment } X = \frac{\Pr}{100\left[1 - \left(\frac{100}{100 + r}\right)^n\right]}$$

$$= \frac{400000 \times 12}{100\left[1 - \left(\frac{100}{112}\right)^3\right]}$$

$$= \frac{4000 \times 12}{(1 - 0.71)} = \frac{4000 \times 12}{0.29}$$

$$\cong \frac{4000 \times 12}{0.29} = \frac{4000 \times 12}{0.3} = 160000.$$

As the actual denominator is slightly less than 0.3 the actual answer should be slightly more than ₹160000.

Actual answer is ₹165000 as per the options.

Hence, the correct option is (B).

12. Let the first, second and third sums be ₹f, ₹s and ₹t respectively.

$$f\left(1 + \frac{5}{100}\right) = s\left(1 + \frac{5}{100}\right)^2 = t\left(1 + \frac{5}{10}\right)^3 = A \quad \text{(say)}$$

$$f : s : t = \frac{A}{\frac{21}{20}} : \frac{A}{\frac{441}{400}} : \frac{A}{\frac{9261}{8000}}$$

$$= \frac{20}{21} : \frac{400}{441} : \frac{8000}{9261}$$

$$= 8820 : 8400 : 8000 = 441 : 420 : 400$$

Hence, the correct option is (D).

13. Suhaas borrows at 28% compound interest and pays back in 1 year (let him borrow P_1)

$$P_1\left(1 + \frac{28}{100}\right)^1 = 38400$$

$$P_1\ (1.28) = 38400$$

$$P_1 = 30000$$

Let Bhanu borrow P_2

Given $P_1 + P_2 = 54000$;

$$P_2 = 24000$$

$$24000\left(1 + \frac{4 \times r}{100}\right) = 38400$$

$4r/100 = 1.6 – 1 = 0.6$

$\Rightarrow r = 15\%$

Hence, the correct option is (C).

14. Sum = P; C.I. = $r\%$; In 3 years, amount = $27P$

$$P\left(1 + \frac{r}{100}\right)^3 = 27P$$

$$\Rightarrow \left(1 + \frac{r}{100}\right)^3 = 27$$

$$\Rightarrow 1 + \frac{r}{100} = 3 \quad r = 200.$$

Compounding half yearly, the amount will be

$$P\left(1 + \frac{100}{100}\right)^6 = P \times 2^6 = 64P$$

Additional Amount received = $64P - 27P = 37P$

Hence, the correct option is (B).

15. Let $P_1 = ₹x$

$P_2 = ₹4x$.

$P_3 = ₹5x$

$P_4 = ₹2x$

Simple interests on P_1, P_2, P_3 and P_4 are $(x)\left(\frac{10}{100}\right)(1)$

$(4x)\left(\frac{20}{100}\right)(5)$, $(5x)\left(\frac{2}{100}\right)(4)$

and $(2x)\left(\frac{6}{100}\right)(10)$ i.e.,

₹01x, ₹4x, ₹0.4x and ₹1.2x respectively

$4x - 0.1x = 7800$ $x = 2000$

The total simple interest = $5.7x = ₹11400$

Hence, the correct option is (C).

16. As the rate of interest is increasing by 20 percentage points annually, the rate of interest per half year increases by 10 percentage points.

Half year	Principal	Interest for the half year	Interest of the end of half year	Amount
1	40000	2000	2000	42000
2	42000	6300	8300	48300
3	48300	12075	20375	60375

∴ The required interest will be ₹20375.

Hence, the correct option is (C).

17. Let the present value be ₹x.

$$x = \frac{87880}{\left(1+\frac{30}{100}\right)^3} = 40000$$

Hence, the correct option is (B).

18. ₹20000 deposited at the beginning of the first year becomes, in 2 years, an amount equal to

$20000 \times (1.05)^2 = ₹22050$

(i.e., at the beginning of the third year)

₹20000 deposited at the beginning of the second year becomes in 1 year $(20000) \times (1.05) = ₹21000$

∴ Amount at the beginning of the third year

$= 22050 + 21000 = ₹43050$

Now ₹10000 is withdrawn

∴ There is ₹33050 in the account on which 5% interest for one more year is earned.

Final amount = $33050 \times 1.05 = ₹34702.5$

Hence, the correct option is (D).

19. Value of ₹72000 at the end of the first year = ₹86400. Since ₹x was repaid at the end of the first year, in order to clear the loan at the end of the second year, the amount to be repaid must be

$$(86400 - x) + \frac{20}{100}(86400 - x)$$

$1.2\ (86400 - x) = 57600$

$x = 38400$

Hence, the correct option is (C).

20. The amount that ₹1000 becomes under compound interest $= (1000)\left(1+\frac{10}{100}\right)^2 = ₹1210$

The amount that ₹3000 becomes under compound interest $= (3000)\left(1+\frac{30}{100}\right)^2 = ₹5070$

The amount that ₹4000 becomes under compound interest $= (4000)\left(1+\frac{20}{100}\right)^2 = ₹5760$

The interest realized is ₹520 less

Hence, the correct option is (D).

Chapter 6 Averages, Mixtures and Alligations

Exercises

Practice Problems 1

Directions for questions 1 to 25: Select the correct alternative from the given choices.

1. Find the average of all the two digit numbers divisible by 10.
(A) 40 (B) 50 (C) 45 (D) 60

2. Find the average of all odd numbers less than 50.
(A) 26.5 (B) 25.5 (C) 26 (D) 25

3. Find the average of all the multiples of 12 less than 100.
(A) 48 (B) 54 (C) 60 (D) 66

4. The average salary per month of a man for the first four months, next four months, and the last four months of a year are ₹6000, ₹8000, and ₹13000, respectively. Find his average salary per month in that year (in ₹).
(A) 7500 (B) 9000
(C) 10500 (D) 6600

5. In an office there are 20 employees. The average heights of the male employees is 180 cm. The average height of the female employees is 170 cm. Find the average height of all the employees (in cm).
(A) 172
(B) 174
(C) 176
(D) Cannot be determined

6. The average age of the boys in a class is ten years. The average age of the girls in the class is eight years. There are 50% more boys than girls in the class. Find the average age of the class (in years).
(A) 8.4 (B) 8.8 (C) 9.2 (D) 9.6

7. A vessel has 20 litres of a mixture of milk and water having 60% milk. Five litres of pure milk is added to the vessel. Find the percentage of milk in the new solution.
(A) 34% (B) 51%
(C) 68% (D) None of these

8. In what ratio must two kinds of coffee which cost ₹80 per kg and ₹108 per kg be mixed such that the resultant mixture costs ₹96 per kg?
(A) 1 : 2 (B) 2 : 3
(C) 3 : 4 (D) 2 : 1

9. Vessel *A* has 20 litres of a mixture of milk and water having 75% milk. Vessel *B* has *x* litres of a mixture of milk and water having 60% milk. The contents of the vessels are mixed to form a mixture having 66% milk. Find *x*.
(A) 25 (B) 30 (C) 20 (D) 40

10. A milkman has 15 litres of pure milk. How many litres of water have to be added to it so that he gets a 60% profit by selling at cost price?
(A) 9 (B) 10 (C) 8 (D) 12

11. From 90 litres of pure milk, 9 litres is withdrawn and replaced by water. 9 litres of the mixture is then withdrawn and replaced by water. Find the ratio of milk and water in the present mixture.
(A) 19 : 81 (B) 19 : 100
(C) 81 : 19 (D) 81 : 100

12. Just before the last match in a season, the total number of runs scored by Sachin Tendulkar added up to 2100. In his last match, he scored 101 runs. As a result his average score for the season went up by one run. Find the total number of matches he played in that season if he got out in every match.
(A) 31 (B) 5
(C) 71 (D) Either 31 or 71

13. The average weight of all the students of classes I and II equals the average of the average weight of the students of the two classes. There are twice as many students in class II as in class I. The sum of twice the average weight of the students of class I and the average weight of the students of class II is 60 kg. Find the average weight of class I (in kg).
(A) 10 (B) 15 (C) 20 (D) 25

14. Two varieties of wheat are mixed in the proportion of 3 : 4 and the mixture is sold at ₹28 per kg at a profit of 40%. If the second variety of wheat costs ₹3 more than the first variety of wheat, find the cost price of the first variety of wheat.
(A) ₹128/7 per kg (B) ₹120/7 per kg
(C) ₹141/7 per kg (D) ₹149/7 per kg

15. A man buys milk at ₹4 per litre, mixes it with water and sells the mixture at the same price. If his profit is 25%, find the amount of water mixed with each litre of milk.
(A) 0.25 litres (B) 0.5 litres
(C) 0.75 litres (D) 0.6 litres

16. In what proportion can three varieties of sugar priced at ₹10 per kg, ₹12 per kg, and ₹18 per kg, be mixed so that the price of the mixture is ₹14 per kg?
(A) 2 : 2 : 5 (B) 2 : 3 : 4
(C) 1 : 3 : 4 (D) 3 : 4 : 5

17. The ratio of alcohol and water in three mixtures of alcohol and water is 3 : 2, 4 : 1, and 7 : 3. If equal quantities of the mixtures are drawn and mixed, the concentration of alcohol in the resulting mixture will be_____.
(A) 65% (B) 70% (C) 75% (D) 80%

18. In what proportion should milk and water be mixed to reduce the cost of litre of milk from ₹18 per litre to ₹16?
(A) 8 : 1 (B) 6 : 1 (C) 10 : 1 (D) 7 : 1

19. *A*'s weight equals the average weight of *B*, *C,* and *D*. *B*'s weight equals the average weight of *A*, *C,* and *D*. The average weight of *C* and *D* is 30 kg. Find the average weight of *A* and *B*.

(A) 15 kg (B) 30 kg (C) 60 kg (D) 45 kg

20. Of five numbers, the first number is thrice the third, the fourth number is two less than the first, the fifth number is one-seventh of the second and the second number is three less than thrice the first. Find the fifth number, if the average of the numbers is 16.2.
(A) 3 (B) 4 (C) 5 (D) 6

21. There are nine two-digit numbers with distinct tens digits. The units digit of each number is one less than its tens digit. Find the average of the units digits.
(A) 3 (B) 4 (C) 5 (D) 6

22. A sum of ₹7.75 is made up of 100 coins, which are in the denominations of 5 paise and 10 paise. Find the number of 5 paise coins.
(A) 50 (B) 55 (C) 75 (D) 45

23. A businessman lends ₹1800 in two parts, one at 10% and the other at 12% interest. At the end of the year, the average interest he obtained worked out to be 10.5%. Find the interest earned by the businessman from the part which was lent at 10%.
(A) ₹135 (B) ₹150
(C) ₹200 (D) ₹250

24. A vessel is full of a mixture of milk and water, with 9% milk. Nine litres are withdrawn and then replaced with pure water. If the milk is now 6%, how much does the vessel hold?
(A) 27 litres (B) 18 litres
(C) 36 litres (D) 40 litres

25. Three varieties of rice, *A*, *B,* and *C* costing ₹6/kg, ₹9/kg and ₹12/kg are mixed together in a certain ratio. The mixture is sold at $66\frac{2}{3}$ % profit for ₹15 / kg. Of the total of 100 kg of the mixture, 50 kg is variety *B*. Find the quantity of variety *A* (in kgs)
(A) 15 (B) 25 (C) 20 (D) 10

Practice Problems 2

Directions for questions 1 to 25: Select the correct alternative from the given choices.

1. The average age of a board of 10 advisors of a company is the same as it was 3 years ago, on account of the replacement of one of the older advisors by a younger man. How much younger is the new man than the director whom he replaced?
(A) 24 years (B) 40 years
(C) 52 years (D) None of these

2. The average of *n* numbers is *a*. If one of the numbers, *y*, is replaced by *x*, the average becomes *b*. Find the relation between *n*, *a*, *x*, *y,* and *b*.
(A) $\frac{1}{n}=\frac{x-y}{a-b}$ (B) $\frac{1}{n}=\frac{a-b}{y-x}$
(C) $\frac{1}{n}=\frac{x+y}{a+b}$ (D) $\frac{1}{n}=\frac{a+b}{x+y}$

3. The average weight of *N* boys in a group is 36 kg. If 20 other boys whose average weight is 30 kg. join the group, the average weight of the group would be the same as what it would be if 5 boys whose average weight is 40 kg leave the group. Find *N*.
(A) 10 (B) 15 (C) 20 (D) 25

4. The students of three classes, *A*, *B,* and *C* take a test. The average per student marks of the classes *A* and *B* put together is 71. The average marks per student of the classes *B* and *C* put together is 76. The average per student marks of the class *A* and *C* put together is 79. Find the range of the average marks (p) of all the three classes put together.
(A) $72 < p < 76$ (B) $73 < p < 75$
(C) $71 < p < 77$ (D) $73.5 < p < 77.5$

5. All the members of a club meet for lunch every Monday. Last week, just before the bill was presented, two of the club members were called for an official meeting and left. The remaining members were presented with a bill of ₹1440. It was customary to divide the bill equally. To cover the share of the two who left, each member had to pay an extra amount of ₹24. How many people were present for the lunch?
(A) 10 (B) 12 (C) 15 (D) 14

6. The average of n numbers is 16. If $5/8^{th}$ of the numbers are doubled and $3/8^{th}$ of the numbers become 10/3 times their original values, by what percentage does the current average exceed the original average?
(A) 20%
(B) 30%
(C) 25%
(D) Cannot be determined

7. Ajay attempted to add ten two-digit numbers. One of them, *a*, was the reverse of one of the others. If *a* was replaced by another two-digit number *b* and the reverse of *a* was replaced by the reverse of *b* and the average was found, it would be 2.2 more. The sum of the digits in *b* exceeds the sum of the digits in *a* by
(A) 1 (B) 2 (C) 3 (D) 4

8. There are *N* students in a class. Their class teacher gave them a task of finding the average of the first *N* natural numbers and each of them left out a different number and found the average of the remaining numbers. The average of the averages obtained by all the students was 21. Find *N*.
(A) 39 (B) 40 (C) 41 (D) 38

9. In a class, there are 50 students. The average weight of all the girls is 30 kg. The average weight of 30 of the

boys is 40 kg. Which of the following can be a possible value of the average weight of the entire class (in kg)?
(A) 31 (B) 33 (C) 35 (D) 37

10. The average weight of all the students in a class equals the average of the average weight of the boys and that of the girls, which of the following holds true?
(A) The numbers of boys and girls in the class are equal.
(B) The average weights of the boys and the girls are equal.
(C) At least one of (A) and (B)
(D) Neither (A) nor (B)

11. Two alloys A and B contain copper and zinc in the ratio 4 : 9 and 5 : 6, respectively. If equal weights of the two are melted together to form a third alloy, find the ratio of the weights of copper and zinc in the third alloy named C.
(A) 109 : 167 (B) 113 : 164
(C) 109 : 177 (D) 107 : 158

12. A vessel has 10 ml of a solution of milk and water containing 20% milk. x ml of milk was added to the vessel to reverse this ratio. y ml of water was then added to the vessel to reverse the ratio once again. Find $x + y$.
(A) 140 (B) 150 (C) 160 (D) 170

13. There are two containers having mixtures of hydrochloric acid and water. In container 1, the ratio of hydrochloric acid and water is 1 : 2 and in container 2 the ratio of hydrochloric acid and water is 4 : 1. Find the amount of the mixture that should be taken from container 1 in order to make 28 litres of a mixture containing equal amount of water and hydrochloric acid.
(A) 15 litres (B) 14 litres
(C) 20 litres (D) 18 litres

14. A vessel contains a mixture of 100 litres of milk and water. The concentration of milk is 90%. 10% of the contents of the vessel are withdrawn and replaced with an equal amount of water. The minimum number of times that this procedure must be carried out such that the concentration of milk in the vessel is less than $66\frac{2}{3}\%$ is _____.
(A) 1 (B) 2 (C) 3 (D) 4

15. Alloy A has 80% copper and 20% tin. A certain process when applied repeatedly to the alloy decreases the copper quantity in the alloy by one-fifth and the tin quantity in it by one-tenth, each time it is applied. The minimum number of times the process must be applied so that the concentration of copper in the alloy is less than 70% is ____.
(A) 4 (B) 5 (C) 3 (D) 2

16. A basket ball player played nine matches. The average number of points he scored per match was 16. His points in the i^{th} match were two less than that in the $(i-1)^{th}$ match. Find the average number of points scored in the second and the eighth matches.
(A) 8 (B) 12 (C) 16 (D) 18

17. The average score of a class of 32 students is 70. If the top score and the least score, which differ by 70, are excluded, the average score of the class drops by 1. Find the top score.
(A) 110 (B) 120 (C) 130 (D) 100

18. The average age of a group of men is 20 years. Two men whose ages are 23 years and 27 years joined the group. The average age of the group increased by a prime number. The number of men in the original group is
(A) 3 (B) 4 (C) 5 (D) 6

19. There are two classes A and B. The average weight of the students in class A is 40 kg. The average weight of the students in class B is 60 kg. A student, whose weight is x kg left A and joined B. As a result, the average weight of A, as well as that of B decreased. Which of the following must be true?
(A) $35 < x \le 40$ (B) $40 < x < 60$
(C) $60 \le x < 65$ (D) $30 < x \le 35$

20. In an office, the average salary of the employees is ₹8000. If the average salary of all the 50 clerks is ₹3000 and the average salary of all the managers is ₹10000, find the difference in the number of clerks and the number of managers.
(A) 75 (B) 55 (C) 60 (D) 125

21. Four classes P, Q, R, and S take up a test. The ratio of the number of students in P, Q, R, and S is 2 : 3 : 6 : 4. The ratio of the average marks of P, Q, R and S is 5 : 4 : 3 : 2. If the average marks of all combinations of three sections are considered, the greatest is 52. Find the least.
(A) 34 (B) 36.2 (C) 38 (D) 41.8

22. Two vessels contain alcohol and water in the ratio 3 : 4 and 65 : 79. In what ratio should the solution in the first vessel be mixed with the solution in the second, so as to get a solution with alcohol and water in the ratio 4 : 5?
(A) 7 : 16 (B) 16 : 7 (C) 36 : 85 (D) 85 : 36

23. There are eleven people in a group. The average age of the oldest and the youngest is 11 years. If any one person leaves the group, the maximum and the minimum average of the remaining are 12 years and 11 years, respectively. Find the average age of the entire group, in years.
(A) 9 (B) 10
(C) $11\frac{7}{11}$ (D) $11\frac{5}{11}$

24. A vessel is filled to its capacity with pure milk. Nine litres are withdrawn from the vessel and replaced with an equal amount of water. Nine litres of the mixture is again withdrawn and then replaced with an equal amount of water. After these changes, the vessel contains 17.1 litres of milk less than it did initially.

(i) Find the capacity of the vessel.
(A) 120 (B) 150 (C) 90 (D) 75

(ii) What is the least number of such additional replacements required, so that the vessel contains less than 75% milk?
(A) 1 (B) 2 (C) 3 (D) 4

25. A local grocer mixed three qualities of coffee T_1, T_2, and T_3 at ₹56 per kg, ₹64 per kg, and ₹80 per kg in the ratio 1 : 2 : 4. To 4 kg of this mixture, he added p kg of T_1 and $4p$ kg of T_3. The final mixture so formed is sold for ₹87.60 per kg at 20% profit. Find p.
(A) 1/3 (B) 5/9 (C) 2/3 (D) 4/11

HINTS/SOLUTIONS

Practice Problems I

1. The two digit numbers divisible by 10 are 10, 20, 30, 40, 50, 60, 70, 80 and 90.

Sum of the two digit numbers divisible by 10

$= 10 + 20 + 30 + 40 + 50 + 60 + 70 + 80 + 90 = 450$

$\therefore$ required average $= \frac{450}{9} = 50$

Hence, the correct option is (B).

2. There are 25 odd numbers less than 50

Average $= \frac{1+49}{2} = 25$

Hence, the correct option is (D).

3. The last multiple of 12 less than 100 is (12)

Average of all the multiples of 12

$$= \frac{12\,(1 + 2 + 3 + ... + 8)}{8} = \frac{12\,\frac{(8)\,(9)}{2}}{8} = 54$$

Hence, the correct option is (B).

4. Average salary per month

$$= \frac{4\,(6000) + 4\,(8000) + 4(13000)}{12}$$

$$= \frac{6000 + 8000 + 13000}{3} = ₹9000$$

Hence, the correct option is (B).

5. Let the numbers of male employees and female employees be m and f respectively.

Average height of the employees $= \frac{180\ m + 170\ f}{m + f}$

$$= \frac{180\,\frac{m}{f} + 170}{\frac{m}{f} + 1}.$$

As m/f is unknown, the average height cannot be found.

Hence, the correct option is (D).

6. Let the number of girls in the class be g.

Number of boys in the class $= \frac{150}{100}g = \frac{3}{2}g.$

Total of the ages of the boys $= 10\left(\frac{3}{2}g\right). = 15$ g years.

Total of the ages of the girls $= 8\ g$ years.

Total of the ages of the students $= 23$ g years.

Average age of the class $= \frac{23\ g}{g + \frac{3}{2}\ g} = 9.2$ years.

Hence, the correct option is (C).

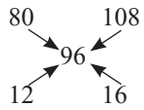
7. Quantity of milk in 20 litres $= (0.6)\ (20) = 12$ litres.

Upon addition of 5 litres of pure milk, the new solution of 25 litres would contain 17 litres as milk.

$\therefore$ Percentage of milk $= \frac{17}{25}(100) = 68\%$

Hence, the correct option is (C).

8.



Ratio = 3 : 4

Hence, the correct option is (C).

9. Quantity of milk in $A = \frac{75}{100}(20) = 15$ litres.

Quantity of milk in $B = \frac{60}{100}x$ litres.

$$15 + \frac{60}{100}x = \frac{66}{100}(20 + x)$$

$15 + 0.6x = 13.2 + 0.66x \Rightarrow x = 30$

Hence, the correct option is (B).

10. Let the cost price of the milk be ₹x/litres.

Let the quantity of water to be added to the milk be y litres.

Selling price of the mixture = ₹ x/litre.

Cost price of the mixture $= \frac{x\ (100)}{100 + 60} = ₹\frac{5}{8}x$/litre.

Cost of water = ₹0/litre

Method 1:

$\therefore \quad 15x + 0 = \frac{5}{8}x\ (15 + y)\ 9 = y$

Method 2:

$$\frac{y}{15} = \frac{x - \frac{5}{8}x}{\frac{5}{8}x - 0} = \frac{3}{5}$$

$y = 9$

Hence, the correct option is (A).

11. Ratio of milk and present mixture $= \left(\frac{90 - 9}{90}\right)^2 = \frac{81}{100}$

$\therefore$ Ratio of milk and water = 81 : 19

Hence, the correct option is (C).

12. Let the total number of matches that Sachin Tendulkar played be x.

$$\frac{2100}{x-1}+1=\frac{2201}{x}$$

Among the choices, choices (A) and (C) satisfy the equation above.

Hence, the correct option is (D).

13. Let the average of classes I and II be A kg and B kg.

Let the number of students in the classes I and II be a and b respectively.

$$\frac{Aa+Bb}{a+b}=\frac{A+B}{2}$$

$2Aa + 2Bb = Aa + Bb + Ab + Ba$

$(a - b)(A - B) = 0$

As $a \neq b$, $A = B$ (3)

and $2A + B = 60$ (4)

solving (3) and (4), we get $A = B = 20$.

Hence, the correct option is (C).

14. Cost price of a mixture of two varieties of wheat

$$=\frac{100}{100+40}x\,28=₹\,20$$

Let the cost price of the first and the second varieties of wheat be ₹y per kg and ₹$(y + 3)$ per kg respectively.

Let the quantities be $3x$ kg and $4x$ kg respectively.

$$\text{Cost price of the mixture} = \frac{y(3x)+(y+3)(4x)}{3x+4x}$$

$$= \frac{7xy+12x}{7x} = y+\frac{12}{7} = 20$$

Cost price of the first variety of wheat

$$= 20-\frac{12}{7} = ₹\frac{128}{7}/\text{kg}$$

Alternate method:

A mixture of two varieties of wheat is sold at ₹28 per kg, at a profit of 40%.

Hence, the cost price of the mixture = (28)/(1.4) = ₹20

Let x and $(x + 3)$ be the prices in rupees of the first and the second varieties respectively.

The quantities are mixed in the ratio 3 : 4 (given)

By alligation equation,

$$\frac{q_1}{q_2}=\frac{p_2-p}{p-p_1};$$

$$\Rightarrow\frac{3}{4}=\frac{(x+3)-20}{20-x}$$

$$\Rightarrow \frac{3}{4}=\frac{x-17}{20-x};$$

$\Rightarrow x = 128/7$

Hence, the correct option is (A).

15. Cost price of each litre of diluted milk

$$=\frac{100}{100+25}(4)=₹\frac{16}{5}$$

By the principle of allegation,

$$\frac{\text{Amount of water used for mixing}}{\text{Amount of milk used for mixing}}$$

$$=\frac{4-\frac{16}{5}}{\frac{16}{5}-0}=\frac{\frac{4}{5}}{\frac{16}{5}}=\frac{1}{4}$$

Hence 0.25 litres of water will be mixed with each litre of milk.

Hence, the correct option is (A).

16. Let the required proportion be $x : y : z$

$$\frac{10x+12y+18z}{x+y+z}=14$$

Going by the options, we have only Choice (D) satisfying the above equation.

Alternate method:

Prices of 3 varieties of sugar are ₹10, ₹12 and ₹18 per kg.

Let them be mixed in the ratio $x : y : z$

Hence, price of the mixture is

$(10x + 12y + 18z)/(x + y + z)$

But this value is given on ₹14.

Hence, $(10x + 12y + 18z)/(x + y + z) = 14$

$\Rightarrow 10x + 12y + 18z = 14x + 14y + 14z$

$\Rightarrow 4x + 2y - 4z = 0;$

$\Rightarrow 2x + y - 2z = 0$

This equation has infinite solution sets. Hence, from among the given options, the one which satisfies the equation will be the solution.

Option (D), i.e., 3 : 4 : 5 satisfies the equation.

Hence, the correct option is (D).

17. In the first, second and third mixtures, $\frac{3}{5}^{\text{th}}$, $\frac{4}{5}^{\text{th}}$ and $\frac{7}{10}^{\text{th}}$ of the contents respectively is alcohol. Let x ml of each of these be drawn and mixed. The resulting mixture of $3x$ ml will have $\left(\frac{3}{5}x+\frac{4}{5}x+\frac{7}{10}x\right)$ ml of alcohol i.e. $\frac{21}{10}x$ ml of alcohol $\therefore$ It will have 70% alcohol.

Hence, the correct option is (B).

18. Applying the alligation equation, Quantity of milk/quantity of water $=\frac{16-0}{18-16}=8:1$

Hence, the correct option is (A).

19. Let A's, B's, C's, and D's weights be a kg, b kg, c kg and d kg respectively.

$$a = \frac{b+c+d}{3} = 3a = b + c + d \quad (5)$$

$$b = \frac{a+c+d}{3} = 3b = a + c + d \quad (6)$$

Adding (5) and (6),

$3(a + b) = a + b + 2(c + d)$

$a + b = c + d$

$$\frac{c+d}{2} = 30 \therefore \frac{a+b}{2} = 30.$$

Hence, the correct option is (B).

20. Let the fifth number be x. Then, from the given data, second number = $7x$

$$\text{first number} = \frac{7x+3}{3}$$

$$\text{fourth number} = \frac{7x+3}{3} - 2$$

$$\frac{7x+3}{3} + 7x + \frac{7x+3}{9} + \left(\frac{7x+3}{3} - 2\right) + x = 5 \times 16.2 = 81$$

Solving, we get $x = 6$

Hence, the correct option is (D).

21. As the tens digits of the 9 numbers are different, the tens digits must be 1, 2, 3, 4, 5, 6, 7, 8 and 9. The units digits of the numbers must be 0, 1, 2, 3, 4, 5, 6, 7 and 8.

$$\text{Their average} = \frac{36}{9} = 4$$

Hence, the correct option is (B).

22. Average value per coin = 775 paise/100 coins 7.75 paise/coin. By the application of the alligation equation.

$$\text{The number of 5 paise coins and the number of 10 paise coins} = \frac{10 - 7.75}{7.75 - 5} = \frac{2.25}{2.75} = \frac{9}{11}$$

Hence the number of 5 paise coins $= (9/20) \times (100) = 45$

Hence, the correct option is (D).

Note: The problem can also be solved using simultaneous equation.

23. Let the part lent at 10% be ₹x and that lent at 12% be $(1800 - x)$.

$$\frac{x}{1800 - x} = \frac{12 - 10.5}{10.5 - 10} = \frac{3}{1}$$

Hence $x = 3/4\ (1800) = 1350$ and $y = 1/4\ (1800) = 450$

Interest earned by the business man from the part lent at 10% $= \frac{1350 \times 1 \times 10}{100} =$ ₹135

Hence, the correct option is (A).

24. Let the capacity of the vessel be x ml.

Amount of milk originally in the vessel = $9x/100$ ml

Amount of milk in the vessel after replacement by water = $6x/100$ ml.

Amount of milk in the 9 litres withdrawn

$= 9(9)/100 = 81/100$ ml

Hence $9x/100 - 6x/100 = 81/100$;

$\Rightarrow 3x/100 = 81/100$

$\Rightarrow x = 27$ litres

Alternate method:

In the case of replacement, the relation between the initial and the final concentration is:

$$C_1\left(\frac{v - x}{v}\right) = C_2$$

C_1, C_2 are the initial and the final concentrations respectively; v is the total volume, x is the volume replaced.

$$\text{Hence, } 9\%\left(\frac{v - 9}{v}\right) = 6\%; \Rightarrow v = 27$$

Hence, the correct option is (A).

25. Let the quantities of A, B and C used for mixing be a kg, b kg and c kg respectively.

$$\text{Cost price of the mixture} = \frac{15(100)}{100 + 66\frac{2}{3}} = ₹9/\text{kg}$$

Total cost price of the mixture = $6a + 9b + 12c = 9(a + b + c) \Rightarrow a = c$

As $b = 50$ and $a + b + c = 100$, $a = 25$ and $b = 25$.

$\therefore$ The quantity of variety A is 25 kg.

Hence, the correct option is (B).

Practice Problems 2

1. Let the present average age of ten advisors as well as their average 3 years ago, be A. Also let the age of the younger man be Y and that of the director replaced by him be D.

$\frac{10A + 30 - D + Y}{10} = A$, as the present average is same as the earlier average.

Hence $D - Y = 30$ years

Hence, the correct option is (D).

2. $an = y + k$ (7)

$bn = x + k$ (8)

where k is the sum of the $(x - 1)$ numbers which are other than y and x.

Subtracting equation (8) from equation (7)

$n(a - b) = y - x \Rightarrow \frac{1}{n} = \frac{a-b}{y-x}$

Hence, the correct option is (B).

3. $\frac{36N + (20)(30)}{N + 20} = \frac{36N - (5)(40)}{N - 5}$

$\Rightarrow N = 10$

Hence, the correct option is (A).

4. Let the number of students in the three classes be n_A, n_B and n_C and the total scores of students in the three classes be T_A, T_B and T_C

$T_A + T_B = 71(n_A + n_B)$

$T_B + T_C = 76\ (n_B + n_C)$

$T_C + T_A = 79\ (n_C + n_A)$

$2T_A + 2T_B + 2T_C = 150n_A + 147n_B + 155n_C$

$\Rightarrow T_A + T_B + T_C = 75n_A + 73.5n_B + 77.5n_C$

Hence $p = \frac{T_A + T_B + T_C}{n_A + n_B + n_C}$

$= \frac{73.5(n_A + n_B + n_C)}{n_A + n_B + n_C} + \frac{1.5n_A + 4n_C}{n_A + n_B + n_C}$

$\Rightarrow \quad p > 73.5$ (9)

p can also be written as,

$p = \frac{T_A + T_B + T_C}{n_A + n_B + n_C}$

$= \frac{77.5(n_A + n_B + n_C)}{n_A + n_B + n_C} - \frac{2.5n_A + 4.n_B}{n_A + n_B + n_C}$

$\Rightarrow \quad p < 77.5$ (10)

Hence $73.5 < p < 77.5$

Hence, the correct option is (D).

5. Let the number of club members who meet for lunch be N. $\frac{1440}{N} + 24 = \frac{1440}{N-2}$; $\frac{1440}{N-2} - \frac{1440}{N} = 24$

Solving, $N = 12$

Hence, the correct option is (B).

6. Total of all numbers = $16n$

Once 5/8th of the numbers are doubled and 3/8th of the numbers are increased by a factor of 10/3,

total of all numbers $= \frac{5n}{8}(2A) + \frac{3n}{8}\left(\frac{10B}{3}\right) + 16n,$

where A and B are the averages of values of the numbers which are respectively doubled and increased by a factor of 10/3.

New average $= \frac{\frac{5n}{4}A + \frac{5n}{4}B + 16n}{n}$.

As A and B are not known, the new average cannot be determined. Hence the percentage increase in the average cannot be determined.

Hence, the correct option is (D).

7. Let $a = 10p + q$ and $b = 10r + s$

Let the sum of all the numbers excluding a and b be x. Let the average of the numbers be A.

$x + 10p + q + 10q + p = 10A$ (11)

$x + 10r + s + 10s + r = 10\ (A + 2.2)$ (12)

Subtracting (11) from (12),

$11\ [(r + s) - (p + q)] = 22$

$\Rightarrow (r + s) - (p + q) = 2$

Hence, the correct option is (B).

8. As each student missed a different number, the sum of the all the numbers added by the students $= (N - 1)$ (sum of the first N natural numbers) $= \frac{(N-1)N(N+1)}{2}$

The number of numbers added by the students in total

$= (N - 1)\ (N)$

$\therefore$ Average $= \frac{(N-1)N(N+1)}{2(N-1)(N)} = 21$

$\Rightarrow \qquad N = 41$

Hence, the correct option is (C).

9. There are at least 30 boys in the class.

$\therefore$ There are more boys than girls in the class.

If the average weight of the entire class is w, $30 < w < 40$ and there are more boys than girls w is closer to 40 than 30, i.e., $w > 35$.

Only Choice (D) satisfies this condition.

Hence, the correct option is (D).

10. Let the numbers of boys and girls in the class be b and g respectively. Let the average weights of the boys and the girls be B and G respectively.

$\frac{Bb + Gg}{b + g} = \frac{B + G}{2}$

$2Bb + 2Gg = Bb + Bg + Gb + Gg$

$B\ (b - g) + G\ (g - b) = 0$

$(B - G)\ (b - g) = 0$

$B = G$ or $b = g$ or both.

Hence, the correct option is (C).

11.

	Alloy A	Alloy B
Copper	$4x$	$5y$
Zinc	$9x$	$6y$
Total wt.	$13x$	$11y$

As the quantity to be drawn from the Alloys A and B to form another alloy C must be a multiple of 13 and 11, let us choose the quantity drawn as LCM of 13 and 11; which is 143. Hence $13x = 11y = 143 \Rightarrow x = 11$ and $y = 13$.

Amount of copper in Alloy $C = 4x + 5y$

$= 4(11) + 5(13) = 109$ kg.

Amount of zinc in alloy $C = 9x + 6y$

$= 9(11) + 6(13) = 99 + 78 = 177$ kg.

Hence the ratio of copper and zinc = 109 : 177

Hence, the correct option is (C).

12. Initial quantity of milk in the vessel = 0.2 (10) = 2 ml

Initial quantity of water in the vessel = 0.8 (10) = 8 ml

After x ml of milk was added, the ratio of milk and water would become 4 : 1

$$x = 4(8) - 2 = 30$$

After y ml of water was added, ratio of milk and water would again become 1 : 4.

$$y = 4\,(4(8)) - 8 = 120$$

$$\therefore \quad x + y = 150$$

Hence, the correct option is (B).

13. The data can be tabulated as:

	Alcohol	**Water**	**Concentration**
Mixture 1	1 part	2 parts	1/3
Mixture 2	4 parts	1 part	4/5
Combined Mixture	1 part	1 part	1/2

If q_1 and q_2 are the quantities of mixture 1 and mixture 2 that are mixed, then by alligation equation,

$$\frac{q_1}{q_2} = \frac{(4/5) - (1/2)}{(1/2) - (1/3)} = \frac{(3/10)}{(1/6)} = \frac{9}{5}$$

It is given that, $q_1 + q_2 = 28$;

Hence $q_1 = \frac{9}{14} \times 28 = 18$ litres

Hence, the correct option is (D).

14. The original mixture had 90 litres of milk and 10 litres of water $\therefore$ It had $\frac{9}{10}$ th of milk

After the first withdrawal of 10 litres, 9 litres of milk and 1 litre of water is lost. $\therefore$ The resulting mixture would have 81 litres of milk. On then adding water the vessel would have $\frac{81}{100}$ th of its contents as milk

We want the milk concentration to be at most $66\frac{2}{3}\%$ (i.e less than or equal to $66\frac{2}{3}\%$), the milk quantity to be at most $66\frac{2}{3}$ litres. After the second replacement the vessel would have $\frac{729}{1000}^{\text{th}}$ of its contents as milk. After the third replacement, the vessel would have $\frac{6561}{10000}^{\text{th}}$ of its contents as milk i.e. 65.61 litres of milk (which is less than $66\frac{2}{3}$ litres)

The least number of times the procedure has to be carried out is 3.

Hence, the correct option is (C).

15. Let the number of times the process is applied be N.

Each application results in the copper quantity dropping by 20% and the tin quantity dropping by 10%. Let the weight of the alloy be 100 kg. This has 80 kg of copper and 20 kg of tin.

The process is applied N times, the quantity of copper in the alloy will be $(0.8)^N 80$ and that of tin in it will be $(0.9)^N (20)$.

$$\frac{(0.8)^N (80)(100)}{(0.8)^N (80) + (0.9)^N (20)} < 70$$

$$\Rightarrow \frac{(0.8)^N (80)}{(0.8)^N 8 + (0.9)^N 2} < 7$$

$$\Rightarrow (0.8)^N (24) < (0.9)^N (14)$$

$$\frac{12}{7} < \left(\frac{9}{8}\right)^N \text{ i.e.} \left(\frac{9}{8}\right)^N > 1\frac{5}{7} \approx 1.7$$

The values of N and the approximate values of $\left(\frac{9}{8}\right)^N$ are tabulated below.

N	$\left(\frac{9}{8}\right)^N$
1	1.125
2	1.27
3	1.42
4	1.60
5	1.80

N must be at least 5 in order that the alloy may have less than 70% copper.

Hence, the correct option is (B).

16. Let the number of goals in the first match be a. The number of goals in successive matches are a, $a + 2$, $a + 4$, $a + 6$, $a + 8$, $a + 10$, $a + 12$, $a + 14$, and $a + 16$

The average number of goals = $\frac{9a+72}{9} = a + 8 = 16$

The average number of goals in the second and eighth matches = $\frac{2a+16}{2} = a + 8 = 16$

Hence, the correct option is (C).

17. Total score = 2240

Total score, excluding the top and the least scores = (69) (30) = 2070

Let the top and the least scores be t and l respectively.

$t + l = 2240 - 2070 = 170$ (13)

$t - l = 70$ (14)

Solving (13) and (14), $T = 120$

Hence, the correct option is (B).

18. Let the number of men in the original group be N.

The total age of the original group = 20 N years

The total age of the new group = (20 N + 50) years

Let the increase in the average be x years.

$20N + 50 = (N + 2)(20 + x)$

$\Rightarrow 10 = x(N + 2)$. Therefore, $N + 2$ and x must be factors of 10. As x is prime, x can be only 2 or 5.

If $x = 2$, $N = 3$. If $x = 5$, $N = 0$, which is not possible.

$\therefore N = 3$

Hence, the correct option is (A).

19. As the average weight of A decreased after the student left, his weight must be more than the average weight of A. As the average weight of B decreased after the student joined, his weight must be less than the average weight of B.

His weight must be between 40 kg and 60 kg

Hence, the correct option is (B).

20. By using the alligation equation,

$$\frac{\text{Number of clerks}}{\text{Number of managers}} = \frac{10000-8000}{8000-3000} = \frac{2}{5}$$

Number of clerks = 50;

$\Rightarrow$ Number of managers = 125

Difference in the number of clerks and managers

= 125 – 50 = 75

Hence, the correct option is (A).

21. Let the number of students in P be $2x$

Let the average marks of P be $5y$

Numbers of students in Q, R and S are $3x$, $6x$ and $4x$ respectively.

Average marks of Q, R and S are $4y$, $3y$ and $2y$ respectively.

Total marks of P, Q, R and S are $10xy$, $12xy$, $18xy$ and $8xy$ respectively. Greatest weighted average

$$= \text{maximum}\left(\frac{10xy+12xy+18xy}{2x+3x+6x}, \frac{10xy+12xy+8xy}{2x+3x+4x}, \frac{12xy+18xy+8xy}{3x+6x+4x}, \frac{10xy+18xy+8xy}{2x+6x+4x}\right)$$

$$\max\left(3\frac{7}{11}y, 3\frac{1}{3}y, 2\frac{12}{13}y, 3y\right) = 3\frac{7}{11}y$$

$$\Rightarrow \quad 3\frac{7}{11}y = 52$$

$$\Rightarrow \quad y = \frac{(52)(11)}{40}$$

Least weighted average = $\frac{38}{13}y = 41.8$

Hence, the correct option is (D).

22. Concentration of alcohol in the 1st mixture = 3/(3 + 4) = 3/7

Concentration of alcohol in the 2nd mixture

= (65)/65 + 79) = (65/144)

Concentration of alcohol in the combined mixture

= 4/(4 + 5) = 4/9

If q_1 and q_2 are the quantities of mixture 1 and mixture 2 that are combined, then, by the equation of alligation,

$$\frac{q_1}{q_2} = \frac{(65/144)-(4/9)}{(4/9)-(3/7)} = \frac{1/144}{1/63} = \frac{63}{144} = \frac{7}{16}$$

Hence, the correct option is (A).

23. Let the average of the ages of the 11 men be A years. Let the ages of the oldest and the youngest be x years and y years respectively.

If any one person leaves the group the maximum and minimum possible average age of the remaining occur if the person aged y years and the person aged x years, respectively, leave

$11A - y = (10)(11) = 110$ (15)

$11A - x = (10)(12) = 120$ (16)

Adding (15) and (16),

$22A - (x + y) = 230$

$$\frac{x+y}{2} = 11A = 11\frac{5}{11}$$

Hence, the correct option is (D).

24. Let the capacity of the vessel be x litres.

(i) quantity of milk in the vessel finally

$$= x\left(\frac{x-9}{x}\right)^2 = x - 17.1$$

$x^2 - 18x + 81 = x^2 - 17.1x$

$x = 90$

Hence, the correct option is (C).

(ii) let the number of further replacements be n.

$$\left(\frac{x-9}{x}\right)^{n+2} \le \frac{75x}{100}$$

$x = 90$

$(0.9)^{n+2} \le 0.75$ (17)

least value of n satisfying (17) is 1

Hence, the correct option is (A).

25. Let x kg be the quantity of T_1 in the mixture.

Cost of the mixture of T_1, T_2 and T_3

$$= \frac{56(x)+64(2x)+80(4x)}{7x} = ₹72 \text{ per kg}$$

Let y kg of T_1 be added to this mixture. The cost of the final mixture

$$= \frac{(72)(4)+56y+80(4y)}{4+5y}$$

This also equals $\frac{87.60(100)}{100+20} = 73$

$$\Rightarrow \frac{288+376y}{4+5y} = 73$$

$$\Rightarrow \frac{4}{11} = y$$

Hence, the correct option is (D).

Chapter 7 Time and Work

Exercises

Practice Problems 1

Directions for questions 1 to 25: Select the correct alternative from the given choices.

1. X men can complete a work in 120 days. If there were 10 men more, the work would be completed in 20 days less. Find the value of X.
(A) 75 (B) 50 (C) 90 (D) 60

2. Nine men can complete a job in 15 days. If a man works thrice as fast as a woman, find the number of days taken by 15 women to complete the job.
(A) 20 (B) 24 (C) 27 (D) 36

3. The ratio of the time taken by A, B, and C to complete a job is 3 : 4 : 6. Find the ratio of the work they can complete in an hour.
(A) 6 : 4 : 3 (B) 4 : 3 : 2
(C) 2 : 3 : 4 (D) 3 : 4 : 6

4. Amar, Bharat, and Charu can complete a job in 12, 24, and 24 days, respectively. If they all work together, how long will they take to complete the same work?
(A) 18 days
(B) 6 days
(C) 20 days
(D) 16 days

5. Adam can complete a job in 25 days. Adam and Chris together can complete it in $9\frac{3}{8}$ days. In how many days can Chris alone complete the job?
(A) $12^5/_8$ (B) 10 (C) 25 (D) 15

6. P and Q can complete a job in 10 days. Q and R can complete it in 12 days. P and R can complete it in 20 days. Who is the slowest of the three workers?
(A) P
(B) Q
(C) R
(D) Cannot be determined

7. Ten men can do a piece of work in 15 days. How many men are needed to complete a work which is five times as large as the first one, in 10 days?
(A) 60 (B) 75 (C) 70 (D) 85

8. Tap X can fill a tank in 10 hours. Tap Y can fill it in 15 hours. If the two taps fill the tank together, what fraction of the tank is filled by X?
(A) 1/10 (B) 1/6 (C) 2/3 (D) 3/5

9. Pipe A can fill an empty tank in 9 hours. Pipe B can empty a full thank in 18 hours. If both pipes are opened simultaneously when the tank is empty, find the time taken to fill the tank (in hours).
(A) 24 (B) 27 (C) 18 (D) 36

10. Raj can build a wall in 18 days and Kiran can do the same in 30 days. After Raj had built half the wall, Kiran joins him. What is the total number of days taken to build the wall?
(A) 24 (B) $14\frac{5}{8}$ (C) $15\frac{1}{2}$ (D) $16\frac{1}{2}$

11. Kaushik is one and a half times more efficient than Ravi. Kaushik can do a piece of work in 20 days. What portion of the total work can both of them together complete in 10 days?
(A) 3/10 (B) 4/5 (C) 9/10 (D) 7/10

12. Had then been one men less, then the number of days required to do a piece of work would have been one more. If the number of mandays required to complete the work is 56, how many workers were there?
(A) 6 (B) 8 (C) 9 (D) 14

13. In 8 days, Peter can do as much work as Pan can do in 12 days. To do a certain job both together take 36 days. In how many days can Pan, working alone, complete the job?
(A) 60 days (B) 80 days
(C) 108 days (D) 90 days

14. X can complete a job in 36 days and Y can complete it in 45 days. Z can complete the job in z days. Z started the job. After 28 days, X and Y joined. The job was completed in 4 more days. Find z.
(A) 40 (B) 35 (C) 30 (D) 50

15. Working in pairs, PQ, QR, and RP can complete a job in 24 days, 20 days, and 30 days, respectively. Find the respective times taken by P, Q, and R individually to complete the same job (in days).
(A) 48, 80, $\frac{240}{7}$ (B) 80, 48, $\frac{240}{7}$
(C) 80, $\frac{240}{7}$, 48 (D) 48, $\frac{240}{7}$, 80

16. A frog was at the bottom of a 80 m deep well. It attempted to come out of it by jumping. In each jump, it covered 1.15 m but slipped down by 0.75 m. Find the number of jumps after which it would out of the well.
(A) 198 (B) 201 (C) 200 (D) 199

17. A man, a woman and a boy can do a piece of work in 2, 4, and 8 days, respectively. How many boys must work together with one man and one woman to complete the work in one day?
(A) 5 (B) 4 (C) 2 (D) 1

18. A machine of type A which has to produce a set of 1500 bolts, can do so in 30 days. The machine breaks down after 10 days. A machine of type B completes the remaining work in 10 days. In 30 days how many bolts can both of them together produce?
(A) 3000 (B) 4500 (C) 6000 (D) 2500

19. In a farm, each cow eats twice as much grass as each sheep. The cost of grass for 10 cows and 40 sheep for

20 days is ₹ 900. Find the cost of grass for 20 cows and 10 sheep for 18 days (in ₹).
(A) 600 (B) 675 (C) 750 (D) 800

20. The cost of grass for 20 cows and 30 sheep for 30 days is ₹720. If the 30 sheep eat double the grass eaten by the 20 cows, then what is the cost of grass eaten by 20 sheep in 15 days?
(A) ₹200 (B) ₹160 (C) ₹240 (D) ₹100

21. George and Gagan together repair a bridge in 45 days and receive ₹13500. If Gagan is three times as efficient as George, what is the amount of money he earns in 10 days?
(A) ₹2000 (B) ₹2250 (C) ₹2500 (D) ₹2750

22. Two pipes *A* and *B* which can fill a tank in 20 and 30 hours, respectively, were opened simultaneously. But there was a leak and it took 3 hours more to fill the tank. In how many hours can the leak empty the tank?
(A) 60 (B) 50 (C) 30 (D) 40

23. Gokul, Govardhan, and Ganesh can do a piece of work in 10, 20, and 30 days, respectively. They begin a new job of similar nature and each of them works on it for one third of the total period of work. If they get ₹6600 for the new job, how much should Govardhan get, given that the amounts distributed are in proportion to the work done by them?
(A) ₹1800 (B) ₹2200 (C) ₹3300 (D) ₹2400

24. Rakesh and Ramesh take 30 days and 60 days, respectively to complete a job. They work on alternate days to complete it with Rakesh starting the job. Find the time in which the job is completed (in days).
(A) 60 (B) 80 (C) 40 (D) 90

25. If Rakesh and Ramesh had instead taken 10 days and 12 days, respectively, to complete the job, find the time in which the job would have been completed (in days).
(A) $10\frac{1}{3}$ (B) $10\frac{5}{6}$
(C) 11 (D) $10\frac{1}{2}$

Practice Problems 2

Directions for questions 1 to 25: Select the correct alternative from the given choices.

1. A man builds one-eighth part of a wall every day. Out of the length of the wall built per day, 20% falls off at the end of the day till the wall is completely built In how many days can he complete the construction of the wall?
(A) 8 (B) 10 (C) $9\frac{1}{5}$ (D) $9\frac{4}{5}$

2. A group of men are building a wall. After half the wall has been built, double the number of men join the original group. The wall gets completed 6 days earlier than scheduled. What is the total number of days the initial group of men would have taken to complete the wall?
(A) 18 (B) 16 (C) 14 (D) 12

3. Raman can do a piece of work in half the time taken by Kapil. Sunil can do the same work in one-third of the time taken by Raman. All three of them work on it for 30 days after which Kapil leaves. Sunil and Raman complete the remaining work in 18 more days. How many days would it take for Raman alone to complete the total work?
(A) 52 (B) 414 (C) 138 (D) 207

4. A group of 50 salesmen plan to achieve their target for the next 30 days by working 12 hours a day. Due to various reasons, they put in only 10 hours a day for the first 15 days. Now, if 10 men leave and the rest continue working for only 10 hours a day, how many days more than the initially estimated time will they require to meet their target?
(A) $11\frac{1}{4}$ (B) $12\frac{3}{4}$ (C) $13\frac{1}{4}$ (D) $13\frac{3}{4}$

5. *P* can complete a job in 27 days. He starts it and after three days *Q* joins him. They work together for six days. *P* then leaves and *R* takes his place. *Q* and *R* complete the job in twelve more days. If *Q* takes at most 54 days to complete the job, then which of the following cannot be a possible value of the number of days taken by *R* to complete it?
(A) 36 (B) 34 (C) 40 (D) 38

6. Vivek, Rameshwar, and Bhuvan divide a work amongst themselves in the ratio of 2 : 3 : 5. Their rates of work are in the ratio 1 : 2 : 3. It takes Vivek 12 days to complete his part. What is the amount of work completed by them in 8 days from the start?
(A) 29/40 (B) 11/45
(C) 4/5 (D) 31/45

7. A certain structure has to be erected. The workers erecting the structure have to work a fixed number of hours each day. They would be paid fixed lab our wages for each hour of work. If the labor wage per hour of each worker was one-sixth more and the number of hours of work $\frac{1}{35}$ th less, the total wage of the workers would be _____.
(A) $\frac{2}{15}$ th more (B) $\frac{1}{5}$ th more
(C) $\frac{1}{15}$ th more (D) $\frac{2}{15}$ th less

8. Pradeep can work 2/3 times as fast as Abishek and Antony together. Pradeep and Antony together can work twice as fast as Abishek. If Antony alone takes 45 days to complete a job, how long (in days) would Pradeep and Abishek individually take to complete the job, respectively?
(A) 30, 36 (B) 36, 30
(C) 45, 60 (D) 60, 45

9. Jadeja, Bhangar, Balaji, and Dravid can do a piece of work in 8, 16, 32, and 64 days, respectively. Dravid starts the work and Balaji joins him after one-fourth of the work is done, Bhangar joins them after half the work is done and Jadeja joins them after three-fourth work is done. How many days does it take to complete the work?
(A) 30 (B) 32
(C) $25\frac{2}{35}$ (D) $24\frac{24}{35}$

10. The ratio of the rates of doing work of *P*, *Q,* and *R* is 3 : 4 : 5. If they completed a job working together, what part of it did *P* complete?
(A) $\frac{1}{4}$ (B) $\frac{20}{47}$
(C) $\frac{1}{3}$ (D) None of these

11. Machines *P*, *Q,* and *R* can do a piece of work in 20, 30, and 60 days, respectively. Machines *P*, *Q,* and *R* start the work together. Machine *P* goes out of order after 5 days. After three more days machines *Q* and *R* also go out of order. Machine *P* got repaired by then and it completes the remaining work. What portion of the total work did machine *P* do?
(A) 3/4 (B) 11/20 (C) 7/20 (D) 3/5

12. A man started a job. Starting from the second day, each day a new man joined with which the capacity of each man doubled. The job was completed in 6 days. On which day will the job be completed if the joining of a new man on a day results in each man working at thrice the rate as he did on the previous day?
(A) 6th (B) 5th (C) 4th (D) 3rd

13. Anil can complete a piece of work in 6 days and Mukesh can complete it in 8 days each working alone. Consider two cases in which they work on alternate days. The first case when Anil starts and the second when Mukesh starts. What is the difference of the number of days taken in the two cases?
(A) $\frac{1}{2}$ (B) $\frac{1}{4}$ (C) 1 (D) $1\frac{3}{4}$

14. Anwar would have to incur ₹600, ₹900, and ₹1200 as the expenses if he got a job done by *A*, *B,* and *C,* respectively. The daily wages of *A*, *B,* and *C* are ₹100, ₹60, and ₹40, respectively. Find the cost to Anwar of getting the job done by all three of them (in ₹).
(A) 600 (B) 800
(C) 750 (D) 900

15. Six small pumps and three large pumps are fitted to a tank. Each small pump works at two-third the rate of each large pump. If all the pumps work together, what fraction of the time taken by a single large pump, will they take to fill the tank?
(A) 1/6 (B) 1/7 (C) 1/8 (D) 1/9

16. Two pipes *P* and *Q* can fill a cistern in 12 and 18 hours respectively. Both the pipes were opened at 10:00 a.m. and the cistern was full at 6:00 p.m. What could be the minimum possible duration for which one of the pipes must have been closed during that interval?
(A) 4 hours (B) $2\frac{1}{2}$ hours
(C) 2 hours (D) $\frac{4}{3}$ hours

17. A pipe can fill a tank in 4 hours, while a leak which is at one-fourth the height of the tank can empty upto that part in 2 hours. If both are operated simultaneously and initially the tank is full, then when will it be one-fourth full?
(A) 2 hours (B) $2\frac{1}{3}$ hours
(C) $1\frac{1}{2}$ hours (D) 6 hours

18. *N* taps numbered from 1 to *N* are fitted to a tank. The rate at which the nth tap, for $n = 1, 2, \ldots, N$ fills the tank, equals the sum of the rates of all the taps numbered below it. If the sixth tap can fill it in 80 minutes, find the time in which the ninth tap can fill it (in minutes).
(A) 10 (B) 5 (C) 20 (D) 40

19. A piece of work when done by a man, a woman, and a child costs ₹720, ₹810, and ₹1080, respectively. Their respective daily wages are ₹60, ₹45, and ₹30. If a family consisting of a husband, wife, and their child is engaged to complete this work, how much would it cost?
(A) ₹1080 (B) ₹780
(C) ₹810 (D) ₹60

20. There are 5 milk tanks in a dairy farm—*A*, *B*, *C*, *D*, and *E*. Each contains 3300 litres of milk. Milk is pumped from one tank to the other as follows.
From *A* to *C* @ 130 litres/minute
From *B* to *E* @ 100 litres/minute
From *B* to *A* @ 80 litres/minute
From *E* to *C* @ 210 litres/minute
From *D* to *B* @ 30 litres/minute
From *C* to *D* @ 250 litres/minute
Which tank would be the first to get emptied? How long will it take to get emptied?
(A) *D*, 15 minutes (B) *B*, 22 minutes
(C) *A*, 66 minutes (D) *E*, 30 minutes

21. *P* and *Q* are filling pipes which can fill a tank in 15 and 20 minutes, respectively. *R* is an emptying pipe which can empty the full tank in 30 minutes. The three pipes are operated continuously one after the other in the order of *P*, *Q,* and *R*, each being kept opened for 2 minutes until the tank is filled. After how much time will the tank be full?

(A) 30 minutes (B) $32\frac{2}{3}$ minutes

(C) 36 minutes (D) 34 minutes

22. Two taps can normally fill a cylindrical tank in 16 hours and 48 hours. But a leak which can empty the tank in 24 hours is present at three-fourth of the tank's height from the base. Find the time taken to fill the tank if the taps are opened simultaneously (in hours).

(A) 12 (B) 15 (C) 18 (D) 21

23. The efficiency of a man is reduced by half every two hours. At maximum efficiency, he could have completed the job in 150 hours. How many hours does it take him to complete the job, if his efficiency becomes maximum after every 8 hours, and then reduces as mentioned above?

(A) 300 (B) 360

(C) 320 (D) 600

24. *P*, *Q,* and *R* are three machines. They produce electronic gadgets. The ratio of the rates of *P*, *Q,* and R is 3 : 4 : 5. *P* worked for 6 days, *Q* worked for 8 days, and *R* worked for 10 days. They manage to produce 400 gadgets. How many gadgets would they have produced in the same time if *P*'s rate doubled and *Q*'s rate tripled?

(A) 546 (B) 728

(C) 637 (D) 910

25. *P* and *Q* can complete a job in 12 days. *Q* and *R* can complete it in 20 days. *R* and *P* can complete it in 15 days. Find the time taken by *P*, *Q,* and *R* working together, to complete it.

(A) 10 days (B) 20 days

(C) 5 days (D) 8 days

HINTS/SOLUTIONS

Practice Problems I

1. x men $\rightarrow$ 120 days

$x + 10$ men $\rightarrow$ 100 days

$x\,(120) = (x + 10)\,100$

$6x = 5x + 50 \Rightarrow x = 50$.

Hence, the correct option is (B).

2. Job = (15) (9) = 135 mandays = (135) (C) = women days. Time taken by 15 women to complete the job = $\frac{405}{15}$ or 27 days.

Hence, the correct option is (C).

3. Let the job be 1 unit.

Let the times taken by A, B and C the complete it be $3x$ hours, $4x$ hours and $6x$ hours respectively.

Parts of the job completed by A, B and (in a hour are $\frac{1}{3x}$ units, $\frac{1}{4x}$ units and $\frac{1}{6x}$ units respectively.

Required ratio = $\frac{1}{3x} : \frac{1}{4x} : \frac{1}{6x} = 4 : 3 : 2$.

Hence, the correct option is (B).

4. Work done by Amar one day = 1/12

Work done by Bharat in 1 day = 1/24

Work done by Charu in 1 day = 1/24

Work done by 3 of them in one day $= \frac{1}{12} + \frac{1}{24} + \frac{1}{24} = \frac{1}{6}$

$\therefore$ They take 6 days to finish the work.

Hence, the correct option is (B).

5. Per day

Adam = 1/25

Adam + Chris = 8/75

$\therefore$ Chris $= \frac{8}{75} - \frac{1}{25} = \frac{1}{15}$

$\therefore$ Chris takes 15 days.

Hence, the correct option is (D).

6. P and Q take less time to complete the job compared to Q and R. $\therefore$ R is slower than P. We should new compare R and Q (not P and Q).

P and Q take less time to complete the j*o*b compared to P and R. $\therefore$ R is slower than Q.

$\therefore$ R is the slowest of the 3 workers.

Hence, the correct option is (C).

7.

Men	Work	Days
10	1	15
X	5	10

1 men $\rightarrow$ 1/150 (one day's work)

10 days work = 10/150 = 1/15

i.e. 15 men for one work men needed = $15 \times 5 = 75$.

Hence, the correct option is (B).

8. Part of the tank filled in a hour by both taps

$= \frac{1}{10} + \frac{1}{15} = \frac{5}{6}$

$\therefore$ The tank will be filled in 6 hours by them. In this time X can fill $\frac{6}{10} = \frac{3}{5}$th of the tank.

Hence, the correct option is (D).

9. Part of the tank filled in a hour by both pipes

$= \frac{1}{9} - \frac{1}{18} = \frac{1}{18}$

$\therefore$ The tank will be filled in 18 hours by them.

Hence, the correct option is (C).

10. In one day Raj can build $\frac{1}{18}$th of the wall.

Kiran can build $\frac{1}{30}$th of the wall.

To complete $\frac{1}{2}$the work Raj takes 9 days.

Remaining work = $\frac{1}{2}$.

In one day both can together complete

$\frac{1}{18} + \frac{1}{30} = \frac{48}{18 \times 30} = \frac{4}{45}$

To complete $\frac{1}{2}$ the work, it takes them $\frac{1}{2}\left(\frac{45}{4}\right) = \frac{45}{8}$ days.

Total number of days = $\frac{45}{8} + 9 = 14\frac{5}{8}$ days.

Hence, the correct option is (B).

11. Rate of work and time taken to do a work are in inverse proportion.

Hence, Kaushik is one and a half times more efficient than Ravi implies, Ravi takes one and a half time more time than Kaushik, to do the same work.

Kaushik takes 20 days.

Hence Ravi takes $20(1 + 1.5) = 50$ days to do the same work.

In 10 days, amount of work completed by both together

$= \frac{10}{20} + \frac{10}{50} = \frac{70}{100} = \frac{7}{10}$ th of the work.

Hence, the correct option is (D).

12. Initially let there be x men who take y days.

$xy = 56$ (15)

$(x - 1)(y + 1) = xy$ (16)

$\therefore x - y = 1$ (17)

From (15) and (17), as $56 = 8 \times 7$,

$x = 8$ and $y = 7$.

Hence, the correct option is (B).

13. Let Peter and Pan take 8 and 12 days respectively to do 1 unit work. Let the job mentioned in the problem be x units of work. So, they take $8x$ days and $12x$ days to do x units of work respectively. Both together can complete x units of work in $\frac{(8x)\times(12x)}{20x} = 4.8x$ days

Given that $4.8x = 36 \Rightarrow x = 7.5$

$\therefore$ To do x units of work, Pan working alone, takes (12) (7.5) = 90 days.

Hence, the correct option is (D).

14. The total work completed is

$$28\left(\frac{1}{z}\right)+\left(\frac{1}{36}+\frac{1}{45}+\frac{1}{z}\right)4 = 1 \Rightarrow z = 40.$$

Hence, the correct option is (A).

15. Parts of the job done by P and Q, Q and R and P and R in a day are $\frac{1}{24}, \frac{1}{20}$ and $\frac{1}{30}$ respectively

$$\therefore P + Q = \frac{1}{24} \quad (18)$$

$$Q + R = \frac{1}{20} \quad (19)$$

$$P + R = \frac{1}{30} \quad (20)$$

Adding (18) and (19) and subtracting (20) from the sum, $2Q = \frac{7}{120}$

$$Q = \frac{7}{240}$$

From (18), $P = \frac{1}{80}$

From (19), $R = \frac{1}{48}$

$\therefore P$, Q and R take 80 days, $\frac{240}{7}$ days and 48 days to complete the job respectively.

Hence, the correct option is (C).

16. Each jump followed by a slip would have enabled it to cover 0×4 m. In 198 jumps, it would have covered $(0 \times 4)(198) = 79 \times 2$ m. In the next jump, it would have come out of the well.

Hence, the correct option is (D).

17. Let x boys be required to work, then

$$x\left(\frac{1}{8}\right)+\frac{1}{2}+\frac{1}{4} = 1$$

$$\frac{x+6}{8} = 1, x = 2.$$

Hence, the correct option is (C).

18. Number of bolts produced by type A in 10 days

$= \frac{10}{30}(1500) = 500$; Remaining = 1000

Type B produces 1000 bolts in 10 days.

$\Rightarrow$ 3000 bolts are produced in 30 days by type B.

Together they can produce (3000 + 1500)

= 4500 bolts in 30 days.

Hence, the correct option is (B).

19. Let the required cost be ₹x

10 cows = 20 sheep

$\therefore$ 10 cows + 40 sheep = 60 sheep

20 cows + 10 sheep = 50 sheep

Let the cost of grass for each sheep be ₹y/day.

$(60y)(20) = 900$

$$y = \frac{3}{4}$$

$x = (50y)(18) = 675$.

Hence, the correct option is (B).

20. The cost of grass for 20 cows and 30 sheep for 30 days

= ₹720.

Given 30 sheep eat double the grass eaten by 20 cows.

$\Rightarrow$ Cost of grass eaten by 30 sheep in 30 days

$= \frac{2}{3} \times 720 =$ ₹480

The cost of grass eaten by 1 sheep in one day

$= \frac{480}{30\times30} = \frac{8}{15}$.

The cost of grass eaten by 20 sheep in 15 days

$= \frac{8\times20\times15}{15} =$ ₹160.

Hence, the correct option is (B).

21. Money received for one day's work

$= \frac{13500}{45} = 300$

Amount of money received by Gagan $= \frac{3}{4} \times 300$

= 225 per one day.

For 10 days, Gagan receives $225 \times 10 =$ ₹2,250.

Hence, the correct option is (B).

22. Pipe A can fill the tank in 20 hours. Pipe B in 30 hours.

Total time for both A and B together to fill the tank

$= \frac{20\times(30)}{30+20} = 12$ hours.

But it took 3 hours more i.e. 15 hours in total.

Let the number of hours in which the leak can empty the tank be x.

$\Rightarrow \frac{1}{x} = \frac{1}{12} - \frac{1}{15} \Rightarrow x = \frac{60}{5-4} = 60$ hours.

Hence, the correct option is (A).

23. Ratio of work done by Gokul, Govardhan and Ganesh in equal durations of time

$= \frac{1}{10} : \frac{1}{20} : \frac{1}{30}$ respectively i.e., 6 : 3 : 2

$\Rightarrow$ Govardhan gets $\frac{3}{11}(6600) = ₹1{,}800$.

Hence, the correct option is (A).

24. Part of the job completed in the first two days

$= \frac{1}{c} = \frac{1}{10}$ job $= 20\left(\frac{1}{20}\right)$

$\therefore$ time taken to complete the job = 20 (2) = 40 days

Note: The time taken to complete the job would be the same if Ramesh starts the job. Whenever the part of the job completed by two workers working on alternate days in the first two days

$= \frac{1}{n}$ where n is an integer, the time taken to complete the job = $2n$ days.

Hence, the correct option is (C).

25. Part of the job completed in the first 2 days = $\frac{11}{60}$

After S cycles of 2 days, part of the job completed = $\frac{11}{12}$.

This is completed in 10 days. Remaining part = $\frac{1}{12}$

On the 11th day, Rakesh would work. He would complete the remaining part in $\frac{5}{6}$ th of that day.

$\therefore$ Total time = $\frac{11}{2c} = \frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \frac{11}{20}$.

Hence, the correct option is (B).

Practice Problems 2

Solutions for questions 1 to 25:

1. Portion of wall built per day = $\frac{1}{8}$ th = 12.5%

20% of this (i.e. 2.5% of the wall) falls off.

Therefore 10% of the wall is completed every day.

To determine whether the whole of the 10th day is needed for work, consider work done in 9 days.

In 9 days 90% of the wall is built. On the 10th day, one-tenth has to be built. So, he takes = $\frac{1/10}{1/8} = \frac{4}{5}$ days

So, he takes $9\frac{4}{5}$ days to construct the wall.

Hence, the correct option is (D).

2. Let x be the number of men initially engaged.

Let d be the number of days for which they worked. Hence, x men completed half of the work in d days (21)

Because $2x$ men joined, total number of working men = $x + 2x = 3x$.

This new group completes the remaining work 6 days earlier; i.e., $3x$ men complete half of the work in $(d-6)$ days (22)

As the product of men and number of days is the same in both cases, being equal to work done,

$(x)(d) = 3x(d-6)$;

$\Rightarrow d = 3(d-6), 2d = 18$ (23)

But, $2d$ is the duration required by x men to complete the work.

Hence, the correct option is (A).

3. Let the number of days taken by Kapil be x.

Work done by Kapil in one day = $\frac{15}{1} = 8\frac{3}{4}$

Work done by Raman in one day = $\frac{2}{x}$

Work done by Sunil in one day = $\frac{2\times3}{x} = \frac{6}{x}$

Work done by 3 of them together in 30 days.

$= \frac{30}{x} + \frac{2\times30}{x} + \frac{6\times30}{x} = \frac{270}{x}$

Work done by Sunil and Raman in 18 days = $\frac{1}{24}$

Total work = $\frac{270}{x} + \frac{144}{x} = 1 \Rightarrow x = 414$

$\therefore$ Time taken by Raman = $\frac{x}{2}$ i.e. 207 days.

Hence, the correct option is (D).

4. Total work = 30 (12) (50) man hours = 18000 man hours.
Amount of work done in 15 days putting 10 hours a day = 15 (10) (50) = 7500 man hours.

Work remaining = 18000 − 7500 = 10,500 man hours.

Number of men remaining = 40

Number of hours = 10 per day.

Number of days needed $= \frac{10500}{40(10)} = 26\frac{1}{4}$ days.

$\therefore$ They require $11\frac{1}{4}$ more days to complete the work.

Hence, the correct option is (A).

5. P works for a total of 9 days. Q works for a total of 18 days. R works for a total of 12 days. Let the times taken by Q and R to complete the job be q days and r days respectively.

$$\frac{9}{27}+\frac{18}{q}+\frac{12}{r}=1$$

$$q \le 54$$

$$\therefore \quad r \ge 36$$

Only Choice (B) violates this condition.

Hence, the correct option is (B).

6. From the ratio (2 : 3 : 5),

Vivek does $\frac{1}{a}=\frac{3}{10}$ of work in 12 days.

$\Rightarrow$ Vivek does full work in $12 \times \frac{10}{2}$ i.e., 60 days.

From the ratio 1 : 2 : 3, it can be decided that Rameshwar does in 30 days. Bhuvan does in 20 days.

In 8 days they complete

$$\frac{8}{60}+\frac{8}{30}+\frac{8}{20}=\frac{11}{6a}=\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=\frac{11}{20}.$$

Hence, the correct option is (C).

7. Let the number of days that the workers have to work to complete the erection of the structure be D. Let the hourly wage be W rupees per worker. Let us say they work H hours per day. Let the number of workers be N.

Total wages of the workers presently $= D(N\,W\,H) = N\,D\,W\,H$.

If the changes mentioned take place, the new total wages will be

$$ND\left(\frac{7W}{6}\right)\left(\frac{34H}{35}\right) \quad \text{i.e.,} \frac{17}{15} \; N\,D\,W\,H.$$

This is $\frac{2}{15}$ th more than the present total wages of the workers.

Hence, the correct option is (A).

8. Let the work done by Pradeep, Abishek and Antony in one day be a, b and c respectively.

Then, $a = \frac{2}{3}(b+c)$ (24)

$$a + c = 2b \quad (25)$$

$$c = \frac{1}{45} \quad (26)$$

From (24), $3a - 2b = 2c$ or $3a - 2b = \frac{2}{45}$

and from (25), $2b - a = \frac{1}{45}$

Adding, we get $\therefore 2a = \frac{3}{45}$ $a = \frac{1}{30}$ and $b = \frac{1}{36}$

So, Pradeep takes 30 days and Abishek takes 36 days.

Hence, the correct option is (A).

9. Dravid starts the work and completes 1/4th work in 16 days. Balaji joins and together they complete 1/4th work more. For them to complete 1/4th it takes

$$\frac{1/4}{1/64+(1/32)}=\frac{16}{3} \text{ days.}$$

Then Bhangar joins and Dravid, Balaji and Bhangar complete another 1/4th of the work. For them to complete 1/4th it takes

$$\frac{1/4}{1/64+1/32+1/16}=\frac{16}{7}.$$

Then Jadeja joins and all four of them complete the remaining (1/4)th work.

For them to complete 1/4th, it takes

$$\frac{1/4}{1/64+1/32+1/16+1/8}=\frac{16}{15} \text{ days.}$$

Total number of days

$$= 16\left(1+\frac{1}{3}+\frac{1}{7}+\frac{1}{15}\right)=24\frac{24}{35} \text{ days.}$$

Hence, the correct option is (D).

10. P's rate $= \frac{1}{4}$ (Combined rate of P, Q and R)

$\therefore$ P must have done $\frac{1}{4}$ th of the job.

Hence, the correct option is (A).

11. In one day, Machine P does $\frac{1}{20}$ th of the work.

Machine Q does $\frac{1}{30}$ th of the work

Machine R does $\frac{1}{60}$ th of the work

Together in 5 days they complete

$$\frac{5}{20}+\frac{5}{30}+\frac{5}{60}=\frac{(1+2+3)}{60}(5)=\frac{1}{2} \text{ portion of the work.}$$

In 3 days Q and R complete $\frac{3}{30}+\frac{3}{60}=\frac{3+6}{60}=\frac{9}{60}$

$$\text{Remaining} = 1 - \frac{30}{60}-\frac{9}{60}=\frac{21}{60}$$

Portion of work done by $P = \frac{5}{20}+\frac{21}{60}=\frac{36}{60}=\frac{3}{5}$.

Hence, the correct option is (D).

12. Let the work done by Rohit on the first day be 1 unit

Job = 1 + 2 (2) + 3 (D) + 4 (8) + 5 (16) + 6 (32) = 312 units.

If the men worked at twice the rate, part of the job completed in the first 4 days = 1 + 2 (C) + 3 (9) + 4 (27) = 142 units. On the 5th day, 5 (81)

= 405 units can be done.

∴ The job will be completed on the 5th day.

Hence, the correct option is (B).

13. **Case 1:** Anil starts

Anil $\frac{1}{6}$ 2 days

Mukesh $\frac{1}{8}$

In a period of 2 days $\frac{7}{24}$ work is done

In 3 such time periods i.e. 6 days $\frac{21}{24}$

i.e. $\frac{7}{8}$ work is done. Remaining work is $\frac{1}{8}$ and it is done by Anil in $\frac{6}{8}=\frac{3}{4}$.

Hence a total of $6\frac{3}{4}$ days.

By a similar calculation, if Mukesh starts, a total of 7 days are required. Difference $=\frac{1}{4}$ days.

Hence, the correct option is (B).

14. Time in which A can complete the job $=\frac{600}{100}=6$ days.

Time in which B can complete the job $=\frac{900}{60}=15$ days

Time in which C can complete the job $\frac{1200}{40}=30$ days

Part of the job which can be done by A, B and C in a day

$=\frac{1}{6}+\frac{1}{15}+\frac{1}{30}=\frac{4}{15}$

∴ They would take $\frac{15}{4}$ days to complete it.

Cost to Anwar $=\left(\frac{15}{4}\right)(100+60+40)=₹750$.

Hence, the correct option is (C).

15. Let the rate at which each large pump fills the tank be l litres/hr.

Rate at which each small pump fills the tank

$=\frac{2}{3}l$ litres/ hr.

Rate at which six small pumps and three large pumps fill the tank $=6\left(\frac{2}{3}l\right)+3l=71$ litres/hr.

∴ They will take $\frac{1}{7}^{\text{th}}$ the time taken by a single large pump to fill it.

Hence, the correct option is (B).

16. The time from 10:00 a.m. to 6:00 p.m. = 8 hours.

So, one of the pipes worked for 8 hours and the other pipe worked for x hours.

$$\frac{8}{12}+\frac{x}{18}=1 \quad (27)$$

or $$\frac{x}{12}+\frac{8}{18}=1 \quad (28)$$

We get $x = 6$ from equation (27)

So, pipe Q was closed for 2 hours or

From Eqn (28), $x=12\left[1-\frac{8}{18}\right]=\frac{20}{3}$

So, pipe P was closed for $\left[8-\frac{20}{3}\right]=\frac{4}{3}$ hrs

Minimum possible time = $1\frac{1}{3}$ hours.

Alternate method:

Part of the tank filled, in 1 hour, by the two pipes

$=\frac{1}{12}+\frac{1}{18}=\frac{5}{36}$ th part.

Hence, time required to fill $=\frac{36}{5}=7.2$ hour.

Actual time taken to fill = 6:00 p.m. – 10:00 a.m. = 8 hours.

Extra time taken 8 – 7.2 = 0.8 hours

Extra time was needed because one pipe was closed for some time.

Hence, part filled by 2 pipes in 0.8 hours = Part not filled due to closure of one tap

i.e. Part not filled $=0.8\times\frac{5}{36}=\frac{1}{9}$ th part.

Minimum time of closure is to be found out;

⇒ The faster pipe was closed.

Rate of filling of the faster pipe = 1/12th/hour.

Hence duration of closure = (1/9)/(1/12) = (12/9)

= (4/3) hours.

Hence, the correct option is (D).

17. To fill $\frac{3}{4}$ th part, the filling pipe takes 3 hours.

To empty $\frac{3}{4}$ th part, the emptying pipe takes 2 hours.

∴ When both are operated simultaneously, they will empty $\frac{3}{4}$ th part of the full tank in $\frac{1}{2}-\frac{1}{3}=\frac{1}{6}$

i.e., 6 hours.

Hence, the correct option is (D).

18. Let the rate of filling of the first tap be x litres/hr

Let the rate of filling of the nth tap be denoted by t_n

$t_2 = x$

$t_3 = 2x$

$t_4 = 4x = 22x$

$t_5 = 8x = 23x$

$\therefore t_n = 2^n - 2x$

$\frac{t_6}{t_9} = \frac{2^3 x}{2^6 x} = \frac{1}{8}$

∴ Time taken by the ninth tap $= \frac{1}{8}(80) = 10$ minutes.

Hence, the correct option is (A).

19. Number of days required:

by a man $= \frac{720}{60} = 12$

by a woman $= \frac{810}{45} = 18$; and by a child $= \frac{1080}{30} = 36$

If 1 man, 1 woman and 1 child work together,

$\frac{1}{12}+\frac{1}{18}+\frac{1}{36}$ work will be done in 1 day $= \frac{1}{6}$ th

∴ 6 days will be required.

Cost = 6 × (60 + 45 + 30) = ₹810.

Hence, the correct option is (C).

20. Effective out flow from A = −130 + 80 = −50 litres/minute.

Effective outflow from B = −100 − 80 + 30 = −150 litres/minute.

Effective outflow from C = −250 + 130 + 210 = +70 litres/minute. Effective inflow occurs into C).

Effective outflow from D = −30 + 250 = 220 litres/minute (effective inflow occurs into D). Effective out flow from E = −210 + 100 = − 110 litres / minutes.

Effective out flow occurs from A, B and E only.

Maximum effective outflow occurs from B.

∴ B would be the first to get emptied time taken to empty

$B = \frac{3000}{150} = 22$ minutes.

Hence, the correct option is (B).

21. In one cycle of 6 minutes the part of the tank that is filled $= \frac{2}{15}+\frac{2}{20}-\frac{2}{30}=\frac{10}{60}=\frac{1}{6}$

In order to decide whether the 6th cycle is needed in full or in part, consider the situation after 5 cycles.

$5 \times \frac{1}{6} = \frac{5}{6}$ th of the tank is filled, in the first five cycles.

5 × 6 = 30 minutes are over.

Part of tank to be filled $= \frac{1}{6}$ th.

After next 2 minutes, the part of the tank that is empty

$= \frac{1}{6}-\frac{2}{15}=\frac{1}{30}$, as pipe P fills (2/15)th part.

Q can fill this part in $20 \times \frac{1}{30} = \frac{2}{3}$ minutes,

⇒ 6th cycle is not required in full.

∴ The tank is filled in $30 + 2 + \frac{2}{3} = 32\frac{2}{3}$ minutes.

Hence, the correct option is (B).

22. Required time = Time taken to fill the bottom $\frac{3}{4}$ th + time taken to fill the top $\frac{1}{4}$ th

The leak will not affect the filling of the bottom $10\sqrt{3}$ cm

∴ Time taken to fill the bottom part is $10\sqrt{3}$ cm (time taken by the taps to fill the tank). Time taken to fill the top part is $5\sqrt{3}$ cm (time taken by the taps to fill the tank along with the leak)

Time taken to fill the bottom part

$= \frac{3}{4}\left(\frac{(16)(48)}{16+48}\right) = 9$ hours

Part of the tank which can be filled by the taps and the leak each hour = $5\sqrt{3}$ cm

∴ Time taken to fill the top $\frac{1}{4}$ th $= \frac{1}{4}(24) = 6$ hours

∴ Required time = 15 hours.

Hence, the correct option is (B).

23. Given that the man can complete the jo*b* in 150 hours at maximum efficiency. i.e. at maximum efficiency he can do $\frac{1}{15}$ of the work per hour.

(He works each 2 hours with the same efficiency)

∴ In 8 hours he can do :

$2\left[\frac{1}{150}+\frac{1}{2(150)}+\frac{1}{4(150)}+\frac{1}{8(150)}\right]$

$= \frac{2\times 15}{8\times 150} = \frac{1}{40}$ th of the work.

Hence, total number of hours = 8(40) = 320.

Hence, the correct option is (C).

24. Let the efficiencies of P, Q and R be $3x$ gadgets/day, $4x$ gadgets/day, $5x$ gadgets/day.

$(3x)(6) + (4x)(8) + (5x)(10) = 400 \; x = 4$

Required number of gadgets

$= (6x)(6) + (12x)(8) + (5x)(10) = 728.$

Hence, the correct option is (B).

25. Parts of the job, which can be completed, by P and Q, Q and R and R and P in a day are $\frac{1}{12}$ th, $\frac{1}{20}$ th and $\frac{1}{15}$ th respectively.

$\therefore$ 2 P's, 2 Q's and 2 R's can complete $\frac{1}{12} + \frac{1}{20} + \frac{1}{15}$

$= \frac{1}{5}$ th of the job in a day.

$\therefore$ P, Q and R can complete $\frac{1}{10}$ th of the job in a day.

$\therefore$ They can complete the job in 10 days.

Hence, the correct option is (A).

Chapter 8 Time and Distance

Exercises

Practice Problems 1

Directions for questions 1 to 30: Select the correct alternative from the given choices.

1. Convert the following speeds into meters per second

(a) 36 km/hr

(A) 10 (B) 12 (C) 15 (D) 20

(b) 12.6 km/hr

(A) 3.5 (B) 4 (C) 0.35 (D) 6

(c) 252/35 km/hr

(A) 2.2 (B) 2.4 (C) 2 (D) 2.6

2. If a man runs at 6 metres per second, what distance (in km) will he cover in 3 hours and 45 minutes?

(A) 81 (B) 96 (C) 91 (D) 27

3. Travelling at 5/6th of his usual speed a man is 10 minutes late. What is the usual time he takes to cover the same distance?

(A) 50 minutes (B) 70 minutes

(C) 1 hour (D) 75 minutes

4. X and Y are 270 km apart. At 9:00 a.m, buses A and B left X and Y for Y and X, respectively. If the speeds of A and B are 50 kmph and 40 kmph, respectively, find their meeting time.

(A) 11:00 a.m (B) 12:00 p.m

(C) 1: 00 p.m (D) 2:00 p.m

5. Car A left X for Y at 9:00 a.m. Car B left Y for X at 10:00 a.m. XY = 180 km. Speeds of A and B are 30 kmph and 20 kmph, respectively. Find their meeting time.

(A) 12:36 p.m. (B) 1:36 p.m.

(C) 1:00 p.m (D) 2:00 p.m

6. Ashok left X and reached Y in 4 hours. His average speed for the journey was 90 kmph. Find the distance between X and Y (in km).

(A) 180 (B) 360 (C) 720 (D) 900

7. Alok travelled from Hyderabad to Tirupati at 60 kmph and returned at 90 kmph. Find his average speed for the journey (in kmph).

(A) 72 (B) 75 (C) 66 (D) 78

8. What is the time taken by a train 650 $\overline{m}$ long travelling at 72 km/hr to cross a 750 $\overline{m}$ long platform?

(A) 60 sec (B) 65 sec

(C) 70 sec (D) 75 sec

9. What is the time taken by a 750 $\overline{m}$ long train travelling at 99 km/hr to cross a boy running at 9 km/hr towards the train?

(A) 30 sec (B) 33 sec

(C) 36 sec (D) 25 sec

10. In a 200 $\overline{m}$ race, Eswar gives Girish a start of 10 $\overline{m}$ and beats him by 10 $\overline{m}$. Find the ratio of their speeds.

(A) 1 : 1 (B) 9 : 10 (C) 10 : 9 (D) 19 : 20

11. In a 100 $\overline{m}$ race, Ganesh beats Harish by 10 $\overline{m}$ or 2 seconds. Find Harish's speed (in m/sec).

(A) 5 (B) $5\frac{5}{9}$

(C) $4\frac{1}{2}$ (D) 6

12. In a 100 $\overline{m}$ race, Akbar gives Birbal a start of 2 seconds. Birlbal covers 10 $\overline{m}$ by the time Akbar starts. If both of them finish together, find Akbar's speed. (in m/sec)

(A) 5 (B) $5\frac{5}{9}$

(C) $4\frac{1}{2}$ (D) 4

13. In a race, P beats Q by 20 seconds. Q beats R by 30 seconds. By how many seconds did P beat R?

(A) 44 (B) 25 (C) 50 (D) 36

14. In a 100 $\overline{m}$ race, A beats B by 10 $\overline{m}$ and B beats C by 20 m. Find the distance by which A beats C (in m).

(A) 30 (B) 28 (C) 32 (D) 36

15. Anand can row a boat in still water at a speed of 5 kmph. The speed of the stream is 3 kmph. Find the time taken by him to row 40 km downstream (in hours).

(A) 5 (B) 20 (C) 8 (D) 10

16. Ram, Shyam, and Tarun started cycling from a point on a circular track 600 $\overline{m}$ long with speeds of 10 m/sec, 15 m/sec, and 20 m/sec, respectively. Find the time taken by them to meet at the starting point for the first time (in seconds).

(A) 120 (B) 60 (C) 240 (D) 600

17. Ashwin and Bhaskar started running simultaneously from a point on a 300 $\overline{m}$ long circular track. They ran in opposite directions with speeds of 6 m/sec and 4 m/sec, respectively. After meeting for the first time, they exchange their speeds. Who will reach the starting point first?

(A) Ashwin

(B) Bhaskar

(C) Both reach simultaneously

(D) Cannot be determined

18. A man reaches his destination which is 16 km away, 9 min late, if he travels at 8 kmph. What should his speed be if he wishes to reach 15 minutes ahead of the right time?

(A) 10 kmph (B) 3 m/sec

(C) 20/9 m/sec (D) 12 kmph

19. The distance between two points P and Q is 84 km. Two persons start at the same time but one travelling from P towards Q and the other travelling from Q towards P. If their respective speeds are 36 kmph and 27 kmph, where do they meet each other?

(A) 48 km from Q (B) 24 km from P

(C) 36 km from P (D) 48 km from P

20. Towns P and Q are 80 km apart. Cars A and B are stationed at towns P and Q respectively. If they start simultaneously towards each other, they would meet in an hour. If both start simultaneously in the same direction, the faster car would overtake the slower car in 4 hours. Find the speed of the faster car (in kmph).
(A) 50 (B) 55 (C) 60 (D) 65

21. A cat on seeing a dog 100 m away turns around and starts running away at 24 kmph. The dog spots him one minute later and starts chasing the cat at a speed of 33 kmph. After how much time, from the start of the cat's run, will the chase end?
(A) 160 s (B) 220 s
(C) 260 s (D) 280 s

22. Train A starts at 6 a.m. from city P towards city Q at a speed of 54 kmph. Another train 'B' starts at 9 a.m. from P towards Q at 72 kmph. If the distance between P and Q is 1440 km, find at what distance from Q would the two trains meet each other?
(A) 648 km (B) 792 km
(C) 486 km (D) 954 km

23. Mahesh travelled from Hyderabad to Tirupati at a certain speed and returned at a certain speed. His average speed for the entire trip was the average of his onward and return speeds. He travelled a total distance of 1200 km in 12 hours. Find his onward speed (in kmph).
(A) 100 (B) 80 (C) 60 (D) 40

24. Two cars left simultaneously from two places P and Q, and headed for Q and P, respectively. They crossed each other after x hours. After that, one of the cars took y hours to reach its destination while the other took z hours to reach its destination. Which of the following always holds true?
(A) $x = \frac{y+z}{2}$ (B) $x = \frac{2yz}{y+z}$
(C) $x = \sqrt{yz}$ (D) $x = \frac{y^2+z^2}{y+z}$

25. A boat travels 30 km upstream in 5 hours and 24 km downstream in 3 hours. Find the speed of the boat in still water and the speed of the water current
(A) 7 kmph, 2 kmph (B) 14 kmph, 1 kmph
(C) 7 kmph, 1 kmph (D) 8 kmph, 2 kmph

26. Amar, Akbar, and Anthony start running in the same direction and from the same point, around a circular track with speeds 7 m/sec, 11 m/sec, and 22 m/sec, respectively. If Akbar can complete 5 revolutions around the track in 40 sec, when will they meet for the first time after they start?
(A) 56 s (B) 88 s (C) 118 s (D) 79 s

27. If Ashok travelled at $4/5^{th}$ of his usual speed, he would reach his destination 15 minutes late. By how many minutes would he be early if he travelled at $6/5^{th}$ of his usual speed?
(A) 12 (B) 10 (C) 15 (D) 20

28. In a 500 ft race, Habib beats Akram by 60 ft. If Habib takes 5 paces for every 4 paces taken by Akram, what is the ratio of the length of Habib's pace to that of Akram?
(A) 10 : 11 (B) 11 : 10
(C) 25 : 22 (D) 22 : 25

29. Girish takes 1 minute to complete a round around a circular track. Harish is twice as fast as Girish, Suresh is thrice as fast as Harish. All three start at the same point. Find the time taken by them to meet at the starting point for the first time (in minutes).
(A) 1 (B) 2
(C) 6 (D) 12

30. Two cars C and D start from a junction along two perpendicular roads at 8:00 a.m. and 9:00 a.m., respectively. If at 12 noon, the cars, which travel at the same speed, are 150 km apart, then, find the speed of each car.
(A) 15 kmph
(B) 45 kmph
(C) 60 kmph
(D) 30 kmph

Practice Problems 2

Directions for questions 1 to 23: Select the correct alternative from the given choices.

1. Ram covers a distance of 100 $\overline{m}$ in a certain time. If he increases his speed to 3 times his original speed, the time taken reduces by 40 seconds. Find the original time taken (to cover the same distance of 100 $\overline{m}$).
(A) 30 seconds (B) 120 seconds
(C) 180 seconds (D) 60 seconds

2. A person covered 50% of a certain distance at 15 kmph, 60% of the remaining distance at 18 kmph and the rest at 45 kmph. The total travel time of the person is 9.8 hours. Find the total distance covered.
(A) 60 km (B) 120 km
(C) 150 km (D) 180 km

3. P and Q are two stations. Train A started from P towards Q at 6:00 a.m. at 90 kmph. At the same time, train B started from R, an intermediate station 60 km from P, and travelled towards Q at 60 kmph. Train C started from Q towards P at 7:00 a.m. at 120 kmph. All the trains crossed each other simultaneously. Find PQ (in km).
(A) 300 (B) 360 (C) 330 (D) 390

4. P and Q are two points, 10 km apart. Anand started from P towards Q and at the same time, Ashok started from Q towards P. They crossed each other after 1 hour. After that, Anand reduced his speed by 2 kmph and Ashok increased his speed by 2 kmph. They reached their destinations simultaneously. Find Anand's initial speed (in kmph).
(A) 5.5 (B) 6.5 (C) 4.0 (D) 6.0

5. A person travels one-third of a certain distance *AB* at *x* kmph, one-fourth of the remaining distance at 2*x* kmph and the remaining distance at 3*x* kmph. If his average speed for the entire journey is (*x* + 2) kmph, then find the total distance he covers.
(A) 60 km
(B) 50 km
(C) 40 km
(D) Cannot be determined

6. Rahul ran around a square plot *ABCD* of side 0.48 km once. He ran the distance *AB*, *BC*, *CD,* and *DA* at speeds of 4 kmph, 6 kmph, 4 kmph, and 6 kmph, respectively.
(i) Find his average speed from *A* to *C* (in kmph).
(A) 2.4 (B) 3.6 (C) 4.8 (D) 7.2
(ii) Find his average speed for the entire distance (in kmph).
(A) 2.4 (B) 3.6 (C) 4.8 (D) 7.2
(iii) Find the time taken for the entire distance (in hours).
(A) 0.05 (B) 0.53
(C) 0.4 (D) 0.27

7. A car had to travel a total distance of 600 km. After travelling a part of the distance, it developed an engine problem. It travelled the remaining distance at 4/5th of its usual speed. It arrived an hour late at its destination. Had the engine problem occurred after it had travelled 150 km more, the car would have arrived half an hour earlier at its destination. Find the distance it travelled without any problem (in km).
(A) 150 (B) 250 (C) 200 (D) 300

8. Everyday, Ashwin starts at 3.00 pm from his home to pick up his son from school. They reach their house at 5.00 p.m. One day, school was over at 3.00 p.m. Ashwin, not aware of this, started from home as usual. He met his son on the way and they reached home 20 minutes earlier than usual. If the speed of his car is 55 kmph, find his son's speed (in kmph).
(A) 10 (B) 11 (C) 12 (D) 15

9. Two persons *R* and *S* had to meet at a place 36 km from where they were at that moment. *S*, who was given a head start of 2 hours, reached the destination 12 minutes earlier than *R*. If *R* travelled 18 kmph faster than S, at what speed did R travel?
(A) 25 kmph (B) 24 kmph
(C) 30 kmph (D) 16 kmph

10. *M* and *N* are two points that are 8 $\overline{m}$ apart. Anand and Ajay started simultaneously from *M* and *N,* respectively. Anand moved towards N at 3 m/sec. Ajay moved towards M at 5 m/sec. After reaching their destinations, both turned back at their original speeds towards their starting points. Find the total distance (in m) that Ajay would have travelled before crossing Anand for the second time.
(A) 12 (B) 13 (C) 15 (D) 14

11. In a kilometre race, Ram beats Shyam by one minute and Shyam beats Tarun by 30 seconds. If Ram beats Tarun by 250 *m* in the same race, find the time taken by Ram to run the race (in seconds).
(A) 180 (B) 270 (C) 360 (D) 330

12. A train takes 30 seconds to cross a 200 $\overline{m}$ long platform and 40 seconds to cross a 300 $\overline{m}$ long platform. Find its length (in m).
(A) 150 (B) 225 (C) 200 (D) 100

13. A train takes 2 minutes to overtake a cyclist traveling at 18 kmph and one and a half minutes to overtake a cyclist traveling at 9 kmph. Find its length (in m).
(A) 1200 (B) 800 (C) 1500 (D) 900

14. A train 360 $\overline{m}$ in length, travelling at a uniform speed overtook a car, travelling parallel to the tracks at 72 kmph and passed it in 12 sec. Forty eight minutes later, the train starts overtaking a cyclist and passed him in 9 sec. How much time after the train overtook the cyclist would the car over take him?
(A) 2 hrs 24 min 36 sec (B) 3 hrs 49 min 49 sec
(C) 3 hrs 12 min 9 sec (D) 2 hrs 24 min 27 sec

15. Tony and Harry begin to run in opposite directions on a circular path of radius 35 $\overline{m}$ at 20 m/sec and 11 m/ sec, respectively, from the same point. What is the time taken by them to meet for the third time at the starting point?
(A) 11 min (B) 220 sec
(C) 8 min 40 sec (D) 325 sec

16. *P* and *Q* start cycling simultaneously around a circular track 48000 $\overline{m}$ long with speeds of 10 m/sec and 30 m/sec, respectively, in opposite directions. After every crossing, *P*'s speed increases by 10 m/sec and *Q*'s speed decreases by 10 m/sec. Find the time taken by them to cross for the third time (in seconds).
(A) 3600 (B) 7200
(C) 4800 (D) 9600

17. Akbar takes a certain time to go downstream on a river between two villages. He takes 4 times as much time to cover the same distance upstream. What is the speed of the stream, if the distance between the villages is 10 km, which can be covered in 2 hours if the boat travels in still water?
(A) 2 kmph (B) 5 kmph
(C) 3 kmph (D) 8 kmph

18. A boat travels from point *P* to point *Q* upstream and returns from point *Q* to point *P* downstream *PQ* = 96 km. If the round trip takes 9 hours and the speed of the boat in still water is 8 kmph more than the speed of the stream, find the time taken for the downstream journey.
(A) 3 hours (B) 4 hours
(C) 8 hours (D) 6 hours

19. Rahim and Saleem are competing in a 1000 $\overline{m}$ race. Rahim gives Saleem a head start of 200 $\overline{m}$, but his speed is twice that of Saleem. When Rahim reaches the 600 $\overline{m}$ mark, Rahim reduces his speed by half but still beats Saleem by 20 s. What is Saleem's speed?
(A) 14 m/s (B) 5 m/s (C) 3.5 m/s (D) 7 m/s

20. In a race of 500 $\overline{m}$, *L* beats *M* by 40 seconds and beats *N* by 125 $\overline{m}$. If *M* and *N* run a 500 $\overline{m}$ race, *M* beats *N* by 40 seconds. Find the time taken (in seconds) by *M* to run the race.
(A) 160 (B) 240 (C) 280 (D) 320

21. Anant started answering a mathematics test sometime between 2:00 p.m. and 3:00 p.m. and ended at sometime between 5:00 p.m. and 6:00 p.m. If it is known that the position of the minute hand and the hour hand at the start interchanged with the position at the end, for how long did Anant take the test?
(A) $2\frac{5}{11}$ hours (B) $2\frac{10}{13}$ hours
(C) $2\frac{5}{7}$ hours (D) $2\frac{13}{14}$ hours

22. Prakash and Pramod are running along a circular track having started at the same time from the same point, in the same direction. How much more distance would Prakash have travelled compared to Pramod by the time they meet for the 11th time given that the radius of the track is 7 *m*, Prakash and Pramod run at 22 m/s and 11 m/s, respectively?
(A) 440 $\overline{m}$ (B) 384 $\overline{m}$
(C) 524 $\overline{m}$ (D) 484 $\overline{m}$

23. Peter and Paul are running in opposite directions along a circular track of 600 $\overline{m}$ with initial speeds of 3 m/s and 6 m/s. They start at the same point and at the same time. Whenever they meet, they exchange their speeds and carry on in their respective directions. Find the distance between Peter and Paul (when measured along the track) when Peter completes $4\frac{1}{4}$ rounds.
(A) 150 $\overline{m}$ (B) 200 $\overline{m}$
(C) 300 $\overline{m}$ (D) 120 $\overline{m}$

Directions for questions 24 and 25: These questions are based on the following data.

Train T of length 100 m moving at 54 kmph and train *U* of length 150 m moving at 90 kmph running on parallel tracks enter a 200 m long tunnel at the same instant from opposite directions.

24. Which train exits the tunnel first and at the moment it exits, what length of the other train is still in the tunnel?
(A) T, 50 $\overline{m}$ (B) U, 10 $\overline{m}$
(C) U, 90 $\overline{m}$ (D) T, 75 $\overline{m}$

25. How far is the point where the rear ends of the two trains cross each other, from the point of entry of the slower train?
(A) 40.5 $\overline{m}$ (B) 81 $\overline{m}$
(C) 84.5 $\overline{m}$ (D) 68.75 $\overline{m}$

Directions for questions 26 and 27: Select the correct alternative from the given choices.

26. A thief was running along to a median on a road at 6 kmph. He crossed a police jeep moving at 9 kmph in the opposite direction. The jeep had to continue for 10 more minutes before it could find a gap in the median and start chasing the thief. Find the total time taken by the jeep to catch the thief from the time it crossed him (in minutes).
(A) 45 (B) 50 (C) 55 (D) 60

27. Two runners run in the same direction along a circular track 3 km in length. The faster runner overtakes the slower every 1 hour. Find the speed of the slower runner, if the faster one completes one length of the track 2 minutes sooner than the other.
(A) 20 kmph (B) 18 kmph
(C) 15 kmph (D) 12 kmph

Directions for questions 28 and 29: These question is based on the following data.

Three friends—Anil, Bala, and Chetan, from *X* wanted to travel from town *X* to town *Y* which was 40 km. Anil who had a bike started along with Bala while Chetan started simultaneously on foot. After some time, Anil dropped Bala on the way and went back to pick up Chetan while Bala proceeded to *Y* on foot. Anil picked up Chetan and reached *Y* at the same time as Bala. Anil traveled at 50 km/hr. The speed at which Bala and Chetan walked was 10 km /hr.

28. Find the time after which Anil turned back.
(A) 24 minutes (B) 48 minutes
(C) 36 minutes (D) 30 minutes

29. Find Bala's average speed for the entire trip.
(A) 12 km/hr (B) 24 km/hr
(C) 20 km/hr (D) 25 km/hr

Directions for question 30: Select the correct alternative from the given choices.

30. Two men, *M* and *N* started walking towards each other simultaneously from two places *F* and *G,* respectively, which are 50 km apart. They meet after 5 hours. After their meeting, *M* reduced his speed by 1 kmph and *N* increased his speed by 1 kmph. They arrived at *G* and *F,* respectively, at the same time. Find the initial speed of *M*.
(A) 4.5 kmph (B) 5.5 kmph
(C) 6 kmph (D) 4 kmph

HINTS/SOLUTIONS

Practice Problems I

1. (a) 36 km/hr × 5/18 = 10 m/s

Hence, the correct option is (A).

(b) 12.6 × 5/18 = 3.5 m/s

Hence, the correct option is (A).

(c) 252/35 × 5/18 = 2 m/s

Hence, the correct option is (C).

2. 6 m/s × 18/5 = 108/5 km/hr

$d = s \times t \Rightarrow 108/5 \times 15/4 = 81$ km

Hence, the correct option is (A).

3. As speed and time are inversely proportional, if speed decreases to 5/6th, time taken will be increasing to 6/5th of original time.

$\therefore \frac{6t}{5} - t = 10 \Rightarrow t = 50$ minutes.

Hence, the correct option is (A).

4. Time taken by them to meet $= \frac{270}{\text{Relative speed}}$

$= \frac{270}{50 + 40} = 3$ hours

$\therefore$ The meeting time is 12:00 p.m

Hence, the correct option is (B).

5. Distance that X would have traveled by 10:00 a.m. = 30 k.m. Distance between the cars at 10:00 a.m. = 150 k.m. Meeting time $= \left(\frac{150}{30 + 20}\right)$ hours after 10:00 a.m. i.e. 1:00 p.m.

Hence, the correct option is (C).

6. Average speed $= \frac{\text{Total distance}}{\text{Total time}}$

XY = (4) (90) or 360 km.

Hence, the correct option is (B).

7. Let x km be the distance between Hyderabad and Tirupathi.

Average speed $= \frac{x + x}{\frac{x}{60} + \frac{x}{90}}$

$= \frac{2\,(60)\,(90)}{60 + 90} = 72$ kmph

Hence, the correct option is (A)

8. The distance to be travelled by the train to cross a platform of length 750 m is 750 + 650 i.e., 1400 m. At 72 km/hr. or 20 m/s. the train takes $\frac{1400}{20}$ i.e. 70 seconds to cross the platform.

Hence, the correct option is (C).

9. Relative to the boy the speed of the train is 99 + 9 i.e., 108 km/hr. Hence, the time it takes to cross him

$= \frac{750}{108 \times \frac{5}{8}} = 25.$

Hence, the correct option is (D).

10. When Eswar finishes the race, Girish would have run 200 – (10 + 10) = 180 m.

$\therefore$ Ratio of Eswar's and Girish's speeds $= \frac{200}{180} = \frac{10}{9}$

Hence, the correct option is (C).

11. Ganesh beats Harish by 10 m or 2 seconds.

$\therefore$ Harish must have taken 2 seconds to run the final 10 m.

$\therefore$ Harish's speed $= \frac{10}{2} = 5$ m/sec

Hence, the correct option is (A).

12. Akbar gives Birbal a start of 2 s and Birbal covers 10 m in these 2 s.

$\therefore$ Birbal's speed is5 m/sec.

$\therefore$ He takes 18 seconds, more to cover the 90 m.

$\therefore$ Akbar covered 100 m in 18 seconds.

His speed $= \frac{100}{18}$ m/sec $= 5\frac{5}{9}$ m/sec

Hence, the correct option is (B).

13. Let the time taken by R to run the race is t seconds.

Time taken by Q to run the race = $(t + 30)$ seconds.

Time taken by P to run the race = $(t + 50)$ seconds.

$\therefore$ P beats R by 50 seconds.

Hence, the correct option is (C).

14. Let the speeds of A, B and C be a m/sec, b m/sec and c m/sec respectively.

$\frac{a}{b} = \frac{100}{100 - 10} = \frac{10}{9}$

$\frac{b}{c} = \frac{100}{100 - 20} = \frac{5}{4}$

$\frac{a}{c} = \left(\frac{a}{b}\right)\left(\frac{b}{c}\right) = \frac{25}{18}$

$\therefore$ A beats C by $\frac{25 - 18}{25}(100) = 28$ m.

Hence, the correct option is (B).

15. Speed of Anand's boat downstream = 8 kmph.

Time taken $= \frac{40}{8} = 5$ hours

Hence, the correct option is (A).

16. Time = LCM $\left(\frac{600}{10}, \frac{600}{15}, \frac{600}{20}\right)$

= LCM (60, 40, 30)

= 120 seconds

Hence, the correct option is (A).

17. The relative speed does not change when the two exchange their speeds. ∴ Both will reach the starting point simultaneously.

Hence, the correct option is (C).

18. By formula $\frac{vu}{v-u}(p+q) = \text{distance}$

$\Rightarrow \left(\frac{v\,x\,8}{v-8}\right)\frac{24}{60} = 16$

$\Rightarrow \frac{8v}{v-8} x \frac{2}{5} = 16$

$\Rightarrow v = 5v - 40 \qquad \Rightarrow 40 = 4v \qquad \Rightarrow v = 10$ kmph

Alternate method:

$\frac{d}{s_1} - \frac{d}{s_2} =$ difference of times of travel

$\frac{16}{8} - \frac{16}{s_2} = \frac{15+9}{60} = \frac{24}{60} = \frac{2}{5}$

Solving, $s = 10$ kmph

Hence, the correct option is (A).

19. Time after which they meet $= \frac{84}{36+27} = \frac{4}{3}$ hours

∴ The first person will cover 36 × 4/3 = 48 km in this time.

So, they will meet 48 km from P.

Hence, the correct option is (D).

20. Let the speeds of the faster car and the slower car be x kmph and y kmph respectively.

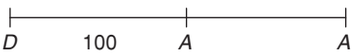
Given, $\frac{80}{x+y} = 1 \Rightarrow x + y = 80$ (4)

$\frac{80}{x-y} = 4 \Rightarrow x - y = 20$ (5)

Solving (4) and (5), $x = 50$

Hence, the correct option is (A).

21.



The cat runs away from an initial point A for 1 minute at a speed of 24 kmph.

∴ $AC = 24 \times 60 \times 5/18 = 400$ m

Now, the dog is separated from the cat by a distance of 100 + 400 ($DA + AC$) = 500 m and this lead has to be covered at a speed of (33 – 24) = 9 kmph i.e., relative speed.

∴ Time $= \frac{500}{9\,x\,\frac{5}{18}} = 200$ sec

But, if the time is required from the start of the cat's run, 1 minute has to be added. i.e. 200 + 60 = 260 seconds.

Hence, the correct option is (C).

22.

When 'B' starts from city P towards city Q, the distance which 'A' would already have covered = 54 × 3 = 162 km.

At 9 a.m., train B is separated from train 'A' by a distance of 162 km.

Train B overtakes train A after a time of

$\frac{162}{(72-54)} = \frac{162}{18}$ hrs = 9 hrs.

In 9 hrs, the distance travelled by B = 72 × 9 = 648 km.

Distance from city Q, when they meet

= (1440 – 648) = 792 km

Hence, the correct option is (B).

23. Let his forward and return speeds be x kmph and y kmph respectively.

Average speed $= \frac{2xy}{x+y}$

Given, $\frac{2xy}{x+y} = \frac{x+y}{2}$

$\Rightarrow (x+y)^2 - 4xy = 0$

$\Rightarrow (x-y)^2 = 0 \Rightarrow x = y$

$\therefore \qquad \frac{2xy}{x+y} = x$

∴ Forward speed = Average speed $= \frac{1200}{12} = 100$ kmph

Hence, the correct option is (A).

24. Let the speeds of the cars leaving P and Q be p kmph and q kmph respectively.

$$px = qy \quad (6)$$

$$pz = qx \quad (7)$$

Dividing (6) by (7),

$$\frac{x}{z} = \frac{y}{x}$$

$$x = \sqrt{yz}\ .$$

Hence, the correct option is (C).

25. Speed of the boat upstream $= \frac{30}{5} = 6$ kmph

Speed of the boat downstream $= \frac{24}{3} = 8$ kmph

Speed of the boat in still water $= \left(\frac{8+6}{2}\right)$ kmph $= 7$ km/hr

Speed of the water current $= \left(\frac{8-6}{2}\right) = 1$ kmph

Hence, the correct option is (C).

26. Akbar completes 1 revolution in 40/5 = 8 sec

Circumference/Akbar's speed = Circumference/11

$\therefore$ Circumference = 88 m

Time taken to meet for the first time is

$\text{LCM of}\left\{\frac{88}{22-11}, \frac{88}{11-7}\right\}$ i.e., $\text{LCM of}\left\{\frac{88}{11}, \frac{88}{4}\right\}$

i.e., LCM of {8, 22} = 88

So, they meet for the first time 88 seconds after the start.

Hence, the correct option is (B).

27. Let his usual time be x minutes. Let the distance be d km

Usual speed $= \frac{d}{x}\ \frac{\text{km}}{\text{minute}}$

If Ashok traveled at $\frac{4}{5}$ th of his usual speed, his usual speed, his time $= \frac{d}{\frac{4}{5}\frac{d}{x}} = \frac{5}{4} \times$ minutes

$$\frac{5}{4}x - x = 15 \qquad x = 60$$

If Ashok traveled at $\frac{6}{5}$ th of his usual speed, his time

$$= \frac{d}{\frac{6}{5}\frac{d}{x}} = \frac{5}{6}x$$

He would be early by $\frac{1}{6}x = 10$ minutes.

The ratio of speeds of Ashok in three instances i.e., usual speed, 4/5th speed and 6/5th speeds, is s : 4s/5 : 6s/5 i.e., 5 : 4 : 6.

As speed $\alpha \frac{1}{\text{time}}$, time taken ratio is $\frac{1}{5} : \frac{1}{4} : \frac{1}{6}$

i.e., 12 : 15 : 10.

Given, 3 parts is representing 15 minutes, hence 2 parts will represent 10 minutes.

Hence, the correct option is (B).

28. Distance travelled by Habib/Distance travelled by Akram = 500/440

Let the length of the pace of Habib be x and that of Akram be y.

Ratio of speeds = ratio of distances covered in equal intervals of time = $(5x/4y)$

$\therefore$ $5x/4y = 500/440$

$x/y = 10/11$

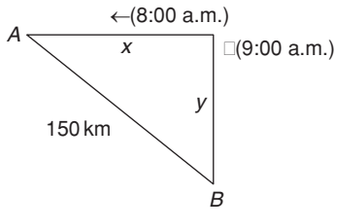
Hence, the correct option is (A).

29. If two or more runners have their speeds as a multiple of the speed of a runner N the time taken by these runners to meet for the first time at the starting point for the first time is the time taken by the N to complete one round.

As the speeds of Harish and Suresh and Multiples of the speed of Girish, required time = Time taken by Girish to complete 1 round = 1 minute.

Hence, the correct option is (A).

30.



The distances covered by C and D are x and y respectively. Since C starts early compared to D, he covers more distance by noon compared to D. Hence $x > y$. If speed of each car is s. $x = 4s$ and $y = 3s$

$x^2 + y^2 = (150)^2$ (by Pythagoras theorem)

Taking $x = 4s$, $y = 3s$ and solving for s we have,

$s = 30$ kmph.

Hence, the correct option is (D).

Practice Problems 2

Solutions for questions 1 to 23:

1. Let the original time taken be 't'. Since, his speed increases to 3 times his original speed, new speed = (3) (original speed) $\Rightarrow$ New time

$= \frac{\text{Original time}}{3}$

Given, $t - \frac{t}{3} = 40$

$\Rightarrow \frac{2t}{3} = 40 \Rightarrow t = 60$ sec.

Hence, the correct option is (D).

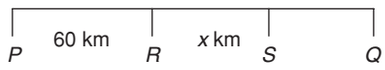
2. Let the total distance covered be 100 d km

$$\frac{50d}{15}+\frac{30d}{18}+\frac{20d}{45}=9.8 \qquad \Rightarrow d=1.8$$

and $100\,d = 180$

Hence, the correct option is (D).

3.



Suppose the trains met at S. Let $RS = x$ km

$$\frac{PS}{90}=\frac{RS}{60}=\frac{SQ}{120}+1$$

$$\frac{60+x}{90}=\frac{x}{60}$$

$$120 = x$$

$$\therefore \ \frac{RS}{60}=2=\frac{SQ}{120}+1$$

$SQ = 120$ km

$PQ = PR + RS + SQ = 300$ km.

Hence, the correct option is (A).

4. Let the speeds of Anand and Ashok before the meeting be x kmph and y kmph respectively

$$1=\frac{10}{x+y} \Rightarrow x+y=10 \qquad (8)$$

Distances travelled by Anand and Ashok before meeting are x km and y km respectively. As they reached their destinations simultaneously,

$$\frac{y}{x-2}=\frac{x}{y+2} \Rightarrow y^2+2y=x^2-2x$$

$$2=x-y \qquad (9)$$

Solving (8) and (9), $x = 6$.

Hence, the correct option is (D).

5. From the given data, which is about the speeds of travel and the average speed, in terms of a variable x, the average speed can be determined. As no information about the total time is available, distance cannot be calculated. The same can be seen from the calculations shown below.

Let the total distance AB be d.

$$\text{Total time} = \frac{\left(\frac{d}{3}\right)}{x}+\frac{\frac{1}{4}\left(\frac{2d}{3}\right)}{2x}+\frac{d-\left(\frac{d}{3}+\frac{1}{4}\left(\frac{2d}{3}\right)\right)}{3x}$$

$$=\frac{d}{x}\left(\frac{1}{3}+\frac{1}{12}+\frac{1}{6}\right)=\frac{d}{x}\left(\frac{7}{12}\right)$$

$$\text{Average speed for the entire journey} = \frac{d}{\frac{d}{x}\left(\frac{7}{12}\right)}$$

$= 12x/7 = x + 2$, $5x/7 = 2$, $x = 14/5$ kmph

Total distance the person covers cannot be determined as total travel time is unknown.

Hence, the correct option is (D).

6. $AB = BC$

(i) $$\text{Average speed} = \frac{(2)(4)(6)}{4+6} = 4\times 8 \text{ kmph}$$

Hence, the correct option is (C).

(ii) Using similar method as shown in (i), average speed of Rahul from C to A $= 4 \times 8$ kmph i.e., same as that from A to C.

$\therefore$ Average speed of Rahul for his journey

$= 4 \times 8$ kmph.

Hence, the correct option is (C).

(iii) $$\text{Time taken for his journey} = \frac{\text{Total distance}}{\text{Average speed}}$$

$$= \frac{(0\cdot 48)(4)}{4\cdot 8} = 0\cdot 4 \text{ hours.}$$

Hence, the correct option is (C).

7. Let the distance it traveled without any problem be d km. Distance it traveled at the reduced speed = (600 − d) km.

Let its usual speed by S kmph

$$\text{Given, } \frac{d}{s}+\frac{600-d}{\frac{4}{5}s}=\frac{600}{s}+1 \qquad (10)$$

$$\Rightarrow \frac{d+150}{S}+\frac{450-d}{\frac{4}{5}s}=\frac{600}{s}+\frac{1}{2} \qquad (11)$$

Subtracting (10) from (11),

$$\Rightarrow \frac{150}{s}-\frac{(150)s}{4s}=\frac{-1}{2}$$

$\Rightarrow S = 75$

Substituting S in (10) or (11),

$d = 300$.

Hence, the correct option is (D).

8. Let the speed of his son be x kmph.

As Ashwin returned home 20 minutes early, he saved a travel of 10 minutes to school from the meeting point and the return journey of 10 minutes. Distance he saved travelling each way = Distance travelled by his son by walk. His son would have walked for 50 minutes.

$$(x)\left(\frac{50}{60}\right)=(55)\left(\frac{10}{60}\right)$$

$$x = 11 \text{ kmph}$$

Hence, the correct option is (B).

9. Since R has given S a start of 2 hours and arrives at the destination 12 minutes after S, he takes 1 hr 48 min (9/5 hrs) less than S to reach the meeting point compared to S. If the speed of R is x, we have,

$$\frac{36}{x-18}-\frac{36}{x}=\frac{9}{5}$$

Solving this equation, we have $x = 30$ kmph

Hence, the correct option is (C).

10. Anand and Ajay would meet for the first time during their onward journeys. They would meet for the second time during their return journeys. Time taken by Anand to travel from M to N $= \frac{8}{3}$ seconds.

Distance travelled by Ajay in this time $= \frac{8}{3}(5) = \frac{40}{3}$ m.

$\therefore$ He would be $\frac{40}{3}-8=\frac{16}{3}$ m from N when Anand reached N.

$\therefore$ Distance between them would then be $8-\frac{16}{3}=\frac{8}{3}$ m.

Anand would cross Ajay in another

$$\frac{\frac{8}{3}}{\text{Relativespeed}}=\frac{1}{3}\text{seconds.}$$

$\therefore$ Ajay would have travelled a distance of

$$\frac{40}{3}+\frac{1}{3}(5)=15 \text{ m}$$

when he crossed Anand for the second time.

Hence, the correct option is (C).

11. Let the time taken by Ram to run the race be t seconds. Time taken by Shyam to run the race $= (t + 60)$ seconds. Time taken by Tarun to run the race

$= (t + 90)$ seconds. Ratio of speeds of Ram and Tarun

$$= \frac{1000}{1000-250}\frac{\frac{1000}{t}}{\frac{1000}{t+90}}$$

$$\frac{4}{3}=\frac{t+90}{t}$$

$t = 270$.

Hence, the correct option is (B)

12. Let the length and the speed of the train be Lm and Sm/ sec respectively

Given, $\frac{L+200}{S}=30 \Rightarrow L + 200 = 30\,S$ (12)

Given, $\frac{L+300}{S}=40 \Rightarrow L + 300 = 40\,S$ (13)

Subtracting (12) from (13), $100 = 10\,S \Rightarrow 10 = S$

From (12) or (13), $L = 100$ m.

Hence, the correct option is (D).

13. Let the length and the speed of the train be Lm and Sm/sec respectively

$$\text{L}=120\left(S-(18)\left(\frac{5}{18}\right)\right) \quad (14)$$

and

$$L=90\left(S-(9)\left(\frac{5}{18}\right)\right) \quad (15)$$

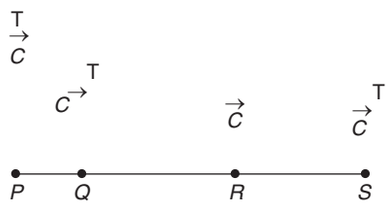
Solving (14) and (15), we have

$S = 12 \times 5$

$\therefore \quad L = 900$

Hence, the correct option is (D).

14.



Let the speed of the train be V m/s

Then $\frac{360}{V-20}=12$; $V = 50$ m/s

Similarly let the speed of the cyclist be V_1m/s

$$\frac{360}{50-V_1}=9$$

$V_1 = 10$ m/s

Let P be the point where the train begins to overtake the car Q be the point where the train completely overtake the car R be the point where the car is, when the train begins to overtake the cyclist.

S be the point where the train begins to overtake the cyclist.

In 48 min, when it is just about to overtake the cyclist, it will cover (48) (60 (50) m, i.e. $QS = 48(60)$ (50) m i.e. $QR = 48(60)$ (20) m

And the car will cover (48) (60) (20) m

At this instant the distance between the car and the cyclist RS is 360 + (48) (60) (50 – 20)

= 360 + (48) (60) (30)

This distance has to be covered with a relative speed of (20 – 10) = 10 m/s

$\therefore$ Time required $= \frac{360+(48)\ (60)\ (30)}{10}$

= 36 s + 144 min = 2 hr 24 min 36 s

As the time required is the time after the train overtook the cyclist, it is 9 seconds less; i.e., 2 hr 24 min 27 s.

Hence, the correct option is (D).

15. Radius = 35 m.

$\Rightarrow$ Circumference = 2 × 22/7 × 35 = 220 m

Time taken for them to meet for the first time at the starting point is LCM [220/20, 220/11]

= LCM [11, 20] = 220 sec

To meet for the third time at the staring point from the start, they need 220 × 3 = 660 sec = 11 minutes

Hence, the correct option is (A).

16. As P's speed increases by the same amount as that by which Q's speed decreases, their relative speed remains unchanged.

$\therefore$ Time taken by them to meet for the third time = 3 (Time taken by them to meet for the first time)

$$= 3\left(\frac{48000}{10+30}\right) = 3600 \text{ sec}.$$

Hence, the correct option is (A).

17. First condition gives $4(b - s) = b + s$ (16)

where, b = speed of the boat in still water (kmph) and s = speed of stream (in kmph)

Second condition gives $10/b = 2 \Rightarrow b = 5$ (17)

From (16), (17)

$4(5 - s) = 5 + s \Rightarrow s = 3$

$\therefore$ Speed of the stream = 3 kmph

Hence, the correct option is (C).

18. Let the speed of the stream be y, then the speed of the boat in still water will be $y + 8$

Upstream speed = $y + 8 - y = 8$ kmph.

Hence, upstream time = (96/2)/8 = 6 hours

Time for downstream = 9 − 6 = 3 hours.

Hence, the correct option is (A).

19. During the time Rahim covers 600 metres, Saleem covers 300 m, as Rahim's speed is double that of Saleem. During the time Rahim covers the balance 400 metres, Saleem covers 400 m, as their speeds are equal. Saleem had a head start of 200 m and he covers (300 + 400) = 700 m. Hence, when Rahim is at finishing point, Saleem is at 200 + 700 = 900 m metres. Saleem takes 20 seconds to cover the balance 100 m. Saleem's speed = 5 m/s.

Hence, the correct option is (B).

20. Let the time taken by L to run 500 m be t_L.

Time taken by M to run 500 m = $t_L + 40$

If M and N run a 500 m race, time taken by $N = t_L + 40 + 40 = t_L + 80$

Since L beats N by 125 m, and as they run for the same time; ratio of speeds of L and N will be

$$= \frac{500}{500-125} \quad (18)$$

L and N, individually cover the distances of 500 m in t_L seconds and $(t_L + 80)$ seconds respectively.

Hence, ratio of their speeds = $\frac{t_L + 80}{t_L}$ (19)

From (18) and (19)

$$\frac{t_L + 80}{t_L} = \frac{500}{375}$$

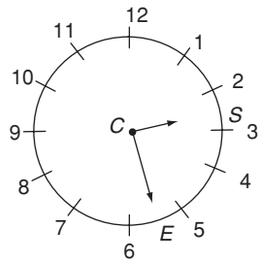
Solving, we have $t_L = 240$ sec.

Time taken by M to run the race = 240 + 40

= 280 sec

Hence, the correct option is (C).

21.



The angle of rotation of minute hand = 6 degrees/minute.

The angle of rotation of hour hand = (1/2) degree/minute.

If the duration of the test is 't' minutes, then the sum of the angles of rotation of the two hands is $6t + t/2 = 13t/2$ degrees (20)

The test started between 2 and 3 o' clock, when the hour hand was pointing at S (in the diagram) and the minute hand was pointing at E (in the diagram).

The test ended between 5 and 6 o' clock, and the position of the two hands got interchanged.

This implies that the duration of the test is less than 3 hours; (because 3 hours would have been completed if the minute hand came to the point E.)

During the test period, the minute hand completes 2 rounds of the dial, and falls short of the angle SCE for the 3rd round; but the hour hand rotates through the angle SCE.

Hence, sum of the angles of rotation of both the hands = angle of rotation for 3 rounds of the dial

= 360 × 3 = 1080 degrees. (21)

From (20) and (21),

$$\frac{13t}{2} = 1080;\ t = \frac{2 \times 1080}{13} \text{ minutes}$$

$$= \frac{2 \times 1080}{13 \times 60} \text{hours} = 2\frac{10}{13} \text{hours}.$$

Alternate method:

Let the time at start be p minutes past 2 and the time at the end be q minutes past 5.

$\therefore$ At the start, angle made by minutes hand = $6p$;

hours hand = $60 + p/2$.

At the end of the test, the angles are:

minute hand = $6q$

hour hand = $150 + q/2$

Since the positions interchange

$6p = 150 + q/2 \rightarrow (1)$

$60 + p/2 = 6q \rightarrow (2)$

$$6(p-q) = 90 - \left(\frac{p-q}{2}\right)$$

$$\frac{13}{2}(p-q) = 90$$

$$(p-q) = \frac{180}{13} \rightarrow (1)$$

We know that the duration from 2 : p to 5 : q is

3 hours + $(q - p)$ minutes = $180 + (q - p)$ minutes

Substituting from (1)

$= 180 - 180/13 = 180 \times 12/13$ minutes

$= (180 \times 12)/(13 \times 60)$ hours

$= 36/13$ hours $= 2\frac{10}{13}$ hours

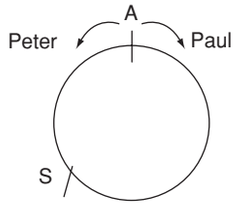
Hence, the correct option is (B).

22. Since Pramod runs at half the speed of Prakash, they meet once for every round Pramod makes or for every 2 round Prakash makes.

$\therefore$ If they meet 11 times, Pramod makes 11 round and Prakash makes 22 round i.e., Prakash covers 11 times the circumference more than Pramod, i.e., 11(22/7) (7) (2)m = 484 m

Hence, the correct option is (D).

23. Since Peter and Paul start at A and have the ratio of their speeds as 1 : 2, first time when they meet say at S, the distances covered by them respectively were in the ratio 1 : 2, i.e., Peter would have covered 200 m and Paul would have covered 400 m



At the point S, the ratio of speeds will now become 2 : 1

So, Peter will cover 400 m and reach A and Paul will cover 200 m and reach A. So, the 2nd meeting point will be at A, the 3rd meeting point will be at S and the 4th meeting point will be at A and so on.

So, by the time they meet for the 8th time i.e., at A, Peter would have covered 4 rounds.

Now, both are at A.

When Peter covers 1/4th round, (i.e., 150 m),

Paul will cover 1/2 round, (i.e., 300 m)

Distance between them is either or 150 + 300

= 450 m or 600 – (150 + 300) = 150 m.

Hence, the correct option is (A).

Solutions for questions 24 and 25:

24. Time taken by Train U to exit the tunnel

$$= \frac{\text{Length of the tunnel + Length of the train U}}{\text{Speed of B}}$$

$$= \frac{200 + 150}{25}$$

Train U exits the tunnel after 350/25 = 14 sec.

In 14 sec, front end of train T's engine travels

$= 14 \times 15 = 210$ mts

That means the front end of engine is 10 mts out of the tunnel.

$\therefore$ The length of the train T still in the tunnel

$= 100 - 10 = 90$ m

Hence, the correct option is (C).

25. Let the point at which the ends cross each other be x mts from the point of entry of the slower train.

$\therefore$ The time elapsed before the ends meet for the two trains will be equal to $\frac{100+x}{15} = \frac{200-x+150}{25}$

$\Rightarrow 500 + 5x = 1050 - 3x$

$8x = 550$

$x = 68.75$ mts

Alternate method:

Distance between the rear ends of the two trains, at the instant of entry = 100 + 200 + 250 = 450 metres.

Time taken for the two ends to meet

= (distance)/(relative speed)

= 450/(15 + 25) = 11.25 seconds.

Distance covered by the slower train

$= 11.25 \times 15 = 168.75$ metres.

The required distance = 168.75 – (length of slower train)

= 168.75 – 100 = 68.75 metres.

Hence, the correct option is (D).

Solutions for questions 26 and 27:

26. Distance ran by the thief in 10 minutes = 1 km

Distance moved by the jeep in 10 minutes = 1×5 km

Distance between the jeep and the thief when the jeep found a gap in the median = 2.5 km

Additional time in which the jeep would overtake the thief $= \frac{2 \cdot 5}{9-6} = \frac{5}{6}$ hours = 50 minutes

$\therefore$ Total time = 60 minutes.

Hence, the correct option is (D).

27. Let the speeds of the faster and the slower runners be x kmph and y kmph respectively. Time to meet for the first time is:

$\frac{3}{x-y}$ hrs, and this is given as 1 hour.

Hence $[3/(x-y)] = 1$ (22)

Time taken, by the faster and the slower runners respectively, for one lap are $\frac{3}{x}$ and $\frac{3}{y}$ hrs. As the difference of the above times is given as 2 minutes,

$$\frac{3}{x} = \frac{3}{y} - \frac{1}{30} \quad (23)$$

From (22), we get $y = x - 3$ which, when substituted in (23) and solved for y, we get $y = 15$ kmph.

Alternate method:

The faster runner overtakes the slower runner once in every hour. This implies that the number of rounds (or laps) of the track made by the faster in 1 hour is more than the rounds made by of the slower by 1. Let the slower complete n rounds in 1 how; then the faster completes $(n + 1)$ rounds in 1 hour.

Time taken by the faster to complete 1 round is less than that of the slower by 2 minutes, i.e., $(1/30)^{th}$ hour.

$\Rightarrow \frac{1}{n} - \frac{1}{n+1} = \frac{1}{30};$

$\Rightarrow \frac{1}{n(n+1)} = \frac{1}{30};$

$\Rightarrow n^2 + n - 30 = 0; (n + 6)(n - 5) = 0$

Ignoring the negative value, $n = 5$.

Track length is 3 km; hence for 5 rounds, the distance covered is 15 km; and the speed is 15 kmph.

Hence, the correct option is (C).

Solutions for questions 28 and 29:

Let us say Anil turned back after t hr. He would have covered 50 t km then. Chetan would have covered 10 t km.

$\therefore$ The two would be 40t km apart. They would meet in

$$\frac{40t}{10+50} \text{ more hr .i.e., } \frac{2t}{3} \text{ hr.}$$

In this time, Chetan would have covered $\frac{20t}{3}$ km while Anil would have covered $\frac{100t}{3}$ km. Also Bala would have covered $\frac{2t}{3}$ km.

When Anil picks up Chetan he would be 40 t km behind Bala. Also the ratio of the speeds of Anil and Bala is 5 : 1. Anil (with Chetan) would catch up with Bala, when he covers $5x$ and Bala covers x

$\therefore 5x - x = 40\ t.$

The bike and Bala would have covered distances of 50 t km and 10 t km respectively

$$XY = 10\ t + \frac{20}{3}t + 50t = \frac{200t}{3} \text{ km} = 40 \text{ km}$$

$$\Rightarrow t = \frac{3}{5}$$

28. Time after which Anil turned back = $\frac{3}{5}$ hours = 36 minutes

Hence, the correct option is (C).

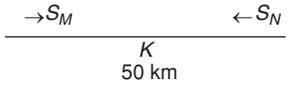
29. Bala's average speed = $\frac{40}{t + \frac{2}{3}t + t} = 25$ km/hr

Hence, the correct option is(D).

Note: The distance covered by Chetan until he was picked up by Anil is the same as that covered by Bala from the point he was dropped.

Solutions for question 30:

30.



In the above diagram, we have S_M and S_N as the speed of M and N and K is the meeting pointing of M and N.

As distance = (Relative speed) (time),

$(S_M + S_N)5 = 50.$

$S_P + S_Q = 10$ (24)

After M and N meet, they move towards their destinations which they reach at the same time; i.e., the times of travel are equal.

Hence $\frac{5S_M}{S_N + 1} = \frac{5S_N}{S_M - 1}$

$S_M^2 - S_M = S_N^2 + S_N$ (25)

Substituting (24) in (25) and solving we have

$S_M = 5.5$ kmph

Alternate method:

If 't' is the time taken to meet, and t_1, t_2 are the durations taken after the meeting to reach the respective destinations then, $t = \sqrt{t_1\ t_2}$. But it is given $t_1 = t_2$.

Hence $t = \sqrt{t_1^2}$ or $t = t_1 = t_2$ (26)

But it is given $t = 5$

$\therefore\ t = t_1 = t_2 = 5$ hours.

M travelled for 5 hours at x kmph and for another 5 hours at $(x - 1)$ kmph.

$\therefore\ 5x + 5(x - 1) = 50 \Rightarrow x = 5.5$ kmph

Hence, the correct option is (B).

EXERCISES

Practice Problems 1

Directions for questions 1 to 30: Select the correct alternative from the given choices.

1. Simplify the following:

$$\left(\frac{243}{1024}\right)^{-2/5}\times\left(\frac{144}{49}\right)^{-1/2}\div\left(\frac{8}{343}\right)^{-2/3}$$

(A) $2^5\times3^{-1}\times7$ (B) $2^5\times3^{-3}\times7^{-2}$
(C) $2^4\times3^{-3}\times7^{-1}$ (D) $2^4\times3^{-1}\times7^{-2}$

2. Simplify the following:

$$\left(\frac{x^2\cdot y^{-3}}{z^4}\right)^{-2}\times\left(\frac{x^2\cdot y}{z^{-2}}\right)^{3}\div\left(\frac{x^{-12}\cdot y^{7}}{z^{-8}}\right)^{-1}$$

(A) $x^{10}.\,y^{16}\,.\,z^{-22}$ (B) $x^{7}.\,y^{-16}\,.\,z^{-22}$
(C) $x^{-7}.\,y^{16}\,.\,z^{-22}$ (D) $x^{-10}.\,y^{16}\,.\,z^{22}$

3. Simplify the following: $\dfrac{1-[1-\{1-(1+y)^{-1}\}]}{(1-y)}$

(A) $\dfrac{y}{(1-y^2)}$ (B) $\dfrac{y}{(1-y)^2}$
(C) $\dfrac{1+y}{(1-y)^2}$ (D) $\dfrac{1+y^2}{1-y^2}$

4. $(x^{a-b})^{(a^2+ab+b^2)}\times(x^{b-c})^{(b^2+bc\,c^2)}\times(x^{c-a})^{(c^2+ac+a^2)}$

(A) 0
(B) 1
(C) $x^{a^3+b^3+c^3}$
(D) $x^{3\left(a^2+b^2+c^2+ac+bc+ca\right)}$

5. $343^{0.12}\times2401^{0.08}\times49^{0.01}\times7^{0.1}=$

(A) 7 (B) $7^{4/5}$ (C) 7^8 (D) $7^{3/5}$

6. Solve for x: $9^{2x+1}=27^{5x-3}$

(A) 1 (B) 2 (C) –1 (D) –2

7. If $\dfrac{p}{q}=\dfrac{r}{s}$ and $p^a=q^b=r^c=s^d$, then $\dfrac{1}{a}-\dfrac{1}{b}=$

(A) $\dfrac{1}{d}-\dfrac{1}{c}$ (B) $\dfrac{1}{c}+\dfrac{1}{d}$
(C) $\dfrac{1}{c}-\dfrac{1}{d}$ (D) $-\left(\dfrac{1}{c}+\dfrac{1}{d}\right)$

8. Which of the following is the largest in value?

(A) $6^{1/2}$ (B) $7^{1/3}$ (C) $8^{1/4}$ (D) $9^{1/5}$

9. $2\sqrt{\dfrac{5}{2}}-5\sqrt{\dfrac{2}{5}}+\sqrt{10}+\sqrt{1000}=$

(A) $9\sqrt{10}$ (B) $8\sqrt{10}$
(C) $8\sqrt{10}$ (D) $11\sqrt{10}$

10. $\left(\dfrac{\sqrt{p}-\sqrt[4]{pq}}{\sqrt[4]{pq}-\sqrt{q}}\right)^{-4}=$

(A) $\dfrac{p}{q}$ (B) $\sqrt{\dfrac{p}{q}}$
(C) $\sqrt{\dfrac{q}{p}}$ (D) $\dfrac{q}{p}$

11. If $y=12+2\sqrt{35}$, then $\sqrt{y}-\dfrac{1}{\sqrt{y}}=$

(A) $\dfrac{\sqrt{7}+\sqrt{5}}{2}$ (B) $\dfrac{3\sqrt{5}-\sqrt{7}}{2}$
(C) $\dfrac{2\sqrt{5}+\sqrt{7}}{2}$ (D) $\dfrac{3\sqrt{5}+\sqrt{7}}{2}$

12. Arrange the following in ascending order.

$a=\sqrt{2}+\sqrt{11}$, $b=\sqrt{6}+\sqrt{7}$,
$c=\sqrt{3}+\sqrt{10}$ and $d=\sqrt{5}+\sqrt{8}$

(A) *abcd* (B) *abdc*
(C) *acdb* (D) *acbd*

13. Arrange the following in descending order.

$a=\sqrt{13}+\sqrt{11}$, $b=\sqrt{15}+\sqrt{9}$, $c=\sqrt{18}+\sqrt{6}$,
$d=\sqrt{7}+\sqrt{17}$.

(A) *abdc* (B) *dcab*
(C) *adcb* (D) *acdb*

14. Solve for x and y:

$3.5^x+2^{y+2}=107$, $5^{x+1}+8.2^y=189$

(A) 3, 2 (B) 5, 7 (C) 7, 5 (D) 2, 3

15. Solve for x, if $\left(5\sqrt{7}\right)^{5x-4}=(35)^3\,(25)^{3/2}$.

(A) 2 (B) 5/4 (C) 7/2 (D) 3

16. If $5^{x+3}-5^{x-3}=78120$, find x.

(A) 4 (B) 3 (C) 5 (D) 6

17. If $a^a\cdot b^b\cdot c^c=a^b\cdot b^c\cdot c^a=a^c\cdot b^a\cdot c^b$ and a, b, c are positive integers greater than 1, then which of the following can NOT be true for any of the possible values of a, b, c?

(A) $abc=8$ (B) $a+b+c=8$
(C) $abc=27$ (D) $a+b+c=27$

18. The ascending order of $16^{\frac{7}{12}}$, $81^{\frac{3}{8}}$, $625^{\frac{2}{3}}$ is _____.

(A) $16^{\frac{7}{12}}$, $81^{\frac{3}{8}}$, $625^{\frac{2}{3}}$ (B) $16^{\frac{7}{12}}$, $625^{\frac{2}{3}}$, $81^{\frac{3}{8}}$,
(C) $625^{\frac{2}{3}}$, $16^{\frac{7}{12}}$, $81^{\frac{3}{8}}$ (D) $81^{\frac{3}{8}}$, $16^{\frac{7}{12}}$, $625^{\frac{2}{3}}$

19. If $2\sqrt{2}+\sqrt{3}=x$, what is the value of $\dfrac{11+4\sqrt{6}}{2\sqrt{2}-\sqrt{3}}$ in terms of x?

(A) $\frac{x^2}{\sqrt{2}}$ (B) x^3

(C) $\frac{x^3}{8}$ (D) $\frac{x^3}{5}$

20. Simplify: $\sqrt{(a+b+c)+2\sqrt{ac+bc}}$.

(A) $\sqrt{a}+\sqrt{b}+\sqrt{c}$ (B) $\sqrt{a+b}+\sqrt{c}$

(C) $\sqrt{ab+bc}$ (D) $\sqrt{abc}$

21. Find the value of $x^2 - y^2$, if $\log_y(x-1) + \log_y(x+1) = 2$.

(A) 2 (B) $2y$ (C) 1 (D) $2xy$

22. If $a > 1$, $\log_a a + \log_{a^{\frac{1}{2}}} a + \log_{a^{\frac{1}{3}}} a + \cdots + \log_{a^{\frac{1}{20}}} a =$

(A) 420 (B) 210 (C) 380 (D) 190

23. If $\log_7(x-7) + \log_7(x^2+7x+49) = 4$, then $x =$

(A) 196 (B) 7 (C) 49 (D) 14

24. If $\frac{\log a}{5} = \frac{\log b}{6} = \frac{\log c}{7}$, then $b^2 =$

(A) ac (B) a^2

(C) bc (D) ab

25. What is the value of $\log_{(1/5)} 0.0000128$?

(A) –7 (B) –5

(C) 5 (D) 7

26. If $(\log\tan5°)(\log\tan10°)(\log\tan15°)\ldots(\log\tan60°)=x$, what is the value of x?

(A) $\log(\sin5°)^{12}$ (B) 1

(C) 0 (D) $\log(\cos60°)$

27. Solve for x, if $\log_x[\log_5(\sqrt{x+5}+\sqrt{x})] = 0$.

(A) 1 (B) 9 (C) 12 (D) 4

28. If a, b, c are distinct values, what is the value of abc if $(\log_b a)(\log_c a) + (\log_a b)(\log_c b) + (\log_a c)(\log_b c) - 3 = 0$?

(A) 2

(B) 1

(C) $1 - \log a - \log b - \log c$

(D) 0

29. If $\log_6 161 = a$, $\log_6 23 = b$, what is the value of $\log_7 6$ in terms of a and b?

(A) a/b (B) $a+b$

(C) $1/(a-b)$ (D) b/a

30. $x = y^2 = z^3 = w^4 = u^5$, then find the value of $\log_x xyzwu$.

(A) $1\frac{47}{60}$ (B) $\frac{111}{120}$

(C) $2\frac{17}{60}$ (D) $2\frac{13}{60}$

Practice Problems 2

Directions for questions 1 to 30: Select the correct alternative from the given choices.

1. Simplify the following: $\frac{5^{2a-5}\times25^{a/2}\times125^{a+3}}{(3125)^{3a/5}\times625^{a+1}\times5^{-a}}$

(A) 5 (B) 25 (C) 1 (D) 125

2. $\frac{(x^a)^{2c}\cdot(x^b)^{2a}\ (x^c)^{2b}}{(x^{a+b})^c\cdot(x^{b+c})^a\cdot(x^{c+a})^b}$

(A) 0 (B) $x^{2a+2b+2c}$

(C) 1 (D) $x^{4ab+4ac+4bc}$

3. Simplify: $\frac{3}{\sqrt{5}+\sqrt{2}} + \frac{1}{\sqrt{6}+\sqrt{5}}$

(A) $\sqrt{6}-\sqrt{2}$

(B) $\sqrt{6}+\sqrt{2}$

(C) $\sqrt{5}+\sqrt{6}$

(D) $\sqrt{2}-\sqrt{6}$

4. $\frac{9}{6^{\frac{2}{3}} - 18^{\frac{1}{3}} + 3^{\frac{2}{3}}} =$

(A) $\left(6^{\frac{1}{3}}+3^{\frac{1}{3}}\right)$ (B) $(1/3)\left(6^{\frac{1}{3}}+3^{\frac{1}{3}}\right)$

(C) $\left(6^{\frac{1}{3}}-3^{\frac{1}{3}}\right)$ (D) $(1/9)\left(6^{\frac{1}{3}}+3^{\frac{1}{3}}\right)$

5. $\left(\sqrt{324+2\sqrt{323}}\right)-\left(\sqrt{324-2\sqrt{323}}\right) =$

(A) 2 (B) 1

(C) 36 (D) $2\sqrt{323}$

6. Arrange the following in ascending order.

$p = \sqrt{26}-\sqrt{23}$, $q = \sqrt{18}-\sqrt{15}$,

$r = \sqrt{11}-\sqrt{8}$, $s = \sqrt{24}-\sqrt{21}$

(A) $rqsp$ (B) $psrq$

(C) $pqrs$ (D) $psqr$

7. If $(3^{x+1}) + (4^{y-1}) = 73$ and $4(3^x) + 3(4^{y-2}) = 60$, find $x + y$.

(A) 5 (B) 4

(C) –1 (D) –2

8. Simplify : $\left(\frac{a}{\sqrt{b}-\sqrt{c}}+\frac{a}{\sqrt{b}+\sqrt{c}}\right)^2$

(A) $\frac{2a^2c}{(b-c)^2}$ (B) $\frac{(b-c)^2}{4a^2b}$

(C) $\frac{4a^2b}{(b-c)^2}$ (D) $\frac{2a^2}{b^2+c^2-a^2}$

9. If $x = \sqrt[3]{55+12\sqrt{21}}$ then the value of $x+\frac{1}{x}$ is _____.

(A) 4 (B) 5

(C) 6 (D) 7

10. If $a = 4(b - 1)$ and $b \neq 2$, then simplify $\sqrt{\frac{b-\sqrt{a}}{b+\sqrt{a}}} + \sqrt{\frac{b+\sqrt{a}}{b-\sqrt{a}}} - \frac{4}{\sqrt{\left(b-\sqrt{a}\right)\left(b+\sqrt{a}\right)}}$.

(A) $\sqrt{b}$ (B) 2

(C) $\frac{b}{b-2}$ (D) $(b-2)^{1/2}$

11. If $x = 5 - \sqrt{21}$, find the value of $\frac{\sqrt{x}}{\sqrt{32-2x-\sqrt{21}}}$.

(A) $\frac{\sqrt{7}-\sqrt{3}}{2}$ (B) $\frac{\sqrt{7}-\sqrt{3}}{\sqrt{3}}$

(C) $\frac{\sqrt{7}-\sqrt{3}}{\sqrt{2}}$ (D) $\frac{\sqrt{7}-\sqrt{3}}{3}$

12. Arrange $a, b, c,$ and d in descending order if $a = \sqrt{13} + \sqrt{9}$, $b = \sqrt{19} + \sqrt{3}$, $c = \sqrt{17} + \sqrt{5}$, $d = \sqrt{12} + \sqrt{10}$.

(A) *b, c, a, d* (B) *d, b, a, c*

(C) *d, b, c, a* (D) *d, a, c, b*

13. If $x = \sqrt{\frac{6}{4+\frac{6}{2+\sqrt{\frac{6}{4+\frac{6}{2+\cdots}}}}}}$, find the value of x.

(A) $\frac{9+\sqrt{33}}{2}$ (B) 1

(C) $\frac{9-\sqrt{43}}{2}$ (D) $\frac{9-\sqrt{33}}{4}$

14. If $\sqrt{x}^{\sqrt{x}^{\sqrt{x}^{\cdots\infty}}} = \frac{1}{4}$, find x.

(A) $\frac{1}{2^{12}}$ (B) $\frac{1}{2^{10}}$ (C) $\frac{1}{2^{16}}$ (D) $\frac{1}{2^{14}}$

15. Find the value of $\sqrt{7-3\sqrt{5}}$.

(A) $\sqrt{7}-2\sqrt{3}$ (B) $\frac{3-\sqrt{5}}{\sqrt{2}}$

(C) $\frac{\sqrt{3}-\sqrt{7}}{2\sqrt{2}}$ (D) $\frac{\sqrt{5}+\sqrt{2}}{3}$

16. If $a = \frac{2}{2+\sqrt{3}}$, $b = \frac{1}{2-\sqrt{3}}$, what is the value of $7b^2 + 11ab - 7a^2$?

(A) $-14 + 21\sqrt{3}$ (B) $11 + 56\sqrt{3}$

(C) $49 + 8\sqrt{3}$ (D) $\sqrt{3} + 11$

17. If $x = \sqrt[3]{2} + \sqrt[3]{4}$, what is the value of $x^3 - 6x$?

(A) 0 (B) 4 (C) –6 (D) 6

18. If $(\sqrt{6} + \sqrt{5})^{x^2-2} + (\sqrt{6} - \sqrt{5})^{x^2-2} = 22$, then $x =$

(A) $\pm\sqrt{2}, 0$ (B) $\pm 2, \pm 1$

(C) $0, \pm 2$ (D) $\pm 2, 1$

19. If $x = 3 + \sqrt{5}$, then find the value of $x^3 - 9x^2 + 22x$.

(A) –15 (B) 12 (C) 42 (D) 45

20. If $a \times 5^2 = 2020.20$, what is the value of $\frac{a \times 10^{-3}}{10^4}$?

(A) 0.0000808080

(B) 0.00000808080

(C) 0.000000808080

(D) 0.0000202020

21. Given that x, y, z are consecutive positive integers, which of the following is the value of $\log(xz + 1)$?

(A) $\log(x + y + z)$ (B) 0

(C) $1 - \log y$ (D) $2 \log y$

22. $5^y = (0.5)^x = 1000$, what is the value of $\left(\frac{1}{y} - \frac{1}{x}\right)$?

(A) 1/3 (B) 0.95 (C) 10 (D) 1

23. If $\log_r 6 = a$, $\log_r 3 = b$, what is the value of $\log_r (r/2)$ in terms of a and b?

(A) $1 - a + b$ (B) $ab - a - b$

(C) $a + b$ (D) 1

24. If $\log_{(x+y)} (x - y) = 3$, then what is the value of $\log_{(x^2-y^2)} \left(x^2 + 2xy + y^2\right)$?

(A) 0 (B) 2

(C) $\frac{\sqrt{3}}{2}$ (D) 1/2

25. Which of the following can never be a value of $\log_{2x} \frac{2x}{3y} + \log_{3y} \frac{3y}{2x}$ given that $2x \geq 3y$ and $y > \frac{1}{3}$?

(A) –2.5 (B) –2 (C) 1 (D) –1.5

26. If $a = \sqrt{b} = \sqrt[3]{c} = \sqrt[4]{d} = \sqrt[5]{e}$, then find the value of $\log_a (abcde)$.

(A) 15 (B) 13 (C) 14 (D) 10

27. If log2 = 0.3010, then find the value of log6250.

(A) 3.294 (B) 3.486

(C) 3.664 (D) 3.796

28. For how many positive integer values of x less than 100, is $\log_{5x} (4x - 15) > \frac{1}{2}$?

(A) 96 (B) 90 (C) 95 (D) 94

29. Find the total number of digits in 1764^{50}, if log 2 = 0.301, log 3 = 0.4771 and log 7 = 0.845.

(A) 161 (B) 163 (C) 162 (C) 164

30. If $\frac{1}{x-1} = \log_{bc} a$, $\frac{1}{y-1} = \log_{ac} b$ and $\frac{1}{z-1} = \log_{ab} c$, then which of the following is true?

(A) $\frac{1}{x} + \frac{1}{y} = \frac{1}{z}$ (B) $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$

(C) $x + y + z = 1$ (D) $xy + yz + zx = 1$

HINTS/SOLUTIONS

Practice Problems I

1. $\left[\frac{3^5}{4^5}\right]^{\frac{-2}{5}} \times \left[\frac{12^2}{7^2}\right]^{\frac{-1}{2}} \times \left[\frac{8}{343}\right]^{\frac{2}{3}}$

$\frac{16}{9} \times \frac{7}{12} \times \frac{4}{49} = 2^4 \times 3^{-3} \times 7^{-1}$

Hence, the correct option is (C).

2. $\frac{x^{-4} \times y^6}{z^{-8}} \times \frac{x^6 \times y^3}{z^{-6}} \times \frac{x^{-12} \times y^7}{z^{-8}}$

$= \frac{x^{-4+6-12} . y^{6+3+7}}{z^{-8-6-8}} = x^{-10} . y^{16} . z^{22}$

Hence, the correct option is (D).

3. $\frac{1-\left[1-\left\{1-\frac{1}{1+y}\right\}\right]}{(1-y)} = \frac{1-\left(1-\frac{y}{(1+y)}\right)}{(1-y)}$

$= \frac{\left(1-\frac{1}{1+y}\right)}{(1-y)} = \frac{y}{(1+y)(1-y)} = \frac{y}{1-y^2}$

Hence, the correct option is (A).

4. $(a-b)(a^2+ab+b^2) = a^3 - b^3$

Similarly the other powers of x become $b^3 - c^3$ and $c^3 - a^3$

$x^{a^3-b^3} \times x^{b^3-c^3} \times x^{c^3-a^3}$

$= x^{a^3-b^3+b^3-c^3+c^3-a^3} = x^0 = 1$

Hence, the correct option is (B).

5. $343^{0.12} \times 2401^{0.08} \times 49^{0.01} \times 7^{0.1}$

$= (7^3)^{0.12} \times (7^4)^{0.08} \times (7^2)^{0.01} \times 7^{0.1}$

$= 7^{0.36} \times 7^{0.32} \times 7^{0.02} \times 7^{0.1}$

$= 7^{0.36+0.32+0.02+0.1} = 7^{0.8} = 7^{8/10} = 7^{4/5}$

Hence, the correct option is (B).

6. $3^{2(2x+1)} = 3^{3(5x-3)}$

$\Rightarrow 2(2x+1) = 3(5x-3)$

$\Rightarrow 4x + 2 = 15x - 9$

$\Rightarrow 11x = 1 \quad \Rightarrow \quad x = 1$

Hence, the correct option is (A).

7. Let

$pa = qb = rc = s^d = k$

$\Rightarrow p = k^{1/a}$

$q = k^{1/b}$

$r = k^{1/c}$, $s = k^{1/d}$

given $\frac{p}{q} = \frac{r}{s}$; $\frac{k^{1/a}}{k^{1/b}} = \frac{k^{1/c}}{k^{1/d}}$.

$k^{\frac{1}{a}-\frac{1}{b}} = k^{\frac{1}{c}-\frac{1}{d}}$

Equating powers of k on both sides, we get

$\frac{1}{a} - \frac{1}{b} = \frac{1}{c} - \frac{1}{d}$

Hence, the correct option is (C).

8. $6^{1/2}$, $7^{1/3}$, $8^{1/4}$, $9^{1/5}$ take the LCM of denominators of the powers of the numbers. LCM = 60

Raise the numbers with this LCM

$(6^{1/2})^{60}$, $(7^{1/3})^{60}$, $(8^{1/4})^{60}$, $(9^{1/5})^{60}$

6^{30}, 7^{20}, 8^{15}, 9^{12}

Between 6^{30} and 7^{20}

$(6^3)^{10}$ and $(7^2)^{10}$

216^{10} and 49^{10}

$216^{10} > 49^{10}$ i.e. $6^{30} > 7^{20}$

Between 6^{30} and 8^{15}

$(6^2)^{15}$ and 8^{15}

$36^{15} > 8^{15}$ i.e. $6^{30} > 8^{15}$

Between 6^{30} and 9^{12}

$(6^5)^6$ and $(9^2)^6$

$(7776)^6 > 81^6$

i.e. $6^{30} > 9^{12}$

$\therefore 6^{1/2}$ is largest in value

Hence, the correct option is (A).

9. $2\sqrt{\frac{5}{2}} - 5\sqrt{\frac{2}{5}} + \sqrt{10} + \sqrt{1000}$

$= \sqrt{2}\sqrt{5} - \sqrt{5}\sqrt{2} + \sqrt{10} + 10\sqrt{10}$

$= 11\sqrt{10}$

Hence, the correct option is (D).

10. $\sqrt{p} - \sqrt[4]{pq} = \sqrt[4]{p}\left(\sqrt[4]{p} - \sqrt[4]{q}\right)$

$\sqrt[4]{pq} - \sqrt{q} = \sqrt[4]{q}\left(\sqrt[4]{p} - \sqrt[4]{q}\right)$

Required value $= \left(\frac{\sqrt[4]{p}\left(\sqrt[4]{p} - \sqrt[4]{q}\right)}{\sqrt[4]{q}\left(\sqrt[4]{p} - \sqrt[4]{q}\right)}\right)^{-4}$

$\left(\sqrt[4]{\frac{p}{q}}\right)^{-4} = \left(\left(\frac{p}{q}\right)^{\frac{1}{4}}\right)^{-4} = \left(\frac{p}{q}\right)^{-4\times\frac{1}{4}} = \left(\frac{p}{q}\right)^{-1} = \frac{q}{p}$

Hence, the correct option is (D).

11. Given $y = 12 + 2\sqrt{35} = \left(\sqrt{7} + \sqrt{5}\right)^2$

$\therefore \sqrt{y} = \sqrt{7} + \sqrt{5}$, $\frac{1}{\sqrt{y}} = \frac{1}{\sqrt{7}+\sqrt{5}} = \frac{\sqrt{7}-\sqrt{5}}{2}$

$$\sqrt{7}+\sqrt{5}-\left(\frac{\sqrt{7}-\sqrt{5}}{2}\right)=\frac{\sqrt{7}+3\sqrt{5}}{2}$$

$$\therefore\ \sqrt{y}-\frac{1}{\sqrt{y}}=\frac{3\sqrt{5}+\sqrt{7}}{2}$$

Hence, the correct option is (D).

12. $a^2 = 13 + 2\sqrt{22}$

$b^2 = 13 + 2\sqrt{42}$

$c^2 = 13 + 2\sqrt{30}$

$d^2 = 13 + 2\sqrt{40}$

$\Rightarrow a^2 < c^2 < d^2 < b^2$

$\therefore a < c < d < b$

Hence, the correct option is (C).

13. Given $a = \sqrt{13}+\sqrt{11}$, $b = \sqrt{15}+\sqrt{9}$,

$c = \sqrt{18}+\sqrt{6}$ and $d = \sqrt{7}+\sqrt{17}$.

In the given surds, the sum of the terms of each of the surds is the same at 24. Then the surd containing the terms as close as possible is the greatest, and the surd containing the terms as far as possible is the smallest.

Hence $a > b > d > c$.

Hence, the correct option is (A)

14. Let $5x = a$, $2^y = b$. $3a + 4b = 107$; $5a + 8b = 189$

Solving we get $a = 5x = 25 = 5^2$

$\therefore x = 2$

$b = 2^y = 8 = 2^3$. $\therefore y = 3$

Hence, the correct option is (D).

15. $(35)^3\ (25)^{3/2} = (7\ x\ 5)^3\ (5)^3$

$\left(\sqrt{7}\right)^6 (5)^6 = \left(5\sqrt{7}\right)^6 = \left(5\sqrt{7}\right)^{5x-4}$ $\therefore\ 5x - 4 = 6$

$x = 10/5 = 2$

Hence, the correct option is (A).

16. Given $5x^{+3} - 5x^{-3} = 78120$

$\Rightarrow 5^x\left[5^3 - \frac{1}{5^3}\right] = 78120$

$\Rightarrow 5x\ [5^6 - 1] = 78120 \times (5^3)$

$\Rightarrow 5x\ [15624] = 78120 \times (5^3)$

$\Rightarrow 5x = 5^4\ \ \therefore x = 4$

Hence, the correct option is (A)

17. $a^a \times b^b \times c^c = a^b \times b^c \times c^a$

$\Rightarrow a^{a-b} \times b^{b-c} \times c^{c-a} = 1$

Since a, b, c are positive integers > 1

$\Rightarrow a - b = 0$, $b - c = 0$ and $c - a = 0 \Rightarrow a = b = c$.

Choice (A) can be true for $a = b = c = 2$

Choice (B) can never be true for any of the possible values of a, b, c, since $a + b + c = 3a$ and $3a \neq 8$ for any integral value of 'a'.

Similarly $abc = 27$ for $a = b = c = 3$ and $a + b + c = 27$ for $a = b = c = 9$

Hence, the correct option is (B).

18. $16^{\frac{7}{12}} = \left(2^4\right)^{\frac{7}{12}} = 2^{\frac{7}{3}}$

$81^{\frac{3}{8}} = \left(3^4\right)^{\frac{3}{8}} = 3^{\frac{3}{2}}$

$625^{\frac{2}{3}} = \left(5^4\right)^{\frac{2}{3}} = 5^{\frac{8}{3}}$

Let us first compare $2^{\frac{7}{3}}$ and $3^{\frac{3}{2}}$

When we raise both numbers to their sixth powers, we get 2^{14} and 3^9

$2^{14} = 2^{10}\ (2^4) = 1024(16) = 16384$

$3^9 = 3^6$, $3^3 = 729(27)$ which is more than $700(27)$ i.e., 18900

$3^9 > 2^{14}$ $\therefore\ 3^{\frac{3}{2}} > 2^{\frac{7}{3}}$

Let us now compare $3^{\frac{3}{2}}$ and $5^{\frac{8}{3}}$

When we raise both to their sixth powers, we get 3^9 and 5^{16}. $3^9 < 5^{16}$, since 3 is a lower base and 9 is a lower index.

$\therefore\ 3^{\frac{3}{2}} < 5^{\frac{8}{3}}$. Alternatively, we directly see that

$3 < 5$ and $\frac{3}{2} < \frac{8}{3}$. $\therefore\ 3^{\frac{3}{2}} < 5^{\frac{8}{3}}$

$2^{\frac{7}{3}} < 3^{\frac{3}{2}} < 5^{\frac{8}{3}}$

Hence, the correct option is (A).

19. It can be noticed that $11 + 4\sqrt{6}$

$= (2\sqrt{2} + \sqrt{3})^2 = x^2$

$\therefore\ \frac{11+4\sqrt{6}}{2\sqrt{2}-\sqrt{3}} = \frac{x^2}{2\sqrt{2}-\sqrt{3}}$;

Multiplying and dividing $2\sqrt{2}+\sqrt{3}$, it is equal to

$$\frac{2\sqrt{2}+\sqrt{3}}{2\sqrt{2}+\sqrt{3}} \cdot \frac{x^2}{2\sqrt{2}-\sqrt{3}} = x^3/5$$

Hence, the correct option is (D).

20. $\sqrt{(a+b+c)+2\sqrt{ac+bc}}$

$= \sqrt{(a+b)+c+2\sqrt{c(b+a)}}$

$= \sqrt{\left(\sqrt{a+b}+\sqrt{c}\right)^2} = \sqrt{a+b}+\sqrt{c}$

Hence, the correct option is (B).

21. $\log_y[(x-1)(x+1)] = 2$

$\Rightarrow \log_y(x^2 - 1) = 2$

$\Rightarrow x^2 - 1 = y^2 \Rightarrow x^2 - y^2 = 1$

Hence, the correct option is (C)

22. $\log_a a + \log_{a1/2} a + \log_{a1/3} a + \ldots. + \log_{a1/20} a$

$$= \frac{\log a}{\log a} + \frac{\log a}{\log a^{\frac{1}{2}}} + \frac{\log a}{\log a^{\frac{1}{3}}} + \ldots.. + \frac{\log a}{\log a^{\frac{1}{20}}}$$

$= 1 + 2 + 3 + \ldots. + 20$

$$= \frac{20\times 21}{2} = 210$$

Hence, the correct option is (B).

23. $\log_7(x-7) + \log_7(x^2 + 7x + 49) = 4$

$\Rightarrow \log_7[(x-7)(x^2 + 7x + 49)] = 4$

$\Rightarrow \log_7 (x^3 - 7^3) = 4$

$\Rightarrow (x^3 - 7^3) = 2401$

$\Rightarrow x^3 = 2744 = (14)^3 \Rightarrow x = 14$

Hence, the correct option is (D).

24. Let $\frac{\log a}{5} = \frac{\log b}{6} = \frac{\log c}{7} = k$

$\Rightarrow \log a = 5k \Rightarrow a = 10^{5k}$

$\log b = 6k \quad \Rightarrow b = 10^{6k}$

$\log c = 7k$

$\Rightarrow c = 10^{7k}$

$b^2 = (10^{6k})^2 = 10^{12k}$

$= 10^{7k} \times 10^{5k} = ac$

Hence, the correct option is (A).

25. $0.0000128 = \frac{128}{10^7} = \frac{2^7}{10^7} = \left(\frac{2}{10}\right)^7 = \left(\frac{1}{5}\right)^7$

$\therefore \log_{\left(\frac{1}{5}\right)} 0.0000128 = \log_{\left(\frac{1}{5}\right)} \left(\frac{1}{5}\right)^7 = 7$

Hence, the correct option is (D).

26. $\tan 45° = 1$

$\log (\tan 45°) = 0$

$\therefore$ Product $= 0$

$\therefore x = 0$

Hence, the correct option is (C).

27. Since $\log_x \log_5 \left(\sqrt{x+5} + \sqrt{x}\right) = 0$

$$\Rightarrow \log_5 \left(\sqrt{x+5} + \sqrt{x}\right) = 1$$

$$\Rightarrow \sqrt{x+5} + \sqrt{x} = 5$$

By observation of the choices, only Choice (D), i.e. $x = 4$ satisfies the above equation.

Alternate method:

$$\sqrt{x+5} = 5 - \sqrt{x}$$

Squaring both sides

$$\Rightarrow x + 5 = 25 - 10\sqrt{x} + x$$

$$\sqrt{x} = \frac{20}{10} = 2 \Rightarrow x = 4$$

Hence, the correct option is (D).

28. The given equation is :

$\log_b a \times \log_c a + \log_c b \log_a b + \log_a c \times \log_b c - 3 = 0$

$$\Rightarrow \frac{(\log a)^2}{\log b \cdot \log c} + \frac{(\log b)^2}{\log c \cdot \log a} + \frac{(\log c)^2}{\log a \cdot \log b} - 3 = 0$$

[(as $\log_b a = (\log a)/\log (b)$]

$\Rightarrow (\log a)^3 + (\log b)^3 + (\log c)^3$

$- 3 \log a \times \log b \times \log c = 0$

when $p^3 + q^3 + r^3 - 3pqr = 0$, either $p = q = r$

or $p + q + r = 0$

If $p = q = r$; $\log a = \log b = \log c$; $\Rightarrow a = b = c$

which is contrary to the data. Hence $p = q = r$ is not acceptable.

$\therefore p + q + r = 0; \qquad \Rightarrow \log a + \log b + \log c = 0$

$\Rightarrow \log abc = 0; \qquad \Rightarrow abc = 1$

Hence, the correct option is (B).

29. Given $a = \log_6 161 = \log_6 (23 \times 7) = \log_6 23 + \log_6 7$

$a = b + \log_6 7$

$a - b = \log_6 7$

$\log_7 6 = 1/(a - b)$

Hence, the correct option is (C).

30. Let $x = y^2 = z^3 = w^4 = u^5 = k$

$x = k$, $y = k^{1/2}$, $z = k^{1/3}$, $w = k^{1/4}$ and $u = k^{1/5}$

$\log_x xyzwu = \log_k k \cdot k^{1/2} \cdot k^{1/3} \cdot k^{1/4} \cdot k^{1/5}$

$$= 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} = \frac{137}{60} = 2\frac{17}{60}$$

Hence, the correct option is (C).

Practice Problems 2

Solutions for questions 1 to 30:

1. $$\frac{5^{2a-5} \times (5^2)^{\frac{a}{2}} \times (5^3)^{a+3}}{(5^5)^{\frac{3a}{5}} \times (5^4)^{a+1} \times 5^{-a}}$$

$$= \frac{5^{2a-5} \times 5^a \times 5^{3a+9}}{5^{3a} \times 5^{4a+4} \times 5^{-a}} = 5° = 1$$

Hence, the correct option is (C).

2. $$\frac{x^{2ac} \cdot x^{2ab} \cdot x^{2bc}}{x^{ac+bc} \cdot x^{ba+ca} \cdot x^{bc+ab}}$$

$$= \frac{x^{2ac} \cdot x^{2ab} \cdot x^{2bc}}{x^{ac+bc+ba+ca+bc+ab}}$$

$$= \frac{x^{2ac+2ab+2bc}}{x^{2ab+2ac+2bc}} = 1$$

Hence, the correct option is (C).

3. $\frac{3}{\sqrt{5}+\sqrt{2}} = \frac{3\left(\sqrt{5}-\sqrt{2}\right)}{\left(\sqrt{5}\right)^2-\left(\sqrt{2}\right)^2} = \sqrt{5}-\sqrt{2}$

$\frac{1}{\sqrt{6}+\sqrt{5}} = \frac{\sqrt{6}-\sqrt{5}}{\left(\sqrt{6}\right)^2-\left(\sqrt{5}\right)^2} = \sqrt{6}-\sqrt{5}$

$\frac{3}{\sqrt{5}+\sqrt{2}} + \frac{1}{\sqrt{6}+\sqrt{5}} = \sqrt{6}-\sqrt{2}$

Hence, the correct option is (A).

4. $\frac{9}{6^{\frac{2}{3}}-18^{\frac{1}{3}}+3^{\frac{2}{3}}} = \frac{\left(6^{\frac{1}{3}}\right)^3+\left(3^{\frac{1}{3}}\right)^3}{\left(6^{\frac{1}{3}}\right)^2-18^{\frac{1}{3}}+\left(3^{\frac{1}{3}}\right)^2}$

$= 6^{\frac{1}{3}}+3^{\frac{1}{3}}$

Hence, the correct option is (A).

5. $\sqrt{324+2\sqrt{323}} = \sqrt{\left(\sqrt{323}+1\right)^2} = \sqrt{323}+1$

$\sqrt{324-2\sqrt{323}} = \sqrt{\left(\sqrt{323}-1\right)^2} = \sqrt{323}-1$

Required value $= \sqrt{323}+1-\left(\sqrt{323}-1\right)$

$= \sqrt{323}+1-\sqrt{323}+1 = 2$

Hence, the correct option is (A)

6. Given $p = \sqrt{26}-\sqrt{23}$, $q = \sqrt{18}-\sqrt{15}$, $r = \sqrt{11}-\sqrt{8}$ and $s = \sqrt{24}-\sqrt{21}$.

In the given surds, the difference between the terms of each of the surds is the same at 3. Then the surd containing the greater terms is the smallest and smaller terms is the greatest.

Hence $p < s < q < r$.

Hence, the correct option is (D).

7. Let $3x = a$ and $4^{y-2} = b$

Given $3a + 4b = 73$ (1)

$4a + 3b = 60$ (2)

$4\,(1) \Rightarrow 12a + 16b = 292$

$3\,(2) \Rightarrow 12a + 9b = 180$

$- \quad -$

Subtracting $7b = 112$

$\Rightarrow b = 16$

From (2), $4a = 60 - 3b = 60 - 48 \Rightarrow a = 3$

$\therefore 3x = 3$ and $4^{y-2} = 16$

$\Rightarrow x = 1$ and $y - 2 = 2$

$\therefore x + y = 5$

Hence, the correct option is (A).

8. $\left(\frac{a}{\sqrt{b}-\sqrt{c}}+\frac{a}{\sqrt{b}+\sqrt{c}}\right)^2$

$= a^2\left(\frac{\sqrt{b}+\sqrt{c}+\sqrt{b}-\sqrt{c}}{b-c}\right)^2$

$= \frac{a^2\,(2\sqrt{b})^2}{(b-c)^2} = \frac{4a^2b}{(b-c)^2}$

Hence, the correct option is (C).

9. $x = \sqrt[3]{55+12\sqrt{21}}$

$x^3 = 55+12\sqrt{21}$

$= \sqrt{3025}+\sqrt{3024}$

$\frac{1}{x^3} = \frac{\left(\sqrt{3025}\right)^2-\left(\sqrt{3024}\right)^2}{\sqrt{3025}+\sqrt{3024}} = \sqrt{3025}-\sqrt{3024}$

$x^3 + \frac{1}{x^3} = 2(55) = 110$

$110 = \left(x+\frac{1}{x}\right)\left(\left(x+\frac{1}{x}\right)^2-3\right)$ (3)

Method 1

Only choice B satisfies this equation.

Method 2

x is positive . $x+\frac{1}{x}$ is positive. The R.H.S. of (3) increases with an increase in $x+\frac{1}{x}$. $\therefore$ Exactly one solution exists for $x+\frac{1}{x}$. When $x+\frac{1}{x} = 5$, (3) is satisfied.

$\therefore$ This is the only solution.

Hence, the correct option is (B).

10. Given expression $= \frac{b-\sqrt{a}+b+\sqrt{a}}{\sqrt{b^2-a}} - \frac{4}{\sqrt{b^2-a}}$

$= \frac{2(b-2)}{\sqrt{b^2-a}} = \frac{2(b-2)}{\sqrt{b^2-4(b-1)}}$;

since $a = 4(b-1)$

$= \frac{2(b-2)}{\sqrt{b^2-4b+4}} = \frac{2(b-2)}{\sqrt{(b-2)^2}} = \frac{2(b-2)}{(b-2)}$

$= 2$,

as it is given that $b \neq 2$.

$\therefore$ From the choices answer is (B)

Hence, the correct option is (B).

11. Given $x = 5-\sqrt{21} \Rightarrow x = \frac{1}{2}\left(10-2\sqrt{21}\right)$

$\sqrt{x} = \frac{1}{\sqrt{2}}\left(\sqrt{7}-\sqrt{3}\right)$

$32 - 2x = 22 + 2\sqrt{21} \;\; \therefore \;\; \sqrt{32-2x} = 1 + \sqrt{21}$

Consider $\dfrac{\sqrt{x}}{\sqrt{32-2x}-\sqrt{21}} = \dfrac{\frac{1}{\sqrt{2}}\left(\sqrt{7}-\sqrt{3}\right)}{\sqrt{21}+\sqrt{1}-\sqrt{21}}$

$= \dfrac{1}{\sqrt{2}}\left(\sqrt{7}-\sqrt{3}\right)$

Hence, the correct option is (C).

12. $a^2 = 22 + 2\sqrt{117}$; $b^2 = 22 + 2\sqrt{57}$;
$c^2 = 22 + 2\sqrt{85}$; $d^2 = 22 + 2\sqrt{120}$

$\therefore$ We get $d^2 > a^2 > c^2 > b^2$

$\therefore$ Descending order is d, a, c, b.

Hence, the correct option is (D).

13. Given $x = \sqrt{\dfrac{6}{4+\dfrac{6}{2+\sqrt{\dfrac{6}{4+\dfrac{6}{2+.....\infty}}}}}}$

$\Rightarrow \quad x = \sqrt{\dfrac{6}{4+\dfrac{6}{2+x}}}$

$\Rightarrow \quad x = \sqrt{\dfrac{6(2+x)}{4(2+x)+6}} = \sqrt{\dfrac{6(2+x)}{4x+14}}$

Squaring both sides

$x^2 = \dfrac{6x+12}{4x+14}$

$\Rightarrow 4x^3 + 14x^2 - 6x - 12 = 0$

By observation $x = 1$ satisfies the above equation.

$\Rightarrow (x - 1)(4x^2 + 18x + 12) = 0$

$\Rightarrow x = 1$ (or) $4x^2 + 18x + 12 = 0$

If $4x^2 + 18x + 12 = 0$, then the value of x must be negative. But from the given question it is clear that x cannot be negative.

Hence $x = 1$ is the only solution.

Hence, the correct option is (B).

14. $\sqrt{x}^{\sqrt{x}^{\sqrt{x}^{\cdots^{\infty}}}} = \dfrac{1}{4}$

$\Rightarrow \sqrt{x}^{\frac{1}{4}} = \dfrac{1}{4}$

$\Rightarrow \sqrt{x} = \left(\dfrac{1}{4}\right)^4$

$\Rightarrow x = \left(\dfrac{1}{4}\right)^8 = \dfrac{1}{2^{16}}$

Hence, the correct option is (C).

15. $\sqrt{7-3\sqrt{5}} = \sqrt{\dfrac{14-2\sqrt{45}}{2}}$

$= \sqrt{\dfrac{9+5-2\sqrt{9}\sqrt{5}}{2}} = \dfrac{\sqrt{9}-\sqrt{5}}{\sqrt{2}} = \dfrac{3-\sqrt{5}}{\sqrt{2}}$

Hence, the correct option is (B).

16. $a = \dfrac{1}{2+\sqrt{3}} = 2-\sqrt{3}$; $b = \dfrac{1}{2-\sqrt{3}} = 2+\sqrt{3}$

$7b^2 + 11ab - 7a^2 = 7(b + a)(b - a) + 11(a)(b)$

$= 7(2 + \sqrt{3} + 2 - \sqrt{3})(2 + \sqrt{3} - 2 + \sqrt{3})$

$+ 11(2 - \sqrt{3})(2 + \sqrt{3}) = 7(4)(2\sqrt{3}) + 11(4 - 3)$

$= 56\sqrt{3} + 11$

Hence, the correct option is (B).

17. $x = \sqrt[3]{2} + \sqrt[3]{4}$; $x - \sqrt[3]{2} = \sqrt[3]{4}$

$x^3 - 2 - 3x\sqrt[3]{2}\left(x - \sqrt[3]{2}\right) = 4$

$x^3 - 2 - 3x\sqrt[3]{2}\left(\sqrt[3]{4}\right) = 4$

$x^3 - 2 - 3x\sqrt[3]{8} = 4$

$x^3 - 2 - 6x = 4$

$x^3 - 6x = 6$

Alternate method:

$x = \sqrt[3]{2} + \sqrt[3]{4}$; on cubing both sides,

$x^3 = \left(\sqrt[3]{2}\right)^3 + \left(\sqrt[3]{4}\right)^3 + (3)\left(\sqrt[3]{2}\right)\left(\sqrt[3]{4}\right)\left(\sqrt[3]{2} + \sqrt[3]{4}\right)$

$\Rightarrow x^3 = 2 + 4 + (3)\sqrt[3]{8}\,(x)$

$\Rightarrow x^3 = 6 + 6x; \Rightarrow x^3 - 6x = 6$

Hence, the correct option is (D).

18. Let $\left(\sqrt{6}+\sqrt{5}\right)^{x^2-2} = a$

$\left(\sqrt{6}-\sqrt{5}\right)^{x^2-2} = \dfrac{1}{\left(\sqrt{6}+\sqrt{5}\right)^{x^2-2}} = \dfrac{1}{a}$

$\therefore$ the equation becomes $a + \dfrac{1}{a} = 22$

$\Rightarrow a^2 - 22a + 1 = 0$

$\Rightarrow a = \dfrac{22 \pm \sqrt{484-4}}{2}$

$= \dfrac{22 \pm 4\sqrt{30}}{2} = 11 \pm 2\sqrt{30}$

Case (i) $a = 11 + 2\sqrt{30}$

$\Rightarrow \left(\sqrt{6}+\sqrt{5}\right)^{x^2-2} = 11 + 2\sqrt{30} = \left(\sqrt{6}+\sqrt{5}\right)^2$

$\Rightarrow x^2 - 2 = 2 \Rightarrow x^2 = 4 \Rightarrow x = \pm 2$

Case (ii)

$a = 11 - 2\sqrt{30}$

$\Rightarrow \left(\sqrt{6}+\sqrt{5}\right)^{x^2-2} = \left(\sqrt{6}-\sqrt{5}\right)^{2} = \left(\sqrt{6}+\sqrt{5}\right)^{-2}$

$\Rightarrow x^2 - 2 = -2 \Rightarrow x = 0$

Hence, the correct option is (C).

19. $x = 3 + \sqrt{5}$

$\Rightarrow (x-3) = \sqrt{5}$

$\Rightarrow (x-3)^3 = 5\sqrt{5}$

$\Rightarrow x^3 - 9x^2 + 27x - 27 = 5\ \sqrt{5} = 5(x-3)$

$\Rightarrow x^3 - 9x^2 + 22x = 12$

Hence, the correct option is (B).

20. $a \times 5^2 = 2020.20$

$a \times (10/2)^2 = 2020.20$

$a \times (10)^2 = 8080.80$

$a = 8080.80/100 = 80.8080$

$$\frac{a\times 10^{-3}}{10^4} = \frac{a}{10^4 \times 10^3} = \frac{a}{10^7}$$

$$= \frac{80.8080}{10^7} = 0.00000808080$$

Hence, the correct option is (B).

21. $x = y - 1$

$y = y$

$z = y + 1$

$\log(xz + 1) = \log[y^2 - 1 + 1] = 2\log y$

Alternate method:

Such questions can be solved by numerical method. Assume the smallest numerical values satisfy the given conditions and substitute in the function.

1, 2 and 3 are three consecutive positive integers.

$\log(xz + 1) = \log[(1 \times 3) + 1] = \log 4$

For the values $x = 1$, $y = 2$, $z = 3$, first option $\log(x + y + z) = \log 6$;

Second option is zero; third option is $(1 - \log 2)$; and fifth option is $2\log 2$, which is $\log 2^2 = \log 4$. This is equal to $\log(xz + 1)$.

Hence, the correct option is (D).

22. Taking logarithms to the base 5, the given equation becomes, $\log_5 1000 = \log_{10} 1000$, $\log_5 10 = 3\log_5 10 = y$

$\log_{0.5} 1000 = 3\log_{0.5} 10 = x$

$$-\frac{1}{x} + \frac{1}{y} = \frac{-\log_{10} 0.5}{3} + \frac{\log_{10} 5}{3} = \frac{\log_{10} 10}{3} = 1/3$$

Alternate method:

Given that $5^y = 1000$,

$\Rightarrow 5 = 10^{(3/y)}$ (4)

$(0.5)x = 1000, \quad \Rightarrow 10^{(3/x)} = 0.5,$

$\Rightarrow \frac{5}{10} = 10^{(3/x)}$

$\Rightarrow 5 = 10(3/x + 1)$ (5)

(4) and (5) are equal; $10^{(3/y)} = 10^{(3/x + 1)}$;

$\Rightarrow \frac{3}{y} = \frac{3}{x} + 1, \Rightarrow \left(\frac{1}{y} - \frac{1}{x}\right) = \frac{1}{3}$

Hence, the correct option is (A)

23. $\log_r (r/2) = 1 - \log_r 2$

$1 - \log_r 6 + \log_r 3 = 1 - a + b$

Hence, the correct option is (A).

24. $\log_{x^2-y^2}\left(x^2 + 2xy + y^2\right)$

$$\frac{2\log(x+y)}{\log(x+y)+\log(x-y)} \quad [(\because \log_b a = (\log a)/(\log b)]$$

Considering the logarithm to the base $(x + y)$ the given function is $\frac{2}{1+\log_{(x+y)}(x-y)} = \frac{2}{1+3} = \frac{2}{4} = \frac{1}{2}$

Hence, the correct option is (D).

25. $\log_{2x}\frac{2x}{3y} + \log_{3y}\frac{3y}{2x}$

$= 1 - \log 2x3y + 1 - \log_{3y} 2x$

$= 2 - (\log 2x3y + \log_{3y} 2x)$

let $\log_{3y} 2x = p$

It is given that $y > \frac{1}{3}$

$\Rightarrow$ the base $3y > 1$

Also $2x \geq 3y$

$\Rightarrow \log_{3y} 2x \geq 1$

$\Rightarrow p \geq 1$

Consider $2 - (\log 2x\ 3y + \log_{3y} 2x)$

$$= 2 - \left(\frac{1}{p} + p\right)$$

since $p \geq 1$, the value of $\left(p + \frac{1}{p}\right) \geq 2$

Hence the maximum value of $2 - \left(p + \frac{1}{p}\right) = 0$

From the given options 1 cannot be the value of the given expression.

Hence, the correct option is (C).

26. Given $a = \sqrt{b} = 3\sqrt{c} = 4\sqrt{d} = 5\sqrt{e}$

$\Rightarrow b = a^2$, $c = a^3$ $d = a^4$ and $e = a^5$

$\therefore \log_a (abcde)$

$= \log_a (a)(a^2)(a^3)(a^4)(a^5)$

$= \log_a a^{15} = 15$

Hence, the correct option is (A).

27. $\log 6250$

$= \log 5^4 (10)$

$= 4\log 5 + \log 10$

$= 4 \log \frac{10}{2} + \log 10$

$= 4 (\log 10 - \log 2) + \log 10$

$= 4 (1 - 0 \times 301) + 1$

$= 4 (0 \times 699) + 1$

$= 2 \times 796 + 1 = 3.796$

Hence, the correct option is (D).

28. As $x = 1, 2, \ldots . 99$, $5x \geq 5$.

Given $\log_{5x} (4x - 15) > \frac{1}{2}$

$\Rightarrow (4x - 15) > \left(5x^{\frac{1}{2}}\right)$

$\because$ For a strong base (> 1) the log increases with number

$\Rightarrow (4x - 15)^2 > 5x$

$\Rightarrow 16x^2 - 120x + 225 > 5x$

$\Rightarrow 16x^2 - 125x + 225 > 0$

$\Rightarrow 16x^2 - 80x - 45x + 225 > 0$

$\Rightarrow 16x (x - 5) - 45 (x - 5) > 0$

$\Rightarrow (16x - 45) (x - 5) > 0$

$\Rightarrow x \in \left(-\infty, \frac{45}{16}\right) \cup (5, \infty)$

Since $4x - 15 > 0$

$\Rightarrow x > 3 \ \therefore x \in (5, \infty)$

Hence x can take values from 6 to 99 i.e. a total of 94 values.

Hence, the correct option is (D).

29. $N = 1764^{50}$

$N = (42)^{100}$

$N = [(2 \times 3 \times 7)]^{100}$

$\log N = 100 [\log 2 + \log 3 + \log 7]$

$\log N = 100 [1 \times 6231]$

$\log N = 162 \times 31$

Hence the number of digits in N is 163.

Hence, the correct option is (B)

30. Given $\frac{1}{x-1} = \log_{bc} a$

$\Rightarrow x - 1 = \log_a bc$

$\Rightarrow x = 1 + \log_a bc = \log_a a + \log_a bc$

$\Rightarrow x = \log_a abc$

Similarly $y = \log_b abc$ and

$z = \log_c abc$

$\therefore \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \log_{abc} abc = 1$

Hence, the correct option is (B).

Chapter 10 Quadratic Equations

Exercises

Practice Problems 1

Directions for questions 1 to 25: Select the correct alternative from the given choices.

1. The roots of the quadratic equation $2x^2 - 7x + 2 = 0$ are
(A) Rational and unequal (B) Real and equal
(C) Imaginary (D) Irrational

2. Find the nature of the roots of the quadratic equation $2x^2 + 6x - 5 = 0$.
(A) Complex conjugates
(B) Real and equal
(C) Conjugate surds
(D) Unequal and rational

3. Construct a quadratic equation whose roots are one third of the roots of $x^2 + 6x + 10 = 0$.
(A) $x^2 + 18x + 90 = 0$ (B) $x^2 + 16x + 80 = 0$
(C) $9x^2 + 18x + 10 = 0$ (D) $x^2 + 17x + 90 = 0$

4. A quadratic equation in x has its roots as reciprocals of each other. The co-efficient of x is twice the co-efficient of x^2. Find the sum of the squares of its roots.
(A) 5 (B) 4 (C) 3 (D) 2

5. If one root of the quadratic equation $4x^2 - 8x + k = 0$, is three times the other root, find the value of k.
(A) 3 (B) 9 (C) -3 (D) -6

6. The roots of the quadratic equation $(m - k + \ell)\, x^2 - 2mx + (m - \ell + k) = 0$ are
(A) $1, \dfrac{\ell + m - k}{k + m - \ell}$ (B) $1, \dfrac{2m}{\ell + m - k}$
(C) $1, \dfrac{k + m - \ell}{\ell + m - k}$ (D) $1, \dfrac{2k}{k - m + \ell}$

7. The expression $\dfrac{4ac - b^2}{4a}$ represents the maximum/minimum value of the quadratic expression $ax^2 + bx + c$. Which of the following is true?
(A) It represents the maximum value when $a > 0$.
(B) It represents the minimum value when $a < 0$.
(C) Both (A) and (B)
(D) Neither (A) nor (B)

8. Find the signs of the roots of the equation $x^2 + x - 420 = 0$.
(A) Both are positive.
(B) Both are negative.
(C) The roots are of opposite signs with the numerically larger root being positive.
(D) The roots are of opposite signs with the numerically larger root being negative.

9. If k is a natural number and $(k^2 - 3k + 2)\,(k^2 - 7k + 12) = 120$, find k.
(A) 7 (B) 6 (C) 5 (D) 9

10. Both A and B were trying to solve a quadratic equation. A copied the co-efficient of x wrongly and got the roots of the equation as 12 and 6. B copied the constant term wrongly and got the roots as 1 and 26. Find the roots of the correct equation.
(A) 6, 16 (B) $-6, -16$
(C) 24, 3 (D) $-3, -24$

11. If the roots of the equation $(x - k_1)\,(x - k_2) + 1 = 0$, k_1 and k_2 are integers, then which of the following must be true?
(A) k_1, k_2 are two consecutive integers
(B) $k_2 - k_1 = 2$
(C) $k_1 - k_2 = 2$
(D) Either (B) or (C)

12. The roots of the equation $ax^2 + bx + c = 0$ are k less than those of the equation $px^2 + qx + r = 0$. Find the equation whose roots are k more than those of $px^2 + qx + r = 0$.
(A) $ax^2 + bx + c = 0$
(B) $a(x - 2k)^2 + b(x - 2k) + c = 0$
(C) $a(x + 2k)^2 + b(x + 2k) + c = 0$
(D) $a(x - k)^2 + b(x - k) + c = 0$

13. If one root of the equation $x^2 - 10x + 16 = 0$ is half of one of the roots of $x^2 - 4Rx + 8 = 0$. Find R such that both the equations have integral roots.
(A) 1 (B) 2/3 (C) 3/2 (D) 4

14. If $x + y = 4$, find the maximum/minimum possible value of $x^2 + y^2$.
(A) Minimum, 8 (B) Maximum, 8
(C) Maximum, 16 (D) Minimum, 16

15. Find positive integral value(s) of p such that the equation $2x^2 + 8x + p = 0$ has rational roots.
(A) 8 (B) 4
(C) 6 (D) (A) or (C)

16. Two equations have a common root which is positive. The other roots of the equations satisfy $x^2 - 9x + 18 = 0$. The product of the sums of the roots of the two equations is 40. Find the common root.
(A) 1 (B) 2 (C) 3 (D) 4

17. If one root of the equation $x^3 - 11x^2 + 37x - 35 = 0$ is $3 - \sqrt{2}$, then find the other two roots.
(A) $5, 3 - \sqrt{2}$ (B) $-5, 3 + \sqrt{2}$
(C) $5, 3 + \sqrt{2}$ (D) $-5, 3 - \sqrt{2}$

18. The roots [the values of x (and not $|x|$)] of the equation $|x|^2 + 6|x| - 55 = 0$ are α and β. One of the roots of $py^2 + qy + r = 0$ is $\alpha\beta$ times the other root. Which of the following can be concluded?
(A) $25q^2 = -576pr$ (B) $25pr = -576q^2$
(C) $25q^2 = 576pr$ (D) $25pr = 576q^2$

19. The sides of a right-angled triangle are such that the sum of the lengths of the longest and that of the shortest side is twice the length of the remaining side. Find

the longest side of the triangle if the longer of the sides containing the right angle is 9 cm more than half the hypotenuse.
(A) 30 cm (B) 25 cm (C) 20 cm (D) 15 cm

20. Solve for x: $2\{3^{2(1+x)}\} - 4(3^{2+x}) + 10 = 0$
(A) $-1, \log_3\left(\frac{5}{3}\right)$ (B) $-1, \log_3 2$
(C) $-1, \frac{5}{3}$ (D) $-1, \log_3\left(\frac{3}{5}\right)$

21. If $\sqrt{x^2-2x-3}+\sqrt{x^2+5x-24}=\sqrt{x^2+7x-30}$, then find x.
(A) 2 (B) 3 (C) 4 (D) 6

22. Two software professionals Ranjan and Raman had 108 floppies between them. They sell them at different prices, but each receives the same sum. If Raman had sold his at Ranjan's price, he would have received ₹722 and if Ranjan had sold his at Raman's price, he would have received ₹578. How many floppies did Ranjan have?
(A) 51 (B) 57 (C) 68 (D) 40

23. The sum and product of the roots of a quadratic equation E are a and b, respectively. Find the equation whose roots are the product of first root of E and the square of the second root of E, and the product of the second root of E and the square of the first root of E.
(A) $x^2 - abx + b^3 = 0$
(B) $x^2 + abx + b^3 = 0$
(C) $x^2 + abx - b^3 = 0$
(D) $x^2 - abx - b^3 = 0$

24. Which of the following options represent(s) a condition for the equations $x^2 + ax + b = 0$ and $x^2 + bx + a = 0$ to have exactly one common root, given that the roots of both the equations are real?
(A) $a - b = 1$ (B) $b - a = 1$
(C) $1 + a + b = 0$ (D) Either (A) or (B)

25. If the roots of $2x^2 + (4m + 1)x + 2(2m - 1) = 0$ are reciprocals of each other, find m.
(A) –1 (B) 0 (C) 1 (D) 3/4

Practice Problems 2

Directions for questions 1 to 25: Select the correct alternative from the given choices.

1. If the sum of the roots and the product of the roots of a quadratic equation are 13 and 30, respectively, find its roots.
(A) 10, 3 (B) –10, –3
(C) 10, –3 (D) –10, 3

2. Find the sum and the product of the roots of the equation $\sqrt{5}\,x^2 + 25x + 2\sqrt{5} = 0$.
(A) $-5\sqrt{5}, 2$ (B) $5\sqrt{5}, 2$
(C) $25\sqrt{5}, 2$ (D) $-25\sqrt{5}, 2$

3. The sum of the squares of two consecutive positive integers added to their product is equal to 331. Find the two integers.
(A) 9, 10 (B) 10, 11 (C) 11, 12 (D) 12, 13

4. The sum of squares of three consecutive positive integers is 869. Find the numbers.
(A) 14, 15, 16 (B) 15, 16, 17
(C) 16, 17, 18 (D) 17, 18, 19

5. An integer exceeds its reciprocal by $\frac{143}{12}$. Find the integer.
(A) 6 (B) –12 (C) 12 (D) –6

6. Construct a quadratic equation whose roots are reciprocals of the roots of the equation $2x^2 + 8x + 5 = 0$.
(A) $5x^2 + 8x + 2 = 0$ (B) $8x^2 + 5x + 2 = 0$
(C) $2x^2 + 5x + 8 = 0$ (D) $8x^2 + 2x + 5 = 0$

7. The sum of the roots of a quadratic equation is 33 and the product of its roots is 90. Find the sum of the squares of its roots.
(A) 909 (B) 8034 (C) 36 (D) 729

8. A quadratic equation in x has the sum of its roots as 19 and the product of its roots as 90. Find the difference of its roots.
(A) 9 (B) 10
(C) 1 (D) $\sqrt{7739}$

9. The square of the sum of the roots of a quadratic equation E is 8 times the product of its roots. Find the value of the square of the sum of the roots divided by the product of the roots of the equation whose roots are reciprocals of those of E.
(A) 8 (B) 1/8 (C) 1 (D) 4

10. The quadratic expression $ax^2 + bx + c$ has its maximum/minimum value at
(A) $-\frac{b}{2a}$ (B) $\frac{b}{2a}$
(C) $\frac{-2b}{a}$ (D) $\frac{2b}{a}$

11. The roots of the equation $x^2 - 3x - 108 = 0$ are α and β, where $|\alpha| > |\beta|$. Which of the following holds true?
(A) $\alpha - \beta = 3$ (B) $\alpha - \beta = -3$
(C) $\alpha - \beta = -21$ (D) $\alpha - \beta = 21$

12. If α and β are the roots of the equation $ax^2 + bx + c = 0$ where $c^3 + abc + a^3 = 0$, which of the following is true?
(A) $\alpha\beta^2 = 1$ or $\alpha^2\beta = 1$ (B) $\alpha\beta^3 = 1$ or $\alpha^3\beta = 1$
(C) $\alpha = \beta^2$ or $\alpha^2 = \beta$ (D) $\alpha = \beta^3$ or $\alpha^3 = \beta$

13. Find the equation whose roots are thrice the roots of the equation $2x^2 - 15x + 18 = 0$.
(A) $x^2 + 45x + 324 = 0$ (B) $2x^2 - 45x + 81 = 0$
(C) $x^2 + 45x - 324 = 0$ (D) $2x^2 - 45x + 162 = 0$

14. A person bought a certain number of oranges for ₹70. If the price of each orange was ₹2 less, he would have bought 4 more oranges for the same amount. Find the number of oranges he bought originally.
(A) 12 (B) 10 (C) 18 (D) 15

15. Find the value of p in the equation $x^2 + qx + p = 0$, where one of the roots of the equation is $(2 + \sqrt{3})$ and q and p are integers.
(A) 3 (B) 2 (C) 1 (D) –2

16. If k is a perfect square, the roots of the equation $4kx^2 + 4\sqrt{k}\,x - k = 0$ are
(A) always rational.
(B) rational for only some of the values of k.
(C) always irrational.
(D) always complex.

17. Find the values of k for which the roots of $x^2 + x(14 - k) - 14k + 1 = 0$ are equal integers.
(A) –11, –13 (B) –12, –16
(C) –13, –15 (D) –11, –12

18. If the roots of $2^m x^2 + 8x + 64^m = 0$ are real and equal, find m.
(A) 2/3 (B) 1/2 (C) 7/4 (D) 4/7

19. If the roots of the equation $ax^2 + bx + c = 0$ are α and β, find the value of $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} - 2\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) + 2\alpha\beta$.
(A) $(b^2 + 2ac)/ac$ (B) $(b^2 - 2ac)/ac$
(C) $(b^2 + 4ac)/ac$ (D) None of these

20. If 31 is split up into two parts such that the sum of the squares of the two parts is 481, find the difference between the two parts.
(A) 7 (B) 5 (C) 3 (D) 1

21. Solve for x: $x^4 - 42x^2 + 216 = 0$
(A) $\pm\sqrt{6}, \pm 6$ (B) $\pm 2\sqrt{6}, \pm 6$
(C) $\pm 3\sqrt{6}, \pm 6$ (D) $\pm 4\sqrt{6}, \pm 6$

22. If $x^2 + \frac{1}{x^2} - 2\left(x - \frac{1}{x}\right) - \frac{5}{4} = 0$, which of the following can be the value of $x - \frac{1}{x}$?
(A) 7/2 (B) 1/2
(C) –1/2 (D) –3/2

23. Solve for x: $\sqrt{2x+3} + \sqrt{4x+13} = 8$
(A) 2 (B) –3 (C) 3 (D) 11

24. The roots of the equation $4x^2 + 14x + 3 = 0$ are a and b. Find $\sqrt{\frac{a}{b}} + \sqrt{\frac{b}{a}}$.
(A) $-\frac{7\sqrt{3}}{6}$ (B) $-\frac{7\sqrt{3}}{3}$ (C) $\frac{7\sqrt{3}}{3}$ (D) $\frac{7\sqrt{3}}{6}$

25. The equation $x^2 - 2x - 8 = 0$ will have
(A) the numerically larger root as positive.
(B) the numerically larger root as negative.
(C) both roots as negative.
(D) both the roots as positive.

HINTS/SOLUTIONS

Practice Problems I

1. $2x^2 - 7x + 2 = 0$

Discriminant = $(-7)^2 - 4 \times 2 \times 2 = 33 > 0$ but 33 is not perfect square.

$\therefore$ the roots are irrational.

Hence, the correct option is (D).

2. Discriminant = $6^2 - 4(2)(-5) = 76$.

This is positive but not a perfect square.

$\therefore$ The roots are conjugate surds.

Hence, the correct option is (C).

3. The equation whose roots are p times the roots of $ax^2 + bx + c = 0$ is given by $a\left(\frac{x}{p}\right)^2 + b\left(\frac{x}{p}\right) + c = 0$.

Here $p = 1/3$.

$\therefore$ The required equation is $(3x)^2 + 6(3x) + 10 = 0$

i.e. $9x^2 + 18x + 10 = 0$

Hence, the correct option is (C).

4. Let the equation be $ax^2 + bx + c = 0$.

As the roots are reciprocals of each other, the product of the roots is 1.

$\therefore \frac{c}{a} = 1 \Rightarrow c = a \qquad b = 2a$

$\therefore ax^2 + 2ax + a = 0$

$a(x + 1)^2 = 0$

As $a \neq 0$, $x = -1, -1$

$\therefore$ The sum of the squares of its roots is $(-1)^2 + (-1)^2 = 2$

Hence, the correct option is (D).

5. Let one root be α, other root is 3α.

Sum = $4a = 2$

$\Rightarrow a = \frac{1}{2}, 3a = \frac{3}{2}$

Product = $\frac{k}{4} = \frac{1}{2} \times \frac{3}{2} \Rightarrow k = 3$

Hence, the correct option is (A).

6. From the quadratic equation

Sum = $\frac{2m}{m - k + l}$

Check out with each of the options as for which of them has the same sum and product.

Option (A):

Sum = $1 + \frac{\ell + m - k}{k + m - \ell} = \frac{2m}{k + m - \ell}$

Sum not satisfied

Option (B):

Sum = $1 + \frac{2m}{\ell + m - k\ell} = \frac{\ell + 3m - k}{\ell + m - k}$

Sum not satisfied

Option (C):

Sum = $1 + \frac{k + m - \ell}{\ell + m - k} = \frac{2m}{\ell + m - k}$

Sum satisfied

Product = $1 \times \frac{k + m - \ell}{\ell + m - k} = \frac{k + m - \ell}{\ell + m - k}$

Alternate Solution:

Sum of the coefficients = $m - k + 1 - 2m + m - \ell + k = 0$

$\therefore$ 1 is the one roof of the equation we know that 1 is one root of $ax^2 + +bx + c = 0$, then the other root is c/a

$\therefore$ Hence second root is $\frac{m - \ell + k}{m + \ell - k}$.

Hence, the correct option is (C).

7. The expression $\frac{4ac - b^2}{4a}$ represents the maximum value of the quadratic expression $ax^2 + bx + c$ when $a < 0$ and represents the minimum value of the quadratic expression $ax^2 + bx + c$ when $a > 0$.

$\therefore$ Neither (A) nor (B) is true.

Hence, the correct option is (D).

8. The given equation has the sum of its roots as –1 and the product of its roots as –420. As the sum of the roots as well as the product of the roots are negative, the roots are of opposite signs with the numerically larger root being negative.

Hence, the correct option is (D).

9. $(k^2 - 3k + 2)(k^2 - 7k + 12) = 120$

$(k - 1)(k - 2)(k - 3)(k - 4) = 120 \qquad (5)$

$= (5)(4)(3)(2)$

Comparing the two sides, $k - 1 = 5$

$k = 6$

Alternate method:

Substituting the choices in place of k in the equation (5) above, we see that only choice (B) satisfies it

Hence, the correct option is (B).

10. Since A copied the coefficient of x wrongly he copied the constant term correctly, hence constant term = $12 \times 6 = 72$. B copied the constant term wrongly and hence he copied the coefficient of x correctly, hence coefficient of $x = -(1 + 26) = -27$. Hence, the equation is x^2

$-27x + 72 = 0$

Thus $\Rightarrow (x - 24)(x - 3) = 0$

Thus $x = 24$ or $x = 3$

Hence, the correct option is (C).

11. $(x - k_1)(x - k_2) = -1$;

Given that roots are integers.

$\Rightarrow x$ has integer values.

given that k_1, k_2 are integers.

Hence, $(x - k_1)$ as well as $(x - k_2)$ are integers.

The only way -1 can be resolved into the product of integers is $-1 = 1x - 1$. Hence,

if, $(x - k_1) = +1$
then $(x - k_2) = -1$ } Case I

or

if, $(x - k_1) = -1$
then $(x - k_2) = +1$ } Case II

Case I: gives

$k_1 + 1 = k_2 - 1$ or $k_2 - k_1 = 2$

Case II: gives

$k_1 - 1 = k_2 + 1$ or, $k_1 - k_2 = 2$

Hence, the correct option is (D).

12. As the equations given in the options contain coefficients a, b, and c, consider the equivalent of $px^2 + qx + r = 0$ with co-efficients a, b and c.

$px^2 + qx + r = 0$ is equivalent to

$a(x - k)^2 + b(x - k) + c = 0$ (6)

because the equation whose roots are 'k' more than those of equation (6) is obtained by replacing; 'x' by '$x - k$'.

$a(x - 2k)^2 + b(x - 2k) + c = 0$

Hence, the correct option is (B).

13. $x^2 - 10x + 16 = 0$

$(x - 8)(x - 2) = 0$

$x = 8$ or $x = 2$.

If $x = 8$ is the root of $x^2 - 10x + 16$ which is half of one root (α) of $x^2 - 4Rx + 8 = 0$ then $a = 2 \times 8 = 16$.

Then the other root b of $x^2 - 4Rx + 8 = 0$ would be $\frac{8}{16} = \frac{1}{2}$. Since this is not an integral root this value is not acceptable. If $x = 2$ which is the root of $x^2 - 10x + 16 = 0$, which is half of one root (α) of $x^2 - 4Rx + 8 = 0$ then, the second root b is $\frac{8}{4} = 2$.

The sum of the roots $= -\frac{(-4R)}{1} = 4R = 4 + 2 = 6$,

$R = \frac{6}{4} = \frac{3}{2}$

Hence, the correct option is (C).

14. $x + y = 4$

$x^2 + y^2 = x^2 + (4 - x)^2 = 2x^2 - 8x + 16$

Coefficient of $x^2 = 2$ which is positive

$\therefore$ Minimum value exists.

Minimum value $= \frac{4ac - b^2}{4a}$

As, $a = 2$, $b = -8$ and $c = 16$, the minimum value is

$\frac{128 - 64}{8} = 8$

Hence, the correct option is (A).

15. For the equation $2x^2 + 8x + p = 0$ to have rational roots, $b^2 - 4ac = 8^2 - 4(2)(p) = 64 - 8p$, should be a perfect square.

$64 - 8p$ is a perfect square.

$\Rightarrow 8(8 - p)$ is a perfect square.

$\Rightarrow 8(8 - p) = 0, 1, 4, 9, 16, 25, 36, 49, 64$.

As p is an integer, the value selected should be an integer such that $8(8 - p)$ is divisible by 8. i.e. $8(8 - p) = 0$, 16, 64, …

When $8(8 - p) = 0$; $p = 8$

$8(8 - p) = 16$; $8 - p = 2$, $p = 6$

$8(8 - p) = 64$, $8 - p = 8$, $p = 0$; not admissible.

$8(8 - p) = 144$, $8 - p = 18$, $p = -10$

Hence, $p = 8$ or 6 are the possible values.

Hence, the correct option is (D).

16. Let the common root be a.

$x^2 - 9x + 18 = 0$

$x = 6, 3$

$(6 + a)(3 + a) = 40$

$(a - 2)(a + 11) = 0$ as $a > 0$, $a = 2$

Hence, the correct option is (B).

17. The given equation is $x^3 - 11x^2 + 37x - 35 = 0$ (7)

One root is $3 - \sqrt{2}$. As the coefficients are rational,

The irrational roots occur in pairs, $3 + \sqrt{2}$ is also *a* root of (1)

$\therefore$ Sum of the roots $= -(-11) = 11$ from (7)

Sum of two of the roots is $3 + \sqrt{2} + 3 - \sqrt{2} = 6$

$\therefore$ The third root is $11 - 6 = 5$

$\therefore$ The roots of the equation are $3 \pm \sqrt{2}$ and 5.

Hence, the correct option is (C).

18. $|x|^2 + 6|x| - 55 = 0$

$(|x| + 11)(|x| - 5) = 0$

$|x|$ cannot be negative. $\therefore |x| = 5$. $\therefore x = \pm 5$

$\therefore (\alpha, \beta) = (5, -5)$ or $(-5, 5)$

Let the roots of the second equation be y_1 and y_2.

Let $y_1 = \alpha\beta y_2$

$y_1 + y_2 = (\alpha b + 1)\, y_2$ and $y_1 y_2 = \alpha\beta\, y_2^2$

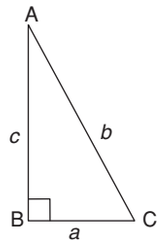
$$\frac{(y_1 + y_2)^2}{y_1 y_2} = \frac{(\alpha\beta + 1)^2}{\alpha\beta}$$

$$\Rightarrow \frac{\left(\frac{-q}{p}\right)^2}{\frac{r}{p}} = \frac{(-25+1)^2}{-25}$$

$$\frac{q^2}{rp} = \frac{-576}{25}$$

Hence, the correct option is (A).

19.



Let the sides be as represented in the figure.

From the data, $b + a = 2c$ (8)

and $c = 9 + \frac{b}{2}$; $\Rightarrow 2c = 18 + b$ (9)

From (8) and (9); $a = 18$ (10)

Applying the theorem of Pythogoras,

$b^2 = c^2 + a^2$ and substituting $a = 2c - b$,

$b^2 = c^2 + (2c - b)^2$; $\Rightarrow b^2 = c^2 + 4c^2 - 4bc + b^2$,

$\Rightarrow 5c^2 - 4bc = 0$; $\Rightarrow 5c = 4b$ (11)

Substituting in (8)

$$b + a = 2c \Rightarrow b + 18 = \left(\frac{4b}{5}\right)^2$$

$\Rightarrow 5b + 90 = 8b \quad \Rightarrow b = 30$

Alternate method:

From the first two equations, $a = 18$ is obtained.

(3 : 4 : 5) is a ratio which satisfies the conditions $b + a = 2c$, because $(5 + 3) = 2(4)$.

Hence, 3 parts of the ratio of (3 : 4 : 5) is 18.

Hence, 5 parts of the ratio is 5(18/3) = 30;

i.e., $b = 30$.

Hence, the correct option is (A).

20. $2(3^2 \cdot 32x) - 4(3^2 \cdot 3x) + 10 = 0$

$2(9)(32x) - 4(9)(3x) + 10 = 0$

$18(32x) - 36(3x) + 10 = 0$;

Dividing by 2 and substituting a for $3x$,

we have $9a^2 - 18a + 5 = 0$

$9a^2 - 3a - 15a + 5 = 0$;

$3a(3a - 1) - 5(3a - 1) = 0$

$a = 1/3$ or $5/3$

Hence $3x = 1/3 = 3^{-1}$ or $5/3$, thus $x = -1$ or $\log_3 (5/3)$

Hence, the correct option is (A).

21. $\sqrt{x^2 - 2x - 3} + \sqrt{x^2 + 5x - 24} = \sqrt{x^2 + 7x - 30}$

$$\sqrt{(x-3)(x+1)} + \sqrt{(x+8)(x-3)} = \sqrt{(x+10)(x-3)}$$

We can see that $(x - 3)$ is common to all the terms of the above equation.

If $x - 3 = 0$, we will have

$$\sqrt{0(x+1)} + \sqrt{(x+8)(0)} = 0 + 0 = \sqrt{(x+10)(0)} = 0$$

Thus the equation is satisfied.

Hence $x - 3 = 0$, i.e. $x = 3$

Other values in the choices do not satisfy the equation.

Hence, the correct option is (B).

22.

	N	P	A
Ranjan	x	p_a	$x \cdot p_a$
Raman	y	p_b	$y \cdot p_b$

When N, P and A stand for the number, the price and the amount respectively.

Given $x + y = 108$

$y\, Pa = 722$ (12)

$x\, Pb = 578$ (13)

$$\Rightarrow \frac{y\, p_a}{x\, p_b} = \frac{722}{578} = \frac{361}{289} \text{ and } xpa = yp_b$$

$$\therefore \frac{p_a}{p_b} = \frac{y}{x} \text{ or } \left(\frac{y}{x}\right)^2 = \left(\frac{19}{17}\right)^2 \Rightarrow \frac{y}{x} = \frac{19}{17}$$

The number of floppies with Ranjan $= \frac{17}{36} \times 108 = 51$

Hence, the correct option is (A).

23. Let the roots of E be a and β

$a = a + \beta$

$b = a\, \beta$

Required equation has its roots as

$a\beta^2$ and $b\alpha^2$

$a\beta^2 + b\alpha^2 = \alpha b\, (a + \beta)$

$(a\beta^2)\, (\beta\alpha^2) = (a\beta)^3$

required equation is $x^2 - abx + b^3 = 0$

Hence, the correct option is (A).

24. $x^2 + ax + b = 0$ (14)

$x^2 + bx + a = 0$ (15)

Let 'k' be a common root, then

$k^2 + ak + b = 0$

$k^2 + bk + a = 0$

$\Rightarrow k(a - b) + (b - a) = 0$ (on subtraction),

$(k - 1)(a - b) = 0$

$a - b \neq 0$, $\therefore$ if $a = b$ the two equations become identical and they will have two common roots.

$\therefore k = +1$

$\Rightarrow 1^2 + a + b = 0$

$\Rightarrow a + b + 1 = 0$.

Hence, the correct option is (C).

25. If the roots of the given equation are a and $\frac{1}{\alpha}$; (α)

$\left(\frac{1}{\alpha}\right) = 1, \frac{2(2m-1)}{2} = 2m - 1 = 1$

$\Rightarrow 1 + 1 = 2m, \Rightarrow m = 1$

Hence, the correct option is (C).

Practice Problems 2

1. Let the equation be

$x^2 - 13x + 30 = 0$

$x^2 - 10x - 3x + 30 = 0$

$x(x - 10) - 3(x - 10) = 0$

$(x - 10)(x - 3) = 0$

$x = 10$ or 3

Hence, the correct option is (A).

2. Sum $= -\frac{25}{\sqrt{5}} = -5\sqrt{5}$

Product $= \frac{2\sqrt{5}}{\sqrt{5}} = 2$

Hence, the correct option is (A).

3. Let the two consecutive positive integers be $(x - 1)$, x.

then, $(x - 1)^2 + x^2 + x(x - 1) = 331$

$\Rightarrow 3x^2 - 3x - 330 = 0$

$\Rightarrow x^2 - x - 110 = 0$

$\Rightarrow (x - 11)(x + 10) = 0$

$\Rightarrow x = 11$ and $x = -10$

As x cannot be negative, the integers are 10, 11

(or) alternatively substitute the options and check.

Hence, the correct option is (B).

4. Let the three consecutive positive integers be $(x - 1)$, x, $(x + 1)$ then, $(x - 1)^2 + x^2 + (x + 1)^2 = 869$

$\Rightarrow 3x^2 + 2 = 869$

$\Rightarrow x = \pm 17$

As x cannot be negative, $x = 17$

The numbers are 16, 17, 18

Hence, the correct option is (C).

5. Let the integers be x,

then $x - \frac{1}{x} = \frac{143}{12}$

$\Rightarrow 12x^2 - 143x - 12 = 0$

$\Rightarrow (12x - 1)(x - 12) = 0$

$\Rightarrow x = \frac{-1}{12}, 12$

As x is integer, $x = 12$

Hence, the correct option is (C).

6. The equation whose roots are reciprocals of the roots of the equation $ax^2 + bx + c = 0$ is given by $cx^2 + bx + a = 0$.

$\therefore$ The required equation is $5x^2 + 8x + 2 = 0$.

Hence, the correct option is (A).

7. A quadratic equation whose sum of the roots is S and whose product of the roots is P has the sum of the squares of its roots given by $S^2 - 2P$.

The sum of the squares of the roots $= 33^2 - 2(90) = 909$.

Hence, the correct option is (A).

8. A quadratic equation whose sum of the roots is 5 and whose product of the roots is P has the difference of its roots given by $\sqrt{S^2 - 4P}$.

The difference of the roots is $\sqrt{19^2 - 4(90)} = 1$

Hence, the correct option is (C).

9. Let E be $ax^2 + bx + c = 0$.

Given: $\left(-\frac{b}{a}\right)^2 = 8\frac{c}{a}$

$b^2 = 8ac$

The equation whose roots are the reciprocals of the roots of E is $cx^2 + bx + a = 0$

As $b^2 = 8ac$

$\frac{b^2}{c^2} = \frac{8ac}{c^2}$

$\therefore \left(-\frac{b}{c}\right)^2 = 8\frac{a}{c}$

$\therefore \frac{\left(-\frac{b}{c}\right)^2}{\frac{a}{c}} = 8$

We don't need the coefficients of either of the equations.

Let the roots be $\alpha.\beta$.

Given : $(a + \beta)^2 = 8\alpha\beta$.

For the second equation, the roots are $1/\alpha$, $1/\beta$. We need to evaluate

$$\left(\frac{1}{\alpha} + \frac{1}{\beta}\right)^2 \frac{1}{(1/\alpha)(1/\beta)} = \frac{(\alpha+\beta)^2}{(\alpha\beta)^2}(\alpha\beta) = \frac{(\alpha+\beta)}{\alpha\beta} = 8$$

Hence, the correct option is (A).

10. The maximum/minimum value of the quadratic expression $ax^2 + bx + c$ occurs at $x = \frac{-b}{2a}$.

Hence, the correct option is (A).

11. $x^2 - 3x - 108 = 0 \Rightarrow (x - 12)(x + 9) = 0$

$x = 12$ or -9

As $|\alpha| > |\beta|$, $\alpha = 12$, $\beta = -9$, $\alpha - \beta = 12 - (-9) = 21$

Hence, the correct option is (D).

12. $c^3 + abc + a^3 = 0$

Dividing each side by a^3, $\left(\frac{c}{a}\right)^3 - \left(\frac{c}{a}\right)\left(-\frac{b}{a}\right) + 1 = 0$

$(\alpha\beta)^3 - (\alpha\beta)(a + \beta) + 1 = 0$

dividing each side by $\alpha\beta$,

$(\alpha\beta)^2 - (a + \beta) + \frac{1}{\alpha\beta} = 0$

$\Leftrightarrow \left(\alpha^2 - \frac{1}{\beta}\right)\left(\beta^2 - \frac{1}{\alpha}\right) = 0 \quad \Leftrightarrow \alpha^2 = \frac{1}{\beta}$ or $\beta^2 = \frac{1}{\alpha}$

Hence, the correct option is (A).

13. $2x^2 - 15x + 18 = 0$

The equation has sum of its roots $= -\left(\frac{-15}{2}\right) = \frac{15}{2}$ and product of the roots $= \frac{18}{2} = 9$

The equation whose roots are thrice the roots of the above equation will have the sum of the roots being thrice its sum of roots and have $3\left(\frac{15}{2}\right) = \frac{45}{2}$ as sum of roots. It will have the product of its roots as nine times the product of the roots of the original equation and hence $9 \times 9 = 81$ as product of roots.

Hence, the required equation whose roots are thrice the roots of $2x^2 - 15x + 18 = 0$ is $x^2 - \frac{45x}{2} + 81 = 0$

or $2x^2 - 45x + 162 = 0$

Alternate method:

In order to find out the equation whose roots are thrice the roots of the given equation, substitute $\frac{x}{3}$ for x, in the given equation.

$2\left(\frac{x}{3}\right)^2 - 15\left(\frac{x}{3}\right) + 18 = 0$

$\Rightarrow 2x^2 - 45x + 162 = 0$

Hence, the correct option is (D).

14. Assume that the person bought x oranges for ₹70. Hence price of each orange is $\frac{70}{x}$. If he bought 4 more oranges for ₹70, the price of each orange would be $\frac{70}{x+4}$ which is 2 less than $\frac{70}{x}$.

Hence $\frac{70}{x+4} = \frac{70}{x} - 2$

$\Rightarrow \frac{70}{x} - \frac{70}{x+4} = 2;$

$\frac{70(x+4) - 70x}{x(x+4)} = \frac{70x + 280 - 70x}{x(x+4)} = 2$; $x(x + 4)$

$= 140 \Rightarrow x^2 + 4x - 140 = 0$; $(x + 14)(x - 10) = 0$

Hence, $x = -14$ or $x = 10$.

Since the number of oranges bought cannot be −ve, x cannot be -14, so $x = 10$. Hence 10 oranges were bought originally.

Hence, the correct option is (B).

15. Given p, q are integers, and one root of the equation is $2 + \sqrt{3}$, the equation must have the conjugate $2 - \sqrt{3}$ as the second root.

The product of the roots $= (2 + \sqrt{3})(2 - \sqrt{3}) = 4 - 3 = 1$

Hence the product of the roots $= p = 1$

Hence, the correct option is (C).

16. The given equation is $4kx^2 + 4\sqrt{k}\,x - k = 0$

The statements presented as options relate to the nature of the roots. Hence, discriminant is to be considered.

Discriminant $(\Delta) = \left(4\sqrt{k}\right)^2 - 4(4k)(-k) = 16k + 16k^2 = 16k(k + 1)$.

As per data, k is *a* perfect square, i.e.+ $k \geq 0$. Hence $\Delta \geq 0$. Hence, roots are definitely real. It is to be decided whether the roots are rational or irrational.

Δ can be equal to *z*ero, if and only if either $k = 0$ or $k = -1$

As k is the coefficient of x^2, it cannot be *z*ero; i.e. $k \neq 0$.

As k is given to be a perfect square, it cannot be equal to (-1); i.e. $k \neq -1$.

Hence $\Delta \neq 0$. $\Rightarrow \Delta > 0$

As k and $(k + 1)$ are two consecutive numbers and k is a perfect square, $(k + 1)$ cannot be a perfect square. Hence, Δ is positive, but not a perfect square. Hence, the roots are always irrational.

Hence, the correct option is (C)

17. If the roots of $x^2 + x(14 - k) - 14k + 1 = 0$ are equal, $(14 - k)^2 - 4(-14k + 1) = 0$;

$\Rightarrow 196 - 28k + k^2 + 56k - 4 = 192 + 28k + k^2 = 0.$

Hence $(k + 16)(k + 12) = 0 \Rightarrow k = -12$ and -16

To check which of the values of k, leads to equal, integer roots, substitute the value of $k = -12$, we get the equation as $x^2 + 26x + 169 = 0$

$(x + 13)^2 = 0$

Both the roots are integers and equal.

If $k = -16$, the equation is $x^2 + 30x + 225 = 0$

$(x + 15)^2 = 0$

Both the roots are integers and equal.

Hence, the correct option is (B).

18. Since the roots of the given equation are equal, the discriminant $b^2 - 4ac = 0$

$b^2 - 4ac = 8^2 - 4(2^m)\,(64)^m = 2^6 - 2^2 2^m 2^{6m}$

$\Rightarrow 2^6 - 2^{2+7m} = 0$, Hence $2^6 = 2^{2+7m}$ i.e., $2 + 7m = 6$ and hence $7m = 6 - 2 = 4$; $m = \dfrac{4}{7}$

Hence, the correct option is (D).

19. $\dfrac{\alpha}{\beta} + \dfrac{\beta}{\alpha} - 2\left(\dfrac{1}{\alpha} + \dfrac{1}{\beta}\right) + 2\alpha\beta$

$= \dfrac{\alpha^2 + \beta^2}{\alpha\beta} - 2\dfrac{(\beta + \alpha)}{\alpha\beta} + 2\alpha\beta$

$= \dfrac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta} - \dfrac{2(\beta + \alpha)}{\alpha\beta} + 2\alpha\beta$

$= \dfrac{\left(\dfrac{-b}{a}\right)^2 - 2\dfrac{c}{a}}{\dfrac{c}{a}} - \dfrac{2\left(-\dfrac{b}{a}\right)}{\dfrac{c}{a}} + 2\dfrac{c}{a}$

$= \dfrac{b^2}{a^2} \times \dfrac{a}{c} - 2 + \dfrac{2b}{c} + \dfrac{2c}{a} = \dfrac{b^2}{ac} + \dfrac{2b}{c} + \dfrac{2c}{a} - 2$

Hence, the correct option is (D).

20. If 31 is split into parts a and b,

$31 = a + b$ and $481 = a^2 + b^2$

$31^2 = (a + b)^2$

Hence $a^2 + b^2 + 2ab = 961$

$2ab = 961 - 481 = 480$

$4ab = 480 \times 2 = 960$

$a^2 + b^2 + 2ab - 4ab = a^2 + b^2 - 2ab$

$= (a - b)^2 = 961 - 960 = 1 \Rightarrow (a - b) = \sqrt{1} = \pm 1$

The difference of $(a - b) = |(a - b)| = 1$

Hence, the correct option is (D).

21. $x^4 - 42x^2 + 216 = 0$

$\Rightarrow (x^2) - 42x^2 + 216 = 0$

$\Rightarrow (x^2 - 6)\,(x^2 - 36) = 0$

$\Rightarrow x^2 - 6 = 0$ or $x^2 - 36 = 0$

$\Rightarrow x = \pm\ \sqrt{6}\ , \pm 6$

Hence, the correct option is (A).

22. $\left(x - \dfrac{1}{x}\right)^2 + 2 - 2\left(x - \dfrac{1}{x}\right) - \dfrac{5}{4} = 0$

Substituting the choices in place of $x - \dfrac{1}{x}$ in the equation above, we see that only choice (B) satisfies it

Hence, the correct option is (B).

23. Squaring on both sides, $2x + 3 + 4x + 13 - 8^2$

$= -2\ \sqrt{2x+3}\sqrt{4x+13}$

$\dfrac{6x - 48}{2} = \sqrt{2x+3}\sqrt{4x+13}$

Squaring on both sides,

$9x^2 - 144x + 576 = 8x^2 + 38x + 39$

$x^2 - 182\,x + 537 = 0$

$(x - 179)\,(x - 3) = 0$

$x = 179$ or 3

But $x = 179$ does not satisfy the equation.

$\therefore\ x = 3$.

Hence, the correct option is (C).

24. Both the roots of the given equation are negative, i.e., $a < 0$, $b < 0$.

$\therefore\ a/b > 0$ and $\sqrt{a/b}\ + \sqrt{b/a} > 0$

$\sqrt{\dfrac{a}{b}} + \sqrt{\dfrac{b}{a}} = \sqrt{\dfrac{|a|}{|b|}} + \sqrt{\dfrac{|b|}{|a|}}$

$= \dfrac{|a| + |b|}{\sqrt{|ab|}} = \dfrac{-(a+b)}{\sqrt{ab}}$

$= \dfrac{\text{Sum of the roots}}{\sqrt{\text{Product of the roots}}} = \dfrac{14}{4}\sqrt{\dfrac{4}{3}} = \dfrac{7\sqrt{3}}{3}$

Hence, the correct option is (C).

25. $x^2 - 2x - 8 = 0$ The sum of the roots $= \left(-\dfrac{2}{1}\right) = 2$ and the product of the roots $= -8$. Since the product of the roots is negative, one of the roots is positive and the other negative. Since the sum of the roots is positive, the numerically larger root is positive.

Hence, the correct option is (A)

CHAPTER 11 INEQUALITIES

EXERCISES

Practice Problems 1

Directions for questions 1 to 25: Select the correct alternative from the given choices.

1. If $a < b$ and $c < 0$, then which of the following is true?

(A) $ac < bc$ (B) $\frac{a}{c} < \frac{b}{c}$

(C) $ac > bc$ (D) None of these

2. If p and q are two real numbers, then which of the following statements is always true?

(A) $\frac{p}{q} < 1 \Rightarrow p < q$

(B) $p > 0, q > 0$ and $\frac{p}{q} > 1 \Rightarrow p > q$

(C) $\frac{p}{q} > 1 \Rightarrow p > q$

(D) All the above

3. If $5x - 8 < 2x + 9$ and $4x + 7 > 7x - 8$, then the range of the values of x that satisfies the inequalities is

(A) $(5, \infty)$ (B) $(-\infty, 5)$

(C) $\left(5, \frac{17}{3}\right)$ (D) $\left(-\infty, \frac{17}{3}\right)$

4. Solve for real values of x; $5x^2 - 3x - 2 \geq 0$.

(A) $\left[\frac{-2}{5}, 1\right]$ (B) $R - \left(\frac{-2}{5}, 1\right)$

(C) $[1, \infty)$ (D) $R - (0, 1)$

5. If $x^2 - 9x - 36$ is negative, then find the range of x.

(A) $(-3, 12)$ (B) $[-3, 12]$

(C) $(-12, 3)$ (D) $[-12, 3]$

6. Which of the following is true?

(A) $|x+y| \leq |x| + |y|$ (B) $\left|\frac{x}{y}\right| = \frac{|x|}{|y|}, y \neq 0$

(C) $|x-y| \geq ||x| - |y||$ (D) All the above

7. If $6x + 8 > 7x - 9$ and $4x - 7 < 6x - 3$, then the values of x is

(A) $(-17, 2)$ (B) $(2, 17)$

(C) $(-2, 17)$ (D) $(-\infty, 17)$

8. The solution set of the inequality $|x-5| < 9$ is

(A) $(0, 14)$ (B) $(-4, 14)$ (C) $(-4, 0)$ (D) $(9, 14)$

9. The number of integral values of x that do not satisfy the inequation $\frac{x+5}{x-2} \geq 0$ is

(A) 7 (B) 5 (C) 6 (D) 4

10. If $(x + 5)(x + 9)(x + 3)^2 < 0$, then the solution set for the inequality is

(A) $(-9, -3)$ (B) $(-9, -5)$

(C) $(-3, \infty)$ (D) $(-9, \infty)$

11. Find the range of the real values of x satisfying $8 - 3x \leq 5$ and $4x + 5 \leq -7$.

(A) $[-3, 1]$ (B) $(-\infty, -3] \cup [1, \infty)$

(C) $(-3, 1)$ (D) ϕ

12. Which of the following is true?

(A) $30^{31} < 31^{30}$ (B) $71^{69} > 70^{70}$

(C) $(155)^{29} < (150)^{30}$ (D) Both (B) and (C)

13. At what value of x is $-|x - 3| + \frac{21}{2}$ maximum?

(A) -3 (B) $\frac{21}{2}$ (C) 0 (D) 3

14. Find the range of all real values of x if $|3x + 5| < 5x - 11$.

(A) $(8, \infty)$ (B) $(-\infty, -5/3) \cup (8, \infty)$

(C) $(-5/3, 8)$ (D) $(-5/3, \infty)$

15. If $ac = bd = 2$, then the minimum value of $a^2 + b^2 + c^2 + d^2$ is

(A) 4 (B) 6 (C) 8 (D) 16

16. If $x, y > 0$ and $x + y = 3$ then

(A) $xy \leq 0.72$ (B) $xy \leq 1.8$

(C) $xy \leq 2.25$ (D) $xy \leq 1.25$

17. Find the complete range of values of x that satisfies $|x - 16| > x^2 - 7x + 24$.

(A) $(0, 2)$ (B) $\left(\frac{3}{2}, \frac{5}{2}\right)$

(C) $(1, 3)$ (D) $(2, 4)$

18. For which of the following range of values of x is $x^2 + x$ less than $x^3 + 1$?

(A) $(-\infty, -1)$ (B) $(1, \infty)$

(C) $(-1, 1) \cup (1, \infty)$ (D) $[-1, 1]$

19. If x, y, z are positive, then the value of

$A = \frac{(4x^2 + x + 4)(5y^2 + y + 5)(7z^2 + z + 7)}{xyz}$ can be

(A) 400 (B) 500 (C) 1000 (D) 1500

20. The range of x for which $2x^2 - 5x - 8 \leq |2x^2 + x|$ is

(A) $\left[-\frac{4}{3}, \infty\right)$ (B) $\left(-\frac{4}{3}, -1\right)$

(C) $[-1, \infty)$ (D) $[-1, 2]$

21. For how many integral values of x, is the inequation $\frac{x-5}{x+7} > 4$ satisfied?

(A) 5 (B) 4 (C) 2 (D) 3

22. If $1 \leq x \leq 3$ and $2 \leq y \leq 5$, then the minimum value of $\frac{x+y}{y}$ is

(A) $\frac{3}{5}$ (B) $\frac{1}{5}$ (C) $\frac{6}{5}$ (D) $\frac{5}{6}$

23. If $|b| \geq 5$ and $x = |a| b$, which of the following is true?

(A) $a - xb > 0$ (B) $a + xb < 0$

(C) $a + xb > 0$ (D) $a - xb \leq 0$

24. Find the number of solutions of the equation $\left|x-\left|x-2\right|\right| = 6$.
(A) 2 (B) 1 (C) 3 (D) 4

25. If x, y and z are positive real numbers, then the minimum value of $\frac{x^2y+y^2z+z^2x+xy^2+yz^2+zx^2}{xyz}$ is
(A) 6 (B) 9 (C) 12 (D) 14

Practice Problems 2

Directions for questions 1 to 25: Select the correct alternative from the given choices.

1. If $2 \le x \le 5$ and $-7 \le y \le -3$, then the minimum value of which of the following is the least?
(A) x^2y (B) xy^2 (C) $3xy$ (D) $-2xy^2$

2. The solution set for the inequation $(x^2 + x + 1)^x < 1$ is
(A) $(0, \infty)$ (B) $(-1, \infty)$
(C) $(-\infty, 0)$ (D) $(-\infty, -1)$

3. The number of integral values of x, that satisfy the inequation $\left|x-3\right|+\left|x-4\right| \le 7$ is
(A) 7 (B) 6 (C) 8 (D) 9

4. The solution of the system of inequation $x^2 - 5x + 6 > 0$ and $x^2 - 3x + 2 > 0$ is
(A) $(-\infty, 2)$ (B) $(-\infty, 1) \cup (3, \infty)$
(C) $(1, 3)$ (D) $(1, \infty)$

5. For how many integral values of x, is $\frac{x-3}{x+2} < 0$?
(A) 3 (B) 5 (C) 4 (D) 7

6. Solve the system of inequations $3x + 17 < 5x - 19$ and $4x + 15 > 9x + 21$.
(A) $(18, \infty)$ (B) R
(C) ϕ (D) $\left(-\infty, \frac{-6}{5}\right)$

7. For which of the following values of x does the expression $\left|x+3\right|+\left|x-5\right| + 7$ have its least value?
(A) 4 (B) –6 (C) 7 (D) –5

8. The maximum value of the expression $10 - \left|3x+5\right|$ is
(A) 10 (B) $\frac{-5}{3}$ (C) 20 (D) 0

9. For how many integral values of x, is the expression $9 - 4x - 5x^2$ non-negative?
(A) 7 (B) 6 (C) 5 (D) 3

10. Find the complete range of values of x for which $|x + 4| < 3x - 7$.
(A) $(11/2, \infty)$ (B) $(3/4, \infty)$
(C) $(-\infty, -3/4)$ (D) $(3/4, 11/2)$

11. If p, q, r are positive and $x=\frac{(2p^2+p+2)(3q^2+q+3)(r^2+r+1)}{15pqr}$, then x cannot be
(A) 8 (B) 10 (C) 7 (D) 6

12. The range of values of k for which $-x^2 + 3kx + 5k + 1 < 0$ is
(A) $(\infty, -2) \cup (-2/9, \infty)$ (B) $\left[-\frac{9}{2}, -2\right]$
(C) $(-2, -2/9)$ (D) $(-\infty, \infty)$

13. The range of x for which $|x^2 + x - 2| \le x^2 - x$ is
(A) $[-1, 1]$
(B) $(-\infty, -1]$
(C) $[1, \infty)$
(D) No such value of x exists

14. If x, y, z are positive real numbers and $xyz = 216$, which of the following is not a possible value of $xy + yz + zx$?
(A) 326 (B) 433 (C) 291 (D) 96

15. The solution set for the inequation $3x + 2 < |2x + 5| < 8x + 9$ is
(A) $\left(-\frac{2}{3}, 3\right)$ (B) $\left(\frac{2}{3}, 3\right)$
(C) $\left[-\frac{2}{3}, 3\right]$ (D) $\left(\frac{-2}{3}, 4\right)$

16. The solution set for the inequality $\frac{x^2-7x+10}{x^2+6x-40} < 1$ is
(A) $\left(\frac{70}{13}, \frac{50}{3}\right)$ (B) $\left(-10, \frac{50}{13}\right) \cup (4, \infty)$
(C) $(4, \infty)$ (D) $R - (-4, 0)$

17. If $x \le 4$, $y \ge -2$, which of the following is necessarily true?
(A) $xy \ge -8$ (B) $x + y > 2$
(C) $x - y \le 6$ (D) (A) and (B)

18. Let $a_1, a_2, \ldots, a_{3n}$ be $3n$ positive numbers such that their product is 1. The minimum value of $a_1 + a_2 + a_3 + \cdots + a_{3n}$ is
(A) n (B) $3^{1/n}$
(C) 3^n (D) $3n$

19. If a, b, c, d are positive real numbers, the minimum value of the product $(a + b + c + d)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}\right)$ is
(A) 16 (B) 32 (C) 24 (D) 36

20. At what value of x is $|2x - 7| - 8$ minimum?
(A) –8 (B) $\frac{7}{2}$
(C) $-\frac{7}{2}$ (D) 15

21. Max $\left[\min(x+6,\ x-3), \min(x+5,\ x-7)\right]$ =

(A) $x+6$ (B) $x+5$
(C) $x-3$ (D) $x-7$

22. If $a > b$ and $c > 0$, then which of the following statements is/are true?

(A) $a+c > b+c$ (B) $a-c > b-c$
(C) $ac > bc$ (D) All the above

23. Solve: $2x - 3 \geq 7$ and $5x - 7 < 3$.

(A) ϕ (B) $[5, \infty)$
(C) $(-\infty, 2)$ (D) $[2, 5]$

24. Which of the following is always true?

(A) $x > y \Rightarrow x^2 > y^2$.
(B) $x > y \Rightarrow x^2 < y^2$.
(C) If $x > 0$, $y > 0$ and $x > y$, then $x^2 > y^2$.
(D) None of the above.

25. If $|x+7| < 10$, then the solution set for the inequality is

(A) $(-\infty, -17) \cup (3, \infty)$ (B) $(3, 17)$
(C) $(-17, -3)$ (D) $(-17, 3)$

HINTS/SOLUTIONS

Practice Problems I

1. When $a < b$ and $c < 0$

Then $ac > bc$ is true.

Hence, the correct option is (C).

2. For any two real numbers, p, q

1. $\frac{p}{q} < 1$

$\Rightarrow p < q$ is not true always.

2. $p > 0, q > 0$ and $\frac{p}{q} > 1$

$\Rightarrow p > q$ is true.

3. if $\frac{p}{q} > 1$

$\Rightarrow p > q$ is not true always.

Hence, the correct option is (B).

3. Given $5x - 8 < 2x + 9$

$\Rightarrow 3x < 17;$

$x < \frac{17}{3};$ (5)

$4x + 7 > 7x - 8$

$\Rightarrow 15 > 3x$

$\Rightarrow x < 5$ (6)

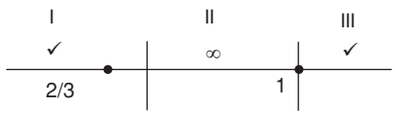
The common solution for (5) and (6) is $x < 5$; $(-\infty, 5)$

Hence, the correct option is (B).

4. Given $5x^2 - 3x - 2 \geq 0$

$(x - 1)(5x + 2) \geq 0$

The critical points are, $1, \frac{-2}{5}$



when $x = 0$, the inequality is not true

I and II regions satisfies the inequality

$\therefore$ solution set is $(-\infty, \frac{-2}{5}] \cup [1, \infty)$

$\therefore R - \left(\frac{-2}{5}, 1\right)$

Hence, the correct option is (B).

5. $x^2 - 9x - 36 < 0$

$\Rightarrow (x - 12)(x + 3) < 0$

$\Rightarrow x \in (-3, 12)$

Hence, the correct option is (A).

6. The given options are properties of modulus,

$\therefore$ all are true.

Hence, the correct option is (D).

7. $6x + 8 > 7x - 9 \Rightarrow 17 > 7x - 6x$

$\Rightarrow 17 > x$

$\Rightarrow x < 17$

$4x - 7 < 6x - 3 \Rightarrow 4x - 6x < 7 - 3$

$\Rightarrow -2x < 4$

$\Rightarrow x > -2$

$\therefore$ Solution set is $(-2, 17)$.

Hence, the correct option is (C).

8. Given $|x - 5| < 9$

We know that if $|x| < a \Rightarrow -a < x < a$

$|x - 5| < 9 \Rightarrow -9 < x - 5 < 9$

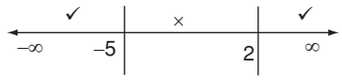
$\Rightarrow -4 < x < 14$

Hence, the correct option is (B).

9. $\frac{x+5}{x-2} \geq 0$

$\Rightarrow (x + 5)(x - 2) \geq 0$

Critical points are $-5, 2$



When $x = 0$, the inequation is not satisfied.

The solution set is $R - (-5, 2]$

$\therefore$ The integral values of x that do not satisfy the inequation is $-4, -3, -2, -1, 0, 1, 2$.

$\therefore$ i.e., 7

Hence, the correct option is (A).

10. Given $(x + 5)(x + 9)(x + 3)^2 < 0$

Since $(x + 3)^2$ is always positive

$(x + 5)(x + 9) < 0$

$\Rightarrow -9 < x < -5$

Solution set is $(-9, -5)$

Hence, the correct option is (B).

11. $8 - 3x \leq 5 \Rightarrow 3x \geq 8 - 5$

$\Rightarrow 3x \geq 3 \Rightarrow x \geq 1$ (7)

$4x + 5 \leq -7 \Rightarrow 4x \leq -7 - 5$

$\Rightarrow 4x \leq -12 \Rightarrow x \leq -3$ (8)

There is no x that satisfies both (7) and (8)

$\therefore$ The range is ϕ, the empty set.

Hence, the correct option is (D).

12. For any $x \geq 1$ we have $2 \leq \left(1 + \frac{1}{x}\right)^x < 2.8$

Consider choice (A)

$\frac{31^{30}}{30^{31}} = \left(\frac{31}{30}\right)^{30} \cdot \frac{1}{30} = \left(\frac{30+1}{30}\right)^{30} \cdot \frac{1}{30}$

$$= \left(1+\frac{1}{30}\right)^{30} \cdot \frac{1}{30} < \frac{2\cdot 8}{30} < 1$$

i.e., $\frac{31^{30}}{30^{31}} < 1$ i.e., $31^{30} < 30^{31}$.

Choice (A) is false

Similarly, it can be seen that (2) is false

Consider choice (C)

$$\frac{(155)^{29}}{(150)^{30}} = \left(\frac{155}{150}\right)^{30} \cdot \frac{1}{155}$$

$$= \left(1+\frac{1}{30}\right)^{30} \cdot \frac{1}{155} < \frac{2\cdot 8}{155} < 1$$

$\therefore \frac{(155)^{29}}{(150)^{30}} < 1$ i.e., $(155)^{29} < (150)^{30}$.

$\therefore$ Choice (C) is true.

Hence, the correct option is (C).

13. $-|x-3| + \frac{21}{2}$ has the maximum value of $\frac{21}{2}$ when $|x-3| = 0$ i.e., when $x = 3$.

Hence, the correct option is (D).

14. $|3x+5| = \begin{cases} 3x+5 \text{ if } x \geq \frac{-5}{3} \\ -3x-5 \text{ if } x < \frac{-5}{3} \end{cases}$

Case 1: If $x \geq \frac{-5}{3}$.

The inequality is $3x + 5 < 5x - 11$ i.e., $2x > 16$

$\Rightarrow x > 8$

$\therefore x > 8$ is a possible set of solutions

Case 2: If $x < \frac{-5}{3}$.

The inequality is $-3x - 5 < 5x - 11$

$\Rightarrow 8x > 6 \Rightarrow x > \frac{3}{4}$

This is not *a* possible set of solutions as this contradicts the assumption, $x < \frac{-5}{3}$

$\therefore$ The solution set is $(8, \infty)$.

Hence, the correct option is (A).

15. We know that $\frac{a^2+c^2}{2} \geq ac$ (A.M of a^2, c^2 compared to G.M of a^2, c^2)

i.e., $a^2 + c^2 \geq 2ac$

Similarly $b^2 + d^2 \geq 2bd$

Adding both these inequalities we get

$a^2 + b^2 + c^2 + d^2 \geq 2(ac + bd)$

But $ac = bd = 2$

$\therefore a^2 + b^2 + c^2 + d^2 \geq 2(2 + 2) = 8$

$\therefore$ The minimum value of $a^2 + b^2 + c^2 + d^2$ is 8.

Hence, the correct option is (C).

16. Given: $x, y > 0$ and $x + y = 3$

Now $\frac{x+y}{2} \geq \sqrt{xy}$ i.e., $(x + y)^2 \geq 4xy$

But $x + y = 3$

$\therefore 4xy \leq 9$ or $xy \leq \frac{9}{4}$ i.e., $xy \leq 2.25$.

Hence, the correct option is (C).

17. **Case (i):-** $x \geq 16$

$\therefore |x - 16| = x - 16$

So the relation becomes

$x - 16 > x^2 - 7x + 24$

$\Rightarrow x^2 - 8x + 40 < 0$

$\Rightarrow x^2 - 8x + 16 + 24 < 0$

$\Rightarrow (x - 4)^2 + 24 < 0$

But $(x - 4)^2 + 24$ is positive for all real x.

So there cannot be any solution in this domain.

Case (ii): $x < 16$

$|x - 16| = 16 - x$

The relation becomes

$16 - x > x^2 - 7x + 24$

$\Rightarrow x^2 - 6x + 8 < 0 \Rightarrow (x - 4)(x - 2) < 0$

$\Rightarrow x \in (2, 4)$

which is consistent with $x < 16$

Hence the range is (2, 4).

Hence, the correct option is (D).

18. Let $a = x^2 + x$, $b = x^3 + 1$

We need to find a range where $a < b$

i.e., $x^2 + x < x^3 + 1$.

$\Rightarrow x^3 - x^2 - x + 1 > 0$ or $x^2(x - 1) - 1(x - 1) > 0$

$\Rightarrow (x^2 - 1)(x - 1) > 0$ or $(x - 1)^2 (x + 1) > 0$

as $(x - 1)^2 > 0$ except at $x = 1$, $x + 1 > 0$ i.e., $x > -1$.

Hence, the range is $(-1, 1) \cup (1, \infty)$.

Hence, the correct option is (C).

19. Consider $4x^2 + x + 4$

$$= x\left(4\left(x+\frac{1}{x}\right)+1\right) \geq 9x \left(\because x+\frac{1}{x} \geq 2\right)$$

similarly $5y^2 + y + 5 \geq 10y$ and $7z^2 + z + 7 \geq 15z$

$$\therefore \frac{\left(4x^2+x+4\right)\left(5y^2+y+5\right)\left(7z^2+z+7\right)}{xyz}$$

$$\geq \frac{(9x)(11y)(15z)}{xyz} = 1485$$

Hence, the correct option is (D).

20. $2x^2 - 5x - 8 \le |2x^2 + x|$ (9)

$2x^2 + x = 0 \Rightarrow x = 0$ or $x = -1/2$

If $x \le -1/2$ or $x \ge 0$, then $2x^2 + x \ge 0$ and $|2x^2 + x| = 2x^2 + x$.

If $-1/2 < x < 0$, then $2x^2 + x < 0$ and $|2x^2 + x| = -2x^2 - x$

Let $x \le -1/2$ or $x \ge 0$

$(9) \Rightarrow 2x^2 - 5x - 8 \le 2x^2 + x$

$\Rightarrow 6x + 8 \ge 0 \Rightarrow x \ge -4/3$

$\therefore x \in [-4/3, \alpha) - (-1/2,0)$ (10)

Let $-1/2 < x < 0$

$(9) \Rightarrow 2x^2 - 5x - 8 \le -2x^2 - x$

$\Rightarrow 4x^2 - 4x - 8 \le 0 \Rightarrow x^2 - x - 2 \le 0$

$\Rightarrow (x - 2)(x + 1) \le 0 \Rightarrow -1 \le x \le 2$

$\therefore x \in (-1/2, 0)$ (11)

From (10), (11) $x \in [-4/3, \infty)$

Hence, the correct option is (A).

21. $\frac{x-5}{x+7} > 4$;

$\frac{x-5}{x+7} - 4 > 0$

$\Rightarrow \frac{x-5-4x-28}{x+7} > 0 \Rightarrow \frac{-3x-33}{x+7} > 0$

$\Rightarrow \frac{x+11}{x+7} < 0 \Rightarrow \frac{(x+11)(x+7)}{(x+7)^2} < 0$

$\Rightarrow (x + 7)(x + 11) < 0$

When $-11 < x < -7$, the above inequation is true.

The number of integral values between -11 and -7 is 3.

$\therefore$ Hence the number of integral solutions is 3.

Hence, the correct option is (D).

22. Given: $1 \le x \le 3$, $2 \le y \le 5$.

$\frac{x}{y}$ is minimum when x is minimum and y is maximum,

$\therefore$ The minimum value of $\frac{x}{y} = \frac{1}{5}$

The minimum value of $\frac{x+y}{y}$

$= 1 + \frac{x}{y}$

$= 1 + \frac{1}{5} = \frac{6}{5}$

Hence, the correct option is (C).

23. $x = |a|b$ and $5 \le |b|$

$\therefore xb = |a|b^2 \ge 25\,|a|$

Consider $a - xb$

From a, we are subtracting a quantity that is greater than or equal to $25\,|a|$. If $a = 0$, this could be 0 or negative.

But if $a \ne 0$, this would be negative.

$\therefore a - xb \le 0$.

Hence, the correct option is (D).

24. Given; $\left|x-\left|x-2\right|\right| = 6$

When $x < 2$, $\left|x-2\right| = -(x - 2)$

$\therefore |x-(-(x - 2)) = 6 \Rightarrow (2x - 2) = \pm 6$

$x - 1 = \pm 3$

$x = 4$ or -2

since $x < 2$; so $x = -2$ is the only solution.

when $x > 2$ the equation is not true.

Hence, the correct option is (B).

25. $\frac{x^2y + y^2z + z^2x + xy^2 + yz^2 + zx^2}{xyz}$

$= \frac{x^2y + x^2z + y^2z + y^2x + z^2x + z^2y}{xyz}$

$= \frac{x^2(y+z) + y^2(z+x) + z^2(x+y)}{xyz}$

$= \frac{x(y+z)}{yz} + \frac{y(z+x)}{xz} + \frac{z(x+y)}{xy}$

$= \frac{x}{z} + \frac{x}{y} + \frac{y}{x} + \frac{y}{z} + \frac{z}{y} + \frac{z}{x}$

$= \left(\frac{x}{z} + \frac{z}{x}\right) + \left(\frac{x}{y} + \frac{y}{x}\right) + \left(\frac{z}{y} + \frac{y}{z}\right)$

Since the sum of a number and its reciprocal is ≥ 2.

$\therefore$ The minimum value of the sum is $= 2 + 2 + 2 = 6$

Hence, the correct option is (A).

Practice Problems 2

1. The 4 expressions in the options involve x, x^2, y, y^2.

$\therefore$ We first determine the range of values for these 4 quantities.

$-7 \le y \le -3$ and $2 \le x \le 5$

$\therefore 9 \le y^2 \le 49$

$\therefore 4 \le x^2 \le 25$

We note that $xy^2 \ge 0$.

$\therefore$ We need to look at only choices (C), (D), (A). The range of possible values for these are tabulated below.

$-175 \le x^2y \le -12$, and $18 \le xy^2 \le 245$

$\Rightarrow -490 \le -2xy^2 \le -36$

$-35 \le xy \le -6$

$\Rightarrow -105 \le 3xy \le -18$

$\therefore$ The minimum value of $-2xy^2$ is the least.

Hence, the correct option is (D).

2. Given: $(x^2 + x + 1)x < 1$

Taking logarithms on both sides to a strong base,

$\log (x^2 + x + 1)x < \log 1$

$\Rightarrow x \log (x^2 + x + 1) < 0$

This could be true if $x > 0$ and $\log(x^2 + x + 1) < 0$, i.e. when $x > 0$ and $x^2 + x + 1 < 1$.

But this is not possible.

$\therefore$ We consider only $x < 0$ and $\log(x^2 + x + 1) > 0$, i.e., $x^2 + x + 1 > 1 \Rightarrow x(x + 1) > 0$ i.e., $x < -1$.

$\therefore$ The solution set is $(-\infty, -1)$.

Hence, the correct option is (D).

3. Given: $|x-3|+|x-4| \le 7$

Case 1: When $x \ge 4$, $|x-3| = (x - 3)$ and $|x-4| = x - 4$

$|x-3|+|x-4| \le 7 \Rightarrow x - 3 + x - 4 \le 7$

$2x \le 14$

$x \le 7$

$\therefore x \in [4, 7]$

Case 2: When $x \le 3$, $|x-3| = 3 - x$ and $|x-4| = 4 - x$

$|x-3|+|x-4| \le 7 \Rightarrow 3 - x + 4 - x \le 7 - 2x \le 0$

$x \ge 0$

$\therefore x\ [0, 3]$

When $x \in (3, 4)$, $|x-3| = x - 3$; $|x-4| = 4 - x$

$|x-3|+|x-4| \le 7 \Rightarrow x - 3 + 4 - x \le 7$

$\Rightarrow 1 \le 7$ is always true

$\therefore$ The solution set $= x \in [0, 7]$

The number of integral solutions of the given inequation is 8.

Hence, the correct option is (C).

4. Given: $x^2 - 5x + 6 > 0$

$(x - 2)(x - 3) > 0$

$x < 2$ or $x > 3$

$x^2 - 3x + 2 > 0$; $(x - 2)(x - 1) > 0$

$x < 1$ or $x > 2$

$\therefore$ The common solution is $x < 1$ or $x > 3$

Hence, the correct option is (B).

5. Given: $\dfrac{x-3}{x+2} < 0$

$\Rightarrow \dfrac{(x-3)(x+2)}{(x+2)^2} < 0$

$\Rightarrow (x + 2)(x - 3) < 0$

$-2 < x < 3$

The number of integral values of x that satisfy is 4.

Hence, the correct option is (C).

6. Given: $3x + 17 < 5x - 19$ and $4x + 15 > 9x + 21$

$3x + 17 < 5x - 19$

$3x - 5x < -19 - 17 \Rightarrow -2x < -36$

$x > 18$ (12)

$4x + 15 > 9x + 21$

$4x - 9x > 21 - 15$

$-5x > 6$

$x < -\dfrac{6}{5}$ (13)

From (12) and (13) we see that there is no common solution.

The solution set is $\{\ \}$

Hence, the correct option is (C).

7. Let $f(x) = |x+3|+|x-5| + 7$

When $f(x) = x > 5$, $|x+3| = x + 3$ and

$|x-5| = x - 5$

$\therefore f(x) = x + 3 + x - 5 + 7 = 2x + 5$

when $x < -3$, $|x+3| = -(x + 3)$ and $|x-5| = -(x - 5)$

$\therefore f(x) = -x - 3 - x + 5 + 7 = 9 - 2x$

when $-3 < x < 5$, $|x+3| = x + 3$

$|x-5| = -(x - 5)$

$f(x) = x + 3 - x + 5 + 7 = 15$.

$\therefore$ The minimum value of $f(x)$ is 15.

$f(x)$ is minimum when $x \in [-3, 5]$

Hence, the correct option is (A).

8. Let $f(x) = 10 - |3x+5|$

We know that $|3x+5|$ is always positive

$10 - |3x+5| \le 10$

$\therefore$ The maximum value of $f(x) = 10$

Hence, the correct option is (A).

9. Given: $9 - 4x - 5x^2 \ge 0$

$5x^2 + 4x - 9 \le 0$

$(x - 1)(5x + 9) \le 0$

$\therefore$ Critical points are $\dfrac{-9}{5}, 1$

X	✓	X
$\frac{-9}{5}$		1

when $x = 0$ the above inequation is true.

The solution is $\left[\dfrac{-9}{5}, 1\right]$

The number of integral values of 'x' is 3.

Hence, the correct option is (D).

10. $|x + 4| < 3x - 7$.

As the modulus of any quantity is non-negative and $3x - 7$ being more than the modulus of a quantity would imply that.

$3x - 7 > 0$

$\Rightarrow x > \frac{7}{3}$

Now when $x > 7/3$

$|x + 4| = x + 4$

So the relation is reduced to $x + 4 < 3x - 7$

$\Rightarrow 2x > 11$

$\Rightarrow x > \frac{11}{2}$

Thus the range of x is $\left(\frac{11}{2}, \infty\right)$.

Hence, the correct option is (A).

11. Consider $2p^2+p+2 = p\left(2\left(p+\frac{1}{p}\right)+1\right) = p\left(2(\geq 2)+1\right) \geq 5p$

$3q^2 + q + 3 \geq 7q$ and $r^2 + r + 1 \geq 3r$

$$\frac{\left(2p^2+p+2\right)\left(3q^2+q+3\right)\left(r^2+r+1\right)}{15pqr} \geq \frac{5p\times 7q\times 3r}{15pqr} = 7$$

$\therefore x$ cannot be 6.

Hence, the correct option is (D).

12. Given $-x^2 + 3kx + 5k + 1 < 0$

$\Rightarrow x^2 - 3kx - 5k - 1 > 0$

The expression is always positive, if $b^2 - 4ac < 0$

$\Rightarrow 9k^2 + 4(5k + 1) < 0$

$\Rightarrow 9k^2 + 20k + 4 < 0$

$\Rightarrow 9k^2 + 18k + 2k + 4 < 0$

$(k + 2)(9k + 2) < 0$

$k \in \left(-2, \frac{-2}{9}\right)$

Hence, the correct option is (C).

13. $|x^2 + x - 2| \leq x^2 - x$ (14)

$x^2 + x - 2 = (x + 2)(x - 1)$

If $x \leq -2$ or $x \geq 1$, then $x^2 + x - 2 \geq 0$ and $|x^2 + x - 2| = x^2 + x - 2$.

If $-2 < x < 1$, then $x^2 + x - 2 < 0$ and $|x^2 + x - 2| = -x^2 - x + 2$.

Let $x \leq -2$ or $x \geq 1$.

(14) $\Rightarrow x^2 + x - 2 \leq x^2 - x$

$\Rightarrow x - 1 \leq 0$

$\therefore x \in (-\alpha, -2]$ (15)

Let $-2 < x < 1$

(14) $\Rightarrow -x^2 - x + 2 \leq x^2 - x$

$\Rightarrow -2x^2 + 2 \leq 0$

$\Rightarrow 2x^2 - 2 \geq 0 \Rightarrow x^2 \geq 1$

$\Rightarrow x \leq -1$ or $x \geq 1$

$\therefore x \in (-2, -1]$ (16)

From (15), (16), $x \in (-\infty, -1]$

Hence, the correct option is (B).

14. For positive numbers, the GM is less than or equal to the AM.

$\therefore \sqrt[3]{(xy)(yz)(zx)} \leq \frac{xy + yz + zx}{3}$

i.e., $xy + yz + yx \geq 3(xyz)^{2/3} = 3\ (216)^{2/3} = 108$

$\therefore$ 96 is not a possible value.

Hence, the correct option is (D).

15. Given: $3x + 2 < |2x+5| < 8x + 9$

Case 1: $x > \frac{-5}{2}$, $|2x+5| = 2x + 5$

$3x + 2 < 2x + 5 < 8x + 9$

consider $3x + 2 < 2x + 5$

$x < 3$ (17)

$2x + 5 < 8x + 9$

$\Rightarrow -6x < 4$

$x > \frac{-4}{6}$ (18)

From (17) and (18) $\frac{-2}{3} < x < 3$

Case: $2x < \frac{-5}{2}$ then $|2x + 5| = -(2x + 5)$

$3x + 2 < -(2x + 5) < 8x + 9$

Consider $3x + 2 < -(2x + 5)$

$3x + 2x < -5 - 2$

$5x < -7$

$x < -\frac{7}{5}$ (19)

$-(2x + 5) < 8x + 9$

$-10x < 9 + 5$

$-10x < 14$

$\Rightarrow x > \frac{-14}{10}$

$x > -\frac{7}{5}$

$x < -\frac{7}{5}$ and $x > -\frac{7}{5}$ such a value of x does not exist.

$\therefore$ The solution set of the inequation is $\frac{-2}{3} < x < 3$

Hence, the correct option is (A).

16. Given:

$$\frac{x^2-7x+10}{x^2+6x-40} < 1$$

$$\Rightarrow \frac{x^2-7x+10}{x^2+6x-40} - 1 < 0$$

$$\Rightarrow \frac{x^2-7x+10-x^2-6x+40}{x^2+6x-40} < 0$$

$$\Rightarrow \frac{-13x+50}{x^2+6x-40} < 0$$

$$\Rightarrow \frac{(13x-50)(x^2+6x-40)}{(x^2+6x-40)^2} > 0$$

$\Rightarrow (13x - 50)\,(x + 10)\,(x - 4) > 0$

The critical points are $\frac{50}{13}, -10, 4$

X	✓	X	✓
−10		$\frac{50}{13}$	4

when $x = 0$ the inequation is true

$\therefore$ The values of x lying in the 2nd and fourth regions satisfy the above inequations.

Solution set is $\left(-10, \frac{50}{13}\right) \cup (4, \infty)$

Hence, the correct option is (B).

17. Given: $x \le 4$ (20)

and $y \ge -2$

$y \ge -2 \Rightarrow -y \le 2$ (21)

Adding (20) and (21) we get

$x - y \le 4 + 2 = x - y \le 6$ [$\therefore$ (C) is true]

If $x = -10$, $y = 1$, $xy = -10$ [$\therefore$ (A) and (B) are false]

Hence, the correct option is (C).

18. If the product of several numbers is constant, their sum is minimum when they are all equal.

As $a_1 a_2 \ldots a_{3n} = 1$, the minimum value of $a_1 + a_2 + \ldots + a_{3n}$ is $1 + 1 + \ldots + 1$ ($3n$ times) $= 3n$.

Hence, the correct option is (D).

19. We know that

AM $(a, b, c, d) \ge$ HM (a, b, c, d)

$$\frac{a+b+c+d}{4} \ge \frac{4}{\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}}$$

$$(a + b + c + d)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}\right) \ge 16$$

$\therefore$ The minimum value is 16.

Hence, the correct option is (A).

20. $|2x - 7| - 8$ is minimum when $|2x - 7|$ is minimum. This happens when $x = \frac{7}{2}$.

Hence, the correct option is (B).

21. $\min(x + 6, x - 3) = x - 3$, for all values of x.

$\min(x + 5, x - 7) = x - 7$, for all values of x.

$\therefore$ Required value $= \max(x - 3, x - 7)$

$= x - 3$ for all values of x

Hence, the correct option is (C).

22. When $a > b$ and $c > 0$

$\Rightarrow a + c > b + c$ is true

$a - c > b - c$ is also true

$ac > bc$ is also true.

Hence, the correct option is (D).

23. $2x - 3 \ge 7 \Rightarrow 2x \ge 10 \Rightarrow x \ge 5$. (22)

$5x - 7 < 3 \Rightarrow x < 2$ (23)

From (22) and (23), there are no common values of x.

$\therefore$ The solution is the empty set.

Hence, the correct option is (A).

24. $-2 > -5$ but $(-2)^2 < (-5)^2$. Option (A) is not true always.

$5 > 2$ but $(5)^2 > (2)^2$. Option (B) is not true always.

But when $x > 0$, $y > 0$, and $x > y \Rightarrow x^2 > y^2$.

Option (C) is always true.

Hence, the correct option is (C).

25. $|x + 7| < 10$

$\Rightarrow -10 < x + 7 < 10 \Rightarrow -17 < x < 3$

$\therefore$ solution set is $(-17, 3)$

Hence, the correct option is (D).

CHAPTER 12 PROGRESSIONS

EXERCISES

Practice Problems I

Directions for questions 1 to 25: Select the correct alternative from the given choices.

1. The sixth term and the eleventh term of an arithmetic progression are 30 and 55, respectively. Find the twenty-first term of the series.

(A) $88\frac{1}{3}$ (B) 105 (C) 110 (D) $92\frac{1}{2}$

2. What is the 15th term of an arithmetic progression whose first term is equal to its common difference and whose 3rd term is 9?

(A) 15 (B) 30 (C) 45 (D) 60

3. If $x + 4$, $6x - 2$, and $9x - 4$ are three consecutive terms of an arithmetic progression, then find x.

(A) 2 (B) 4 (C) 6 (D) 8

4. Find the number of terms and the sum of the terms of the arithmetic progression 32, 28, … 4.

(A) 8; 144 (B) 7; 126
(C) 14; 252 (D) 15; 270

5. Find the sum of the first 31 terms of the arithmetic progression whose first term is 6 and whose common difference is $\frac{8}{3}$.

(A) 1410 (B) 1418 (C) 1426 (D) 1434

6. The sum of five terms of an arithmetic progression is 70. The product of the extreme terms is 132. Find the five terms.

(A) 4, 8, 12, 16, 20 (B) 10, 12, 14, 16, 18
(C) 6, 10, 14, 18, 22 (D) 8, 12, 16, 20, 24

7. The sum to n terms of an arithmetic progression is $5n^2 + 2n$. Find the n^{th} term of the series.

(A) $10n + 5$ (B) $10n - 3$
(C) $5n - 1$ (D) $5n - 2$

8. Which term of the geometric progression 4, $4\sqrt{2}$, 8 … is $64\sqrt{2}$?

(A) 8 (B) 9 (C) 10 (D) 12

9. Find the sixth term of the geometric progression whose first term is 2 and common ratio is 3.

(A) 96 (B) 486 (C) 1458 (D) 162

10. Find the sum of the first 4 terms of a geometric progression whose first term is 6 and whose common ratio is 2.

(A) 90 (B) 84 (C) 96 (D) 102

11. What is the sum of the first 7 terms of a geometric progression whose first term is 1 and 4th term is 8?

(A) 129 (B) 128
(C) 127 (D) None of these

12. If the sum to 37 terms of an arithmetic progression is 703, then find the middle term of the arithmetic progression.

(A) 34 (B) 17 (C) 38 (D) 19

13. Find the sum of the 20 terms of the series 1, (1 + 2), (1 + 2 + 3), (1 + 2 + 3 + 4), (1 + 2 + 3 + 4 + 5), …

(A) 1540 (B) 1435 (C) 1450 (D) 1345

14. If the real numbers a, c and b as well as $a^2 + b^2$, $a^2 + c^2$, and $b^2 + c^2$ are in geometric progression, then which of the following is necessary true?

(A) $a = b$ (B) $b = c$
(C) $a = c$ (D) $a = b = c$

15. How many numbers between 450 and 950 are divisible by both 3 and 7?

(A) 20 (B) 24 (C) 30 (D) 35

16. $S = 2 + 4x + 6x^2 + 8x^3$ … where $|x| < 1$. Which of the following is the value of S?

(A) $\frac{4}{(1-x)^2}$ (B) $\frac{3}{(1-x)^2}$
(C) $\frac{2}{(1-x)^2}$ (D) $\frac{1}{(1-x)^2}$

17. The sum of the first eight terms of a geometric progression. is 510 and the sum of the first four terms of the geometric progression. is 30. Find the first term of the geometric progression, given that it is positive.

(A) 2 (B) 4 (C) 6 (D) 8

18. Find the integer value of y, if $-x$, $2y$, and $2(y + 3)$ are in arithmetic progression and $(x + 2)$, $2(y + 1)$, and $(5y - 1)$ are in geometric progression.

(A) 2 (B) 3 (C) 4 (D) 5

19. Find the number of terms common to the progressions 2, 8, 14, 20, …, 98 and 6, 10, 14, 18, …, 102.

(A) 7 (B) 6 (C) 8 (D) 9

20. Find the sum of the series $2 + 3x + 4x^2 + 5x^3 + \cdots$ to infinity, if $|x| < 1$.

(A) $\frac{2-x}{(1-x)^2}$ (B) $\frac{2+x}{(1+x)^2}$
(C) $\frac{2-x}{(1+x)^2}$ (D) $\frac{2+x}{(1-x)^2}$

21. The mean of the sequence 3, 8, 17, 30, …, 1227 is _____.

(A) 531 (B) 431 (C) 314 (D) 315

22. Find the value of $-1^2 + 2^2 - 3^2 + 4^2 - 5^2 + 6^2 + \cdots - 19^2 + 20^2$

(A) 210 (B) 420 (C) 630 (D) 720

23. Find the sum of the given terms in the following series:

$$\frac{1}{\sqrt{3}+1} + \frac{1}{\sqrt{3}+\sqrt{5}} + \frac{1}{\sqrt{5}+\sqrt{7}} + \cdots + \frac{1}{\sqrt{119}+\sqrt{121}}$$

(A) $2\sqrt{3}+1$ (B) 5
(C) $11 - 2\sqrt{3}$ (D) 10

24. If $\log_3 x + \log_{\sqrt[3]{3}} x + \log_{\sqrt[5]{3}} x + \ldots + \log_{\sqrt[23]{3}} x = 432$, then find x.
(A) 9 (B) 27 (C) $3\sqrt{3}$ (D) 81

25. The sum of the first n terms of two arithmetic progressions S_1 and S_2 are in the ratio $11n - 17 : 5n - 21$. Find the ratio of the 16^{th} terms of S_1 and S_2.
(A) 3 : 2 (B) 162 : 67 (C) 9 : 4 (D) 27 : 8

Practice Problems 2

Directions for questions 1 to 25: Select the correct alternative from the given choices.

1. Thirteen times the thirteenth term of an arithmetic progression is equal to seven times the seventh term of the arithmetic progression. What is the twentieth term?
(A) –1 (B) –3 (C) 0 (D) 4

2. Find the sum of the terms of the arithmetic progression whose first term, last term, and common difference are 3, 101, and 7, respectively.
(A) 750 (B) 720 (C) 780 (D) 810

3. Find the sum of the terms of an arithmetic progression whose first term, last term, and number of terms are –9, 51, and 21, respectively.
(A) 420 (B) 441
(C) 462 (D) 483

4. Find the first four terms of a geometric progression whose n^{th} term is $4(-5)^n$.
(A) –20, 100, –500, 2500
(B) 20, –100, –500, 2500
(C) –20, –100, –500, –2500
(D) 20, 100, 500, 2500

5. Find the sum to infinity of $1, \frac{3}{4}, \frac{9}{16}, \frac{27}{64}, ..$
(A) 2 (B) 4 (C) 4/3 (D) 2/3

6. If x, y, and z are three natural numbers in arithmetic progression, then the x^{th} term, the y^{th} term, and the z^{th} term of any arithmetic progression A are in
(A) Arithmetic progression
(B) Geometric progression
(C) Not necessarily in arithmetic progression or geometric progression.
(D) Arithmetic as well as geometric progression

7. If x, y, and z are three natural numbers in arithmetic progression, then the x^{th} term, the y^{th} term, and the z^{th} term of any geometric progression G, are in
(A) Arithmetic progression
(B) Geometric progression
(C) Not necessarily in arithmetic progression or geometric progression.
(D) Arithmetic as well as geometric progression

8. The sum of the squares of three terms in arithmetic progression is 365. The product of the first and the third terms is 120. Find the sum of the squares of the second term and the common difference.
(A) 145 (B) 170
(C) 122 (D) 197

9. The sum of four numbers in an ascending arithmetic progression. is 160 and the product of the extremes is 1564. Find the smallest of the numbers.
(A) 28 (B) 34
(C) 42 (D) 43

10. Let $A(N)$ denote the sum of the first N natural numbers and $B(N)$ denote the sum of the squares of the first N natural numbers. If $B(N)$ is a multiple of $A(N)$, then which of the following could be true?
(A) N is divisible by 6
(B) $N + 1$ is divisible by 6
(C) $N + 2$ is divisible by 6
(D) $N - 3$ is divisible by 6

11. Find the smallest of the three numbers in arithmetic progression, if the product of the first and the third numbers is 252 and the sum of the three numbers is 48.
(A) 10 (B) 12
(C) 14 (D) 16

12. If the sum of the first 37 terms of an arithmetic progression. is 703, then find the sum of the first 10 terms of the arithmetic progression, given the first term of the arithmetic progression is 1.
(A) 55 (B) 65
(C) 75 (D) 85

13. The first term of an arithmetic progression consisting of 30 terms is 10 and the common difference is 5. Find the ratio of the sum of the 30 terms of the arithmetic progression. to the sum of the last 20 terms of the AP.
(A) 99 : 13 (B) 96 : 17
(C) 93 : 19 (D) 99 : 86

14. The product of three terms in geometric progression. is 1728 and the sum of the products of two of them taken at a time is 1032. Find the smallest of the three terms.
(A) 1 (B) 2
(C) 3 (D) 6

15. The first term of an arithmetic progression is 6 and the common difference is 4. The n^{th} term of the series is 250. Find the value of n.
(A) 60 (B) 62
(C) 66 (D) 64

16. $7^{1+\frac{1}{2}+\frac{1}{4}+\cdots\infty}$ evaluates to
(A) 49 (B) 34 (C) 2401 (D) 343

17. An athlete runs a race, and after every hour, his speed reduces to half the speed with which he travelled for the previous hour. Find the time taken to cover the race, if the person started the race with a speed of 16 km/hr and the length of the race was 31.5 km.

(A) 4 hours (B) 6 hours
(C) 8 hours (D) 5 hours

18. If S_n is the sum of the first n terms of the series $40 + 38 + 36 + \cdots$, then find the maximum value of S_n.
(A) 450 (B) 420
(C) 390 (D) 410

19. $1^3 + 2^3 + 3^3 + \cdots + m^3 = 3025$. Find the value of m.
(A) 8 (B) 9 (C) 10 (D) 11

20. Find the sum of the squares of the first 10 even numbers.
(A) 840 (B) 1540
(C) 1260 (D) 1370

21. Find the sum of the terms of the series
$(1) \times (20), (2) \times (19), (3) \times (18), \ldots (20) \times (1)$.
(A) 1750 (B) 1640
(C) 1540 (D) 1430

22. If the sum of three numbers in a geometric progression is 38 and their product is 1728, then find the smallest number.
(A) 6 (B) 4
(C) 2 (D) 8

23. The common ratio of geometric progression is a positive number less than 1. The first term is 18. The difference of the third and second terms is 4. Find the common ratio.
(A) $\frac{1}{3}$ (B) $\frac{2}{3}$
(C) $\frac{3}{4}$ (D) $\frac{1}{3}$ or $\frac{2}{3}$

24. Find the sum of the first 12 terms of S_n, where $S_n = 1 + 3 + 7 + 13 + 21 + 31 + 43 + \cdots$.
(A) 472 (B) 584
(C) 591 (D) 572

25. S_1 is a square. By joining the midpoints of sides of S_1 another square S_2 is formed. By joining the midpoints of S_2, another square S_3 is formed, and so on. The side of S_1 is 32 cm. Find the sum of the perimeters of all the squares (in cm).
(A) $48\sqrt{2}\left(\sqrt{2}+1\right)$ (B) $128\sqrt{2}\left(\sqrt{2}+1\right)$
(C) $72\sqrt{2}\left(\sqrt{2}+1\right)$ (D) $36\sqrt{2}\left(\sqrt{2}+1\right)$

HINTS/SOLUTIONS

Practice Problems I

1. Let the first term and the common difference of the series be a and d respectively.

Then, $T_6 = a + 5d = 30$ (9)

$T_{11} = a + 10d = 55$ (10)

Solving (9) and (10)

$a = 5, d = 5$

$T_{21} = a + 20d \Rightarrow 5 + 20\,(5) = 105$

Hence, the correct option is (B).

2. Let the first term of the arithmetic progression be a.

$a + 2a = 9$

$a = 3$

15^{th} term $= a + 14a$

$15a = 45$

Hence, the correct option is (C).

3. $x + 4$, $6x - 2$ and $9x - 4$ are three consecutive terms in an A.P.

$\Rightarrow 6x - 2 - (x + 4) = 9x - 4 - (6x - 2)$

$6x - 2 - x - 4 = 9x - 4 - 6x + 2$

$5x - 6 = 3x - 2 \Rightarrow 2x = 4$

$x = 2$

Hence, the correct option is (A).

4. $a = 32, d = -4$

$t_n = 32 + (n - 1)\,(-4) = 4$

$\Rightarrow n = 8$

sum of the series $= \dfrac{8(32+4)}{2} = 144$

Hence, the correct option is (A).

5. Sum of the first n terms of an arithmetic progression whose first term is a and common difference is $d = \dfrac{n}{2}$ $[2a + (n - 1)\,d]$

Sum of the first 31 terms of the arithmetic progression

$= \dfrac{31}{2}[2\,(6) + (31 - 1)\left(\dfrac{8}{3}\right)] = \dfrac{31}{2}\,[92] = 1426$

Hence, the correct option is (C).

6. Let the five terms of the arithmetic progression be $a - 2d$, $a - d$, a, $a + d$, $a + 2d$.

sum $= a - 2d + a - d + a + a + d + a + 2d = 70$

$\Rightarrow 5a = 70 \Rightarrow a = 14$

Product of extremes $= (14 - 2d)\,(14 + 2d) = 132$

$\Rightarrow 196 - 4d^2 = 132 \Rightarrow d = \pm 4$

The five terms are 6, 10, 14, 18, 22 or alternatively, substitute the options and check.

Hence, the correct option is (C).

7. $S_n = 5n^2 + 2n$

$\Rightarrow S_{n-1} = 5n^2 + 2n - 1$

n^{th} term $= S_n - S_{n-1}$

$= 5n^2 + 2n - \{5(n - 1)^2 + 2\,(n - 1)$

$= 10n - 3$

Hence, the correct option is (B).

8. $a = 4r = \sqrt{2}$

let $64\sqrt{2}$ be the n^{th} term of the geometric progression.

$T_n = 4\,(\sqrt{2})^{n-1} = 64\sqrt{2}$

$\Rightarrow 2^{n-1} = 2^{9/2} \Rightarrow n = 10$

Hence, the correct option is (C).

9. Sixth term $= 2\,(3)^5 = 486$

Hence, the correct option is (B).

10. The sum of the first n terms of a geometric progression whose first term is a and whose common ratio is r is given by $\dfrac{a\left(r^n - 1\right)}{r - 1}$

Sum of the first 4 terms $= \dfrac{6\left(2^4 - 1\right)}{2 - 1} = 90$

Hence, the correct option is (A).

11. Let the common ratio of the geometric progression be r.

$r^3 = 8; r = 2$

Sum of the first terms $= \dfrac{1\left(2^7 - 1\right)}{2 - 1} = 127$

Hence, the correct option is (C).

12. Sum of the terms of an A.P. = (number of terms) × (the middle term of the A.P.), if number of terms (n) is odd. Since, n is 37 (odd), we have sum of the terms of the A.P. = 703 = 37(middle term of the A.P.). Middle term of the A.P. = (703/37) = 19.

Hence, the correct option is (D).

13. Given series is 1, (1 + 2), (1 + 2 + 3), (1 + 2 + 3 + 4) …

$\therefore$ The n^{th} bracket is

$1 + 2 + 3 + 4 + \ldots + n$ i.e. $n = \dfrac{n(n+1)}{2}$

$S_n = \Sigma t_n = \Sigma\, \dfrac{n(n+1)}{2}$

$= \dfrac{1}{2}\,[\Sigma n^2 + \Sigma n]$

$= \dfrac{1}{2}\left[\dfrac{n(n+1)(2n+1)}{6} + \dfrac{n(n+1)}{2}\right]$

$$S_n = \frac{n(n+1)(2n+1)}{12} + \frac{n(n+1)}{4}$$

$$\therefore S_{20} = \frac{(20)(21)(41)}{12} + \frac{(20)(21)}{4}$$

$$= 1435 + 105 = 1540$$

Hence, the correct option is (A).

14. As a, c and b are in geometric progression,

$$c^2 = ab \; c^4 = a^2b^2$$

As $a^2 + b^2$, $a^2 + c^2$ and $b^2 + c^2$ are in geometric progression,

$(a^2 + c^2)^2 = (a^2 + b^2)(b^2 + c^2)$

$a^4 + 2a^2c^2 + c^4 = a^2b^2 + a^2c^2 + b^2c^2 + b^4$

$\Rightarrow a^4 + a^2c^2 + c^4 = a^2b^2 + b^2c^2 + b^4$

since $c^4 = a^2b^2$

$a^2(c^2 + a^2 + b^2) = b^2(a^2 + b^2 + c^2)$

$$a^2 = b^2$$

$\Rightarrow a, b$ have the same sign.

$\therefore a = b$ and $c = \pm a$

Hence, the correct option is (A).

15. The first number between 450 and 950 which is divisible by both 3 and 7 i.e., 21 is 462. The last number between 450 and 950 which is divisible by 21 is 945. Hence, from 21×22 to 21×45, there are a total of 24 terms which are divisible by both 3 and 7.

Solving using the concept of Progressions:

First term between 450 and 950 divisible by 21 (both 3 and 7); $a = 462$ (between 450 and 950). The last term divisible by 21 between 450 and 950 (ℓ) = 945. The common difference between successive terms which are divisible by 3 and 7 is 21.

$$\text{Number of terms required} = \frac{945-462}{21} + 1 = \frac{483}{21} + 1$$

$$= 23 + 1 = 24.$$

Hence, the correct option is (B).

16. $S = 2 + 4x + 6x^2 + 8x^3 + \ldots$ (11)

Multiplying by x both sides

$S_x = 2x + 4x^2 + 6x^3 + \ldots$ (12)

Subtracting (12) from (11),

$S(1-x) = 2 + 2x + 2x^2 + 2x^3 + \ldots$

$$S = \frac{2}{\frac{1-x}{1-x}} = \frac{2}{(1-x)^2}$$

Hence, the correct option is (C).

17. If the first term of the G.P. is a and the common ratio is r, we have

$$\frac{a(r^8-1)}{r-1} = 510 \quad (13)$$

and $\dfrac{a(r^4-1)}{r-1} = 30$ (14)

Dividing (13) by (14) we have $\dfrac{a(r^8-1)}{r-1} \Big/ \dfrac{a(r^4-1)}{r-1}$

$$= r^4 + 1 = \frac{510}{30} = 17.$$

$r^4 = 17 - 1 = 16$

$r = \pm\sqrt[4]{16} = \pm 2$

First term of the G.P., $a = \dfrac{510(r-1)}{r^8-1}$

As first term is positive, $r = 2$ is taken.

$$a = \frac{510(2-1)}{2^8-1} = \frac{510(1)}{255} = 2.$$

Hence, the correct option is (A).

18. $2(y + 3) - 2y = 2y + 6 - 2y = 6 =$ Common difference.

As per the G.P. given,

$$\frac{5y-1}{2(y+1)} = \frac{2(y+1)}{x+2} \Rightarrow (5y-1)(x+2) = [2(y+1)]^2$$

$(5y - 1)(6 - 2y + 2) = 4(y + 1)^2$

$\Rightarrow 14y^2 - 34y + 12 = 0 \Rightarrow y = 2.$

Hence, the correct option is (A).

19. Both progressions are A.P.'s. The series of common terms of two A.P.'s is also an A.P. Its common difference is the L.C.M of the common differences of the two progressions.

First common term of the two progressions is 14.

The N^{th} common term of the two progressions

$= 14 + (N - 1)$ L.C.M. $(6, 4) - 14 + (N - 1)\,12$

$14 + (N - 1)\,12 <$ the smaller of the last terms of the two A.P.'s

$= 14 + (N - 1)\,12 = 98$

$\Rightarrow N = 8$

Hence, the correct option is (C).

20. Let $S = 2 + 3x + 4x^2 + 5x^3 + \ldots$ (15)

Then, $S_x = 2x + 3x^2 + 4x^3 + \ldots$ (16)

Subtracting (16) from (15),

$S(1 - x) = 2 + x + x^2 + x^3 + \ldots$

$$S(1-x) = 2 + \frac{x}{1-x}$$

$$S(1-x) = \frac{2(1-x)+x}{1-x} = \frac{2-x}{1-x}$$

$$S = \frac{2-x}{(1-x)^2}$$

Hence, the correct option is (A).

21. The given sequence is 3, 8, 17, 30, …, 1227

The sequence formed by the differences of two successive terms of the above sequence is 8 – 3, 17 – 8, 30 – 17, …

i.e, 5, 9, 13, …., which is an arithmetic progression

∴ The n^{th} term of the given sequence can be taken as $t_n = an^2 + bn + c$

$n = 1, t_1 = a + b + c = 3$ (17)

$n = 2, t_2 = 4a + 2b + c = 8$ (18)

$n = 3, t_3 = 9a + 3b + c = 17$ (19)

Solving (17), (18), and (19) we have $a = 2$, $b = -1$ and $c = 2$

$\therefore t_n = 2n^2 - n + 2 = 1227$

$2n^2 - n - 1225 = 0$

$(2n + 49)(n - 25) = 0$

$n = 25$ as $n > 0$

Sum of the terms is

$$\sum_{n=1}^{n} t_n = \Sigma(2n^2 - n + 2)$$

$$= 2\Sigma n^2 - \Sigma n + 2\Sigma 1$$

$$= \frac{2n(n+1)(2n+1)}{6} - \frac{n(n+1)}{2} + 2n$$

$$= n\left[\frac{2(n+1)(2n+1) - 3(n+1) + 12}{6}\right]$$

$$= \frac{n}{6}\left[4n^2 + 6n + 2 - 3n - 3 + 12\right]$$

$$= \frac{n}{6}\left[4n^2 + 3n + 11\right]$$

When $n = 25$ sum of terms $= \frac{25}{6}\left[4 \times 25^2 + 3(25) + 11\right]$

Mean of 25 terms $= \frac{25}{6}\frac{[2500 + 75 + 11]}{25} = 431$

Hence, the correct option is(B).

22. $-1^2 + 2^2 - 3^2 + 4^2 - 5^2 + 6^2 - 19^2 + 20^2$

$= (-1^2 - 3^2 - 5^2 - \ldots - 19^2) + (2^2 + 4^2 + \ldots + 20^2)$

$= -(1^2 + 3^2 + 5^2 + \ldots + 19^2) + (2^2 + 4^2 + \ldots + 20^2)$

$= -(1^2 + 2^2 + 3^2 + \ldots + 20^2) + (2^2 + 4^2 + \ldots + 20^2) + (2^2 + 4^2 + \ldots + 20^2)$

$= 2(2^2 + 4^2 + \ldots + 20^2) - (1^2 + 2^2 + 3^2 + \ldots + 20^2)$

$= 2\,[2^2\,(1^2 + 2^2 + \ldots + 10^2] - (1^2 + 2^2 + 3^2 + \ldots + 20^2)$

$= \frac{8(10)(11)(21)}{6} - \frac{(20)(21)(41)}{6}$

$= \frac{210(88 - 82)}{6} = \frac{210(6)}{6} = 210.$

Alternate method:

$-1^2 + 2^2 - 3^2 + 4^2 - 5^2 + 6^2 + \ldots + 15^2 + 20^2$

$= (-1 + 4) + (-9 + 16) + (-25 + 36) + \ldots + (-361 + 400)$

$= 3 + 7 + 11 + \ldots + 39$

This is in arithmetic progression.

$a = 3$, n^{th} term $= 39$

Number of terms = 20/2 = 10 (as the given terms are grouped into pairs)

Sum of these 10 terms $= \frac{10}{2}(3 + 39) = 5 \times 42 = 210$

Hence, the correct option is (A).

23. $\frac{1}{1+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{5}} + \frac{1}{\sqrt{5}+\sqrt{7}} + \frac{1}{\sqrt{7}+\sqrt{9}} + \ldots + \frac{1}{\sqrt{119}+\sqrt{121}}$

$= \frac{\sqrt{3}-1}{(\sqrt{3}-1)(\sqrt{3}+1)} + \frac{\sqrt{5}-\sqrt{3}}{(\sqrt{5}-\sqrt{3})(\sqrt{5}+\sqrt{3})} + \frac{\sqrt{7}-\sqrt{5}}{(\sqrt{7}-\sqrt{5})(\sqrt{7}+\sqrt{5})} + \ldots + \frac{\sqrt{121}-\sqrt{119}}{(\sqrt{121}-\sqrt{119})(\sqrt{121}+\sqrt{119})}$

$= \frac{\sqrt{3}-1}{2} + \frac{\sqrt{5}-\sqrt{3}}{2} + \frac{\sqrt{7}-\sqrt{5}}{2} + \ldots + \frac{\sqrt{121}-\sqrt{119}}{2}$

$= \frac{\sqrt{121}-1}{2} = 5$

Hence, the correct option is (B).

24. $\log 3x + \log_3 \frac{1}{3}x + \log_3 \frac{1}{5}x + \ldots + \log_3 \frac{1}{23}x = 432$

$\log_3 x + 3\log_3 x + \log 3x + \ldots + 23\log_3 x = 432$

$\Rightarrow 144 \log 3x = 432 \Rightarrow x = 27$

Hence, the correct option is (B).

25. Let the first term and the common difference of S_1 be a_1 and d_1 respectively. Let the first term and the common difference of S_2 be a_2 and d_2 respectively.

$$\frac{\frac{h}{2}[2a_1 + (n-1)d_1]}{\frac{n}{2}[2a_2 + (n-1)d_2]} = \frac{11n - 17}{5n - 21}$$

$$= \frac{a_1 + \frac{n-1}{2}d_1}{a_2 + \frac{n-1}{2}d_2} = \frac{11n - 17}{5n - 21}$$

The ratio of the 16 terms of S_1 and $S_2 = \frac{a_1 + 15d_1}{a_2 + 15d_2}$

The 16^{th} term is the average of the first 31 terms. The ratio of 16 terms in equal ratio of the sum of the 31 times of the two series.

∴ The ratio of the 16^{th} terms of S_1 and S_2

$$= \frac{11(31) - 17}{5(31) - 21} = \frac{341 - 17}{155 - 21} = \frac{324}{134} = \frac{162}{67}$$

Hence, the correct option is (B).

Practice Problems 2

Solutions for questions 1 to 25:

1. Let the first term and the common difference of the arithmetic progression be a and d respectively.

$13 + t_{13} = 7 \times t_7$

$\Rightarrow 13\ (a + 12d) = 7\ (a + 6d) \Rightarrow a = -19d$

$T_{20} = a + 19\ d = -19d + 19d = 0$

Hence, the correct option is (C).

2. Let the number of terms be n.

$101 = 3 + 7\ (n - 1)$

$15 = n$

Sum of the terms $= \frac{15}{2}[3 + 101] = 780$

Hence, the correct option is (C).

3. Sum to n terms $= \frac{n}{2}[a + 1]$

Sum of the terms $= \frac{21}{2}[-9 + 51] = 441$

Hence, the correct option is (B).

4. Putting $n = 1, 2, 3, 4$ we can get the terms

$t_1 = 4(-5)^1 = -20$

$t_2 = 4(-5)^2 = 100$

Finding two terms is enough to get the answer from options.

Hence, the correct option is (A).

5. The given series represents a geometric progression whose first term is 1 and common ratio is $\frac{3}{4}$.

The sum to infinity of a geometric progression whose first term is a and whose common ratio is $r = \frac{a}{1 - r}$ $(|r| < 1)$

The sum to infinity $= \frac{1}{1 - \frac{3}{4}} = 4$

Hence, the correct option is (B).

6. Let A be an arithmetic progression whose first term is a and whose common difference is d.

x^{th} term of $A = a + (x - 1)\ d$

y^{th} term of $A = a + (y - 1)\ d$

z^{th} term of $A = a + (z - 1)\ d$

As x, y and z are in arithmetic progression, $(x - 1)\ d$, $(y - 1)\ d$ and $(z - 1)\ d$ are in arithmetic progression…. the x^{th} term of A, the y^{th} term of A and the z^{th} term of A are in arithmetic progression.

Hence, the correct option is (A).

7. Let G be a geometric progression whose first term is a and whose common ratio is r.

x^{th} term of $G = arx^{-1}$

y^{th} term of $G = ar^{y-1}$

z^{th} term of $G = ar^{z-1}$

As x, y and z are in arithmetic progression, $y = \frac{x + z}{2}$

$\therefore ar^{y-1} = ar^{\frac{x+z-2}{2}}$

$(ar^{y-1})^2 = a^2 rx^{-1+z-1}$

$= (arx^{-1})\ (ar^{z-1})$

$\therefore$ The x^{th} term, the y^{th} term and z^{th} term of G are in geometric progression.

Hence, the correct option is (B).

8. Let the second term and the common difference be a and d respectively.

First term $= a - d$

Third term $= a + d$

$(a - d)^2 + (a + d)^2 + a^2 = 365$

$3a^2 - 2d^2 = 365$ (20)

$(a - d)\ (a + d) = 120$

$a^2 - d^2 = 120$ (21)

solving (20) and (21),

$a^2 = 121$ and $d^2 = 1$

$a^2 + d^2 = 122$

Hence, the correct option is (C).

9. Let the four terms be $a - 3d$, $a - d$, $a + d$ and $a + 3d$

$a - 3d + a - d + (a + d) + a + 3d = 160$

$4a = 160 \Rightarrow a = 40$;

Given $(a - 3d)\ (a + 3d) = 1564$; $a^2 - 9d^2 = 1564$;

$9d^2 = a^2 - 1564 = 40^2 - 1564$

$d^2 = \frac{36}{9} = 4$; $d = \pm 2$.

Since the A.P. is ascending $d = 2$.

Smallest number $= a - 3d = 40 - 3(2) = 34$.

Hence, the correct option is (B).

10. $A(N) = \frac{N(N+1)}{2}$

$B(N) = \frac{N(N+1)(2N+1)}{6}$

$\frac{B(N)}{A(N)} = \frac{2N+1}{3}$

for $\frac{B(N)}{A(N)}$ to be an integer, N must be in the form $3k + 1$ where k is a whole number, i.e., $6p + 1$ or $6p + 4$

$\therefore N - 1$ or $N + 2$ would be divisible by 6.

Hence, the correct option is (C).

11. Let the three numbers in A.P. be $a - d$, a and $a + d$;

$a - d + a + a + d = 3a = 48$

$\Rightarrow a = \frac{48}{3} = 16.$

Given that $16^2 - d^2 = 252$

$256 - 252 = d^2$

$4 = d^2$

Thus, $d = \pm\sqrt{4} = \pm 2$

Hence, the smallest of the three numbers is $16 - 2 = 14$, Even if $d = -2$ is considered, the smallest number will be 14 only.

Hence, the correct option is (C).

12. Sum upto the first 37 terms is $\frac{37}{2}[2a + (37 - 1)\,d]$

$= \frac{37}{2}[2a + 36d]$ where a is the first term of A.P and d is the common difference.

$= 37[a + 18d] = 703.$

$a + 18d = \frac{703}{37} = 1 + 18d = 19$

$\Rightarrow d = 1$

Sum of the first 10 terms of the A.P.

$= \frac{10}{2}[2\,(1) + (10 - 1)\,1] = +5\,[2 + 9] = 55.$

Hence, the correct option is (A).

13. Sum of the first 30 terms of the A.P. $= \frac{30}{2}[2\,(10) + 29\,(5)]$

$= 15\,[20 + 145] = 15\,[165] = 2475.$

Sum of the first 10 terms of the A.P. $= \frac{10}{2}[2\,(10) + 9(5)]$

$= 5[20 + 45] = 5\,[65] = 325$. Ratio of the sum of the first 30 terms of the A.P. to the sum of the last 20 terms of the A.P. $= (2475) : (2475 - 325) = 2475 : 2150$

$= 5\,[495] : 5\,[430] = 495 : 430 = 99 : 86$

Hence, the correct option is (D).

14. Let the three terms in GP be $\frac{a}{r}$, a and ar

$\therefore \frac{(a)}{r}(a)\,(ar) = 1728;\quad a^3 = 1728$

$\Rightarrow a = 12$

$\frac{a}{r}(a) + a\,(ar) + \frac{a}{r}(ar) = a^2\left(\frac{1}{r} + r\right) + a^2 = 1032$

$\because\ a^2 = 144;\ 144\left(\frac{1}{r} + r\right) + 144 = 1032$

$144\left(\frac{1}{r} + r\right) = 888,\qquad \frac{1}{r} + r = \frac{888}{144} = \frac{111}{18} = \frac{37}{6}.$

$\frac{1}{r} + r = \frac{37}{6} = 6 + \frac{1}{6}$

$\Rightarrow r = 6$ or $\frac{1}{6}$

So, 2, 12, 72 or 72, 12, 2 is the G.P. and 2 is the smallest number.

Hence, the correct option is (B).

15. If a is the first term of an arithmetic progression and the common difference is d, the n^{th} term of the progression is given by $a + (n - 1)$d. The n^{th} term is given as 250, a as 6 and d as 4.

Hence, $250 = 6 + (n - 1)4$.

$(n - 1)4 = 244$

$61 = n - 1.$

Hence, $n = 61 + 1 = 62$.

Hence, the correct option is (B).

16. $1 + \frac{1}{2} + \frac{1}{4} + \cdots$ is an infinite series in geometric progression.

$a = 1,\ r = 1/2$

Sum to infinity $= \frac{a}{1 - r} = \frac{1}{1 - 1/2} = 2$

Hence, required value is $7^2 = 49$.

Hence, the correct option is (A).

17. In the first hour, the distance covered by the athlete $= 16(1) = 16$ km. In the second hour, distance covered by the athlete $= \frac{1}{2}(16)\,(1) = 8$ km.

Assuming that the person travelled for a total of t hours, we have

$$\frac{16\left(1 - \left(\frac{8}{16}\right)^t\right)}{1 - \frac{8}{16}} = 31.5$$

$$= \frac{16\left(1 - \left(\frac{8}{16}\right)^t\right)}{1 - \frac{1}{2}} = 31.5$$

$$1 - \left(\frac{1}{2}\right)^t = \frac{31 \cdot 5}{32}$$

$$\left(\frac{1}{2}\right)^t = 1 - \frac{31 \cdot 5}{32} = \frac{32 - 31 \cdot 5}{32} = \frac{0 \cdot 5}{32} = \frac{1}{64}$$

$$\Rightarrow \left(\frac{1}{2}\right)^t = \left(\frac{1}{2}\right)^6$$

$\Rightarrow t = 6$ hours.

Alternate method:

The sum required is 31.5. First term is 16 and r is 1/2.

Hence, writing down the terms upto the value $\frac{1}{2}$, can be a good method of solving.

The terms are 16, 8, 4, 2, 1, 1/2, 1/4 …

$S_6 = 31.5$; hence $t = 6$ hours.

Hence, the correct option is (B).

18. The given series 40, 38, 36, …, is cm A.P. with common difference of –2.

The sum of the first 20 terms (S_{20}) is 2 (20) (21)/2 = 420

The 21st term is 0.

$\therefore S_{21} = 420$

For $n > 21$ the terms would be negative $S_n < 420$.

$\therefore$ The maximum value of $S_n = 420$.

Hence, the correct option is (B).

19. $1^3 + 2^3 + 3^3 + \ldots + m^3 = \left(\frac{m(m+1)}{2}\right)^2 = 55^2$

Hence, $\frac{m(m+1)}{2} = 55$; $m^2 + m = 110$

$m^2 + m - 110 = 0$

Hence, $m = -11$ or $m = 10$. The number of terms cannot be negative. Thus, $m = 10$.

Hence, the correct option is (C).

20. Sum of the squares of the first 10 even numbers

$= 2^2 + 4^2 + 6^2 + 8^2 + 10^2 + \ldots + 20^2$

$= 2^2 (1^2 + 2^2 + 3^2 + 4^2 + 5^2 + \ldots + 10^2)$

$= \frac{4(10)(11)(21)}{6}$

[Applying the formula $\frac{n(n+1)(2n+1)}{6}$ for the sum of the squares of first n natural numbers]

$= \frac{4 \times 210 \times 11}{6} = 140 \times 11 = 1540.$

Hence, the correct option is (B).

21. The terms of the series are in the form $x\,(21 - x)$

Required sum $= \sum_{x=1}^{20} x\,(21 - x)$

$= \frac{(21)(20)(21)}{2} - \frac{1}{6}\,(20)(21)(41) = 1540$

Hence, the correct option is (C).

22. Let the first term and the common ratio be a and r respectively.

Second term = ar and Third term = ar^2

$a + ar + ar^2 = 38$ and $(a)(ar)(ar^2) = 1728$

$(ar)^3 = 12^3 \Rightarrow ar = 12$

The first 3 terms are $\frac{12}{r}$, 12, $12r$

$\frac{12}{r} + 12 + 12r = 38 \Rightarrow 6r^2 - 13r + 6 = 0 \Rightarrow r = \frac{2}{3}$ or $\frac{3}{2}$

if $r = \frac{2}{3}$, the numbers are 18, 12 and 8.

If $r = \frac{3}{2}$, the same numbers are obtained in the reverse order.

In either case, 8 is the smallest number

Hence, the correct option is (D).

23. Let the common ratio be r. $0 < r < 1$,

$\therefore r > r^2$. The first 3 terms are 18, $18r$, $18r^2$

$18r - 18r^2 = 4 \Rightarrow r = \frac{2}{3}$ or $\frac{1}{3}$

Hence, the correct option is (D).

24. $S_n = 1 + 3 + 7 + 13 + 21 + 31 + 43 + \ldots + t_n$

$S_n = 1 + 3 + 7 + 13 + 21 + 31 + \ldots + t_{n-1} + t_n$

Subtracting, we get

$0 = 1 + [2 + 4 + 6 + 8 + 10 + \ldots$ upto $n - 1$ terms$] - t_n$

$\Rightarrow t_n = 1 + [2 + 4 + 6 + 8 + \ldots$ up to $n - 1$ terms$]$

$\Rightarrow t_n = 1 + \frac{n-1}{2}[2(2) + (n-2)2]$

$= 1 + \frac{(n-1)}{2}[2n] = 1 + n^2 - n$

$\therefore S_N = \sum_{n=1}^{N} t_n = \Sigma(n^2 - n + 1) = \Sigma n^2 - \Sigma n + \Sigma 1$

$= \frac{N(N+1)(2N+1)}{6} - \frac{N(N+1)}{2} + N$

$\therefore S_{12} = \frac{12(13)(25)}{6} - \frac{12(13)}{2} + 12$

$= 650 - 78 + 12$

$= 584$

Hence, the correct option is (B).

25. Side of $S_2 = \sqrt{\left(\frac{\text{side of } S_1}{2}\right)^2}(2) = \frac{1}{\sqrt{2}}$ (side of S_1)

It follows that side of $S_{n+1} = \frac{1}{\sqrt{2}}$ (side of S_n)

Where n is any natural number

Sum of the perimeters $= 4\left(32 + \frac{32}{\sqrt{2}} + \frac{32}{2} + \ldots\right)$

$= \frac{4(32)}{1 - \frac{1}{\sqrt{2}}} = 128\sqrt{2}(\sqrt{2} + 1)$ cm

Hence, the correct option is (B).

Chapter 13 Permutations and Combinations

Exercises

Practice Problems 1

Directions for questions 1 to 25: Select the correct alternative from the given choices.

1. A man has 12 blazers, 10 shirts, and 5 ties. Find the number of different possible combinations in which he can wear the blazers, shirts, and ties.
(A) 27 (B) 300 (C) 240 (D) 600

2. How many different words can be formed by using all the letters of the word INSTITUTE?
(A) $\frac{9!}{2!}$ (B) 9 ! (C) $\frac{9!}{3!}$ (D) $\frac{9!}{3!2!}$

3. In how many ways can a cricket team of 11 members be selected from 15 players, so that a particular player is included and another particular player is left out?
(A) 216 (B) 826 (C) 286 (D) 386

4. A group contains n persons. If the number of ways of selecting 6 persons is equal to the number of ways of selecting 9 persons, then the number of ways of selecting four persons from the group is
(A) 1365 (B) 273 (C) 455 (D) 285

5. The number of ways of arranging 10 books on a shelf such that two particular books are always together is
(A) 9! 2! (B) 9! (C) 10! (D) 8

6. Find the number of ways of inviting at least one among 6 people to a party.
(A) 2^6 (B) $2^6 - 1$ (C) 6^2 (D) $6^2 - 1$

7. An eight-letter word is formed by using all the letters of the word 'EQUATION'. How many of these words begin with a consonant and end with a vowel?
(A) 3600 (B) 10800
(C) 2160 (D) 720

8. A committee of 5 members is to be formed from a group of 6 men and 4 women. In how many ways can the committee be formed such that it contains more men than women?
(A) 180 (B) 186 (C) 126 (D) 66

9. In how many ways can 10 boys and 10 girls be arranged in a row so that all the girls sit together?
(A) 10! (B) 11!
(C) 20! (D) 10! 11!

10. In how many ways can 6 boys and 5 girls be arranged in a row so that boys and girls sit alternately?
(A) $(6!)^2$ (B) $(5!)^2$
(C) 6! 5! (D) 2.5! 6!

11. There are seven letters and corresponding seven addressed envelopes. All the letters are placed randomly into the envelopes—one in each envelope. In how many ways can exactly two letters be placed into their corresponding envelopes?
(A) 44 (B) 924 (C) 308 (D) 189

12. We are given 3 different green dyes, 4 different red dyes, and 2 different yellow dyes. The number of ways in which the dyes can be chosen so that at least one green dye and one yellow dye is selected is
(A) 336 (B) 335 (C) 60 (D) 59

13. Prahaas attempts a question paper that has 3 sections with 6 questions in each section. If Prahaas has to attempt any 8 questions, choosing at least two questions from each section, then in how many ways can he attempt the paper?
(A) 18000 (B) 10125
(C) 28125 (D) 9375

14. Find the number of selections that can be made by taking 4 letters from the word INKLING.
(A) 48 (B) 38 (C) 28 (D) 18

15. A man has $(2n + 1)$ friends. The number of ways in which he can invite at least $n + 1$ friends for a dinner is 4096. Find the number of friends of the man.
(A) 11 (B) 15 (C) 17 (D) 13

16. How many four-digit numbers are there between 3200 and 7300, in which 6, 8, and 9 together or separately do not appear?
(A) 1421 (B) 1420
(C) 1422 (D) 1077

17. Raju has forgotten his six-digit ID number. He remembers the following: the first two digits are either 1, 5 or 2, 6, the number is even and 6 appears twice. If Raju uses a trial and error process to find his ID number at the most, how many trials does he need to succeed?
(A) 972 (B) 2052
(C) 729 (D) 2051

18. A matrix with four rows and three columns is to be formed with entries 0, 1, or 2. How many such distinct matrices are possible?
(A) 12 (B) 36 (C) 3^{12} (D) 2^{12}

19. In how many ways can 4 postcards be dropped into 8 letter boxes?
(A) ${}^8P_4$ (B) 4^8 (C) 8^4 (D) 24

20. In how many ways can 12 distinct pens be divided equally among 3 children?
(A) $\frac{12!}{(3!)^4}$ (B) $\frac{12!}{(4!)^3\,3!}$
(C) $\frac{12!}{3!4!}$ (D) $\frac{12!}{(4!)^3}$

21. If all possible five-digit numbers that can be formed using the digits 4, 3, 8, 6, and 9 without repetition are arranged in the ascending order, then the position of the number 89634 is
(A) 91 (B) 93
(C) 95 (D) 98

22. Manavseva, a voluntary organization, has 50 members who plan to visit 3 slums in an area. They decide to divide themselves into 3 groups of 25, 15, and 10. In how many ways can the group division be made?

(A) 25! 15! 10! (B) $\frac{50!}{25!\,15!\,10!}$

(C) 50! (D) 25! + 15! + 10!

23. In how many ways is it possible to choose two white squares so that they lie in the same row or same column on an 8×8 chessboard?

(A) 12 (B) 48 (C) 96 (D) 60

24. The number of four digit telephone numbers that have at least one of their digits repeated is

(A) 9000 (B) 4464

(C) 4000 (D) 3986

25. There are 4 identical oranges, 3 identical mangoes, and 2 identical apples in the basket. The number of ways in which we can select one or more fruits from the basket is

(A) 60 (B) 59

(C) 57 (D) 55

Practice Problems 2

Directions for questions 1 to 25: Select the correct alternative from the given choices.

1. A five lettered word is formed using some of the letters $\{a, b, h, i, p, r, s\}$. How many of them will be palindromes?

(A) 125 (B) 225 (C) 343 (D) 729

2. Find the number of four-digit numbers that can be formed using the digits 1, 2, 3, 4, 5, 6 when each digit can occur any number of times in each number.

(A) 4^6 (B) ${}^6P_4$ (C) ${}^6P_6$ (D) 6^4

3. Find the number of even numbers formed using all the digits 1, 2, 3, 4, 5 when each digit occurs only once in each number.

(A) ${}^5P_4$ (B) 4! 2 (C) 5! (D) 4!

4. In how many ways can ten students be seated around a circular table so that three students always sit together?

(A) 7! (B) 7! 3! (C) 2.7! (D) 3 (7!)

5. In how many ways can four letters be posted into 5 post boxes?

(A) 350 (B) 125 (C) 625 (D) 1024

6. In how many ways can 7 boys and 6 girls be arranged in a row so that no two girls sit together?

(A) 13! (B) ${}^8P_6 \times 7!$

(C) $6!{}^8P_7$ (D) 12!

7. A man has 7 friends. In how many ways can he invite at least one of his friends for a dinner?

(A) 63 (B) 120 (C) 127 (D) 256

8. The number of squares that can be formed on a 8×8 chessboard is

(A) 204 (B) 220 (C) 240 (D) 210

9. The number of positive integral solutions to the equation $x + y + z = 20$ is

(A) 131 (B) 110 (C) 55 (D) 171

10. In how many ways can the letters of the word 'SUBJECT' be placed in the squares of the figure given below so that no row remains empty?

(A) $5 \times 6!$ (B) $10 \times 6!$

(C) $11 \times 5!$ (D) $13 \times 8!$

11. In how many ways can the letters of the word EUROPE be arranged so that no two vowels are together?

(A) 12 (B) 24

(C) 360 (D) Not possible

12. A four-digit number is formed using the digits 0, 2, 4, 6, 8 without repeating any one of them. What is the sum of all such possible numbers?

(A) 519960 (B) 402096

(C) 133320 (D) 4321302

13. How many four-digit odd numbers can be formed such that every 3 in the number is followed by a 6?

(A) 108 (B) 2592

(C) 2696 (D) 2700

14. How many times does the digit 5 appear in the numbers from 9 to 1000?

(A) 300 (B) 257 (C) 256 (D) 299

15. There are 5 bowls numbered 1 to 5, 5 green balls and 6 black balls. Each bowl is to be filled by either a green or black ball and no two adjacent bowls can be filled by green balls. If the same colour balls are indistinguishable, then the number of different possible arrangements is

(A) 8 (B) 7 (C) 13 (D) 256

16. How many 4-digit numbers can be formed such that the digit in the hundreds place is greater than that in the tens place?

(A) 9000 (B) 10000

(C) 4500 (D) 4050

17. In how many ways can 5 boys and 3 girls sit around a table in such a way that no two girls sit together?

(A) 480 (B) 960 (C) 320 (D) 1440

18. Find the maximum number of ways in which the letters of the word MATHEMATICS can be arranged so that all *M*'s are together and all *T*'s are together.

(A) 11! (B) $\frac{11!}{2!2!2!}$

(C) $\frac{9!}{2!2!2!}$ (D) $\frac{9!}{2!}$

19. Consider the word INSTITUTE.

In how many ways can 5 letters be selected from the word?

(A) 41 (B) 30 (C) 36 (D) 40

20. The letters of the word AGAIN are permuted in all possible ways and are arranged in dictionary order. What is the 28^{th} word?

(A) GAIAN (B) GAINA
(C) GANIA (D) NGAIA

21. In how many ways can 12 distinct pens be divided equally in to 3 parcels?

(A) $\frac{12!}{(4!)^4}$ (B) $\frac{12!}{(4!)^3}$

(C) $\frac{12!}{3!4!}$ (D) $\frac{12!}{(4!)^3\ 3!}$

22. In a certain question paper, a candidate is required to answer 5 out of 8 questions, which are divided into two parts containing 4 questions each. He is permitted to attempt not more than 3 from any group. The number of ways in which he can answer the paper is

(A) 24 (B) 96 (C) 48 (D) 84

23. The sides PQ, QR, and RS of ΔPQR have 4, 5, and 6 points (not the end points), respectively, on them. The number of triangles that can be constructed using these points as vertices is

(A) 455 (B) 34
(C) 425 (D) 421

24. There are 8 different books and 2 identical copies of each in a library. The number of ways in which one or more books can be selected is

(A) 2^8 (B) $3^8 - 1$
(C) $2^8 - 1$ (D) 3^8

25. The number of sequences in which 7 players can throw a ball, so that the youngest player may not be the last is

(A) 4000 (B) 2160
(C) 4320 (D) 5300

HINTS/SOLUTIONS

Practice Problems I

1. The total number of ways that 12 blazers, 10 shirts and 5 ties can be worn is 12(10)(5) = 600 (by fundamental rule).

Hence, the correct option is (D).

2. The given word INSTITUTE contains a total of 9 occurrences – 3*T*'s, 2*I*'s and the rest four are distinct.

The total number of different words that can be formed, is $\frac{9!}{3!2!}$

Hence, the correct option is (D).

3. Since one player is in the team and one player is not in the team, we have to select 10 players from 13. This can be done in ${}^{13}C_{10}$ or ${}^{13}C_{3}$ or 286 ways.

Hence, the correct option is (C).

4. The number of ways of selecting 6 students from *n* students is ${}^{n}C_{6}$. The number of ways of selecting 9 students from *n* students is ${}^{n}C_{9}$.

Given ${}^{n}C_{6} = {}^{n}C_{9} \Rightarrow n = 15$

The number of ways of selecting 4 students from 15 students is ${}^{15}C_{4} = \frac{15(14)(13)(12)}{4(3)(2)(1)} = 1365$

Hence, the correct option is (A).

5. When two books are to be together, we assume those two books as 1 unit. With this one unit, there are 9 books which can be arranged in 9! ways. But these two books can be arranged internally in 2! ways

∴ Required number of arrangements = 9! 2!

Hence, the correct option is (A).

6. Number of ways of inviting at least one of *n* people is given by $2^n - 1$ Here $n = 6$

∴ Required number of ways $2^6 - 1$

Hence, the correct option is (B).

7. The word EQUATION contains 5 vowels and 3 consonants.

Since the words begin with a consonant, the first place can be filled in 3 ways and last place can be filled in 5 ways. The remaining 6 places can be filled with the remaining 6 letters in 6! ways.

∴ The total number of words that can be formed is 3(5)(6!) = 10800.

Hence, the correct option is (B).

8. The possible number of men and women and the corresponding number of ways in which the committee can be selected are tabulated below.

Men 6	Women 4	Number of selections
3	2	${}^{6}C_{3}{}^{4}C_{2}$
4	1	${}^{6}C_{4}{}^{4}C_{1}$
5	0	${}^{6}C_{5}{}^{4}C_{0}$

∴ total number of ways that the committee formed is

$${}^{6}C_{3}.\,{}^{4}C_{2} + {}^{6}C_{4}.\,{}^{4}C_{1} + {}^{6}C_{5}.\,{}^{4}C_{0}$$
$$= 20(6) + 15(4) + 6(1) = 120 + 60 + 6 = 186$$

Hence, the correct option is (B).

9. We treat 10 girls as 1 unit.

Then total number of students = 11

∴ 11 students can be arranged in *a* row in 11! ways

Again the 10 girls can be arranged among themselves in 10! ways.

∴ required number of ways = 10! 11!

Hence, the correct option is (D).

10. Since the number of boys is greater than the number of girls, the first and the last place has to be occupied by boys. 6 boys can be arranged in a row in 6! ways. There are five gaps in between them. The 5 girls can be arranged in these gaps in 5! ways.

∴ Total number of arrangements = 6! 5!.

Hence, the correct option is (C).

11. The number of derangements of *n* objects is

$$D_n = n!\left[\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} + \ldots (-1)^n \frac{1}{n!}\right]$$

Out of seven letters any two letters are placed into its corresponding envelopes and remaining 5 letters, no letter is placed into its corresponding envelope.

i.e. ${}^{7}C_{2}(D_5)$

$$= 21\ (5!)\left(\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!}\right)$$

$= 21 \times (44) = 924$

Hence, the correct option is (B).

12. Number of ways of choosing at least one green dye is $2^3 - 1$.

Number of ways of choosing at least one yellow dye is $2^2 - 1$.

Number of ways of choosing a red dye is 2^4.

∴ Required number of ways = $\left(2^3 - 1\right)\left(2^2 - 1\right)\left(2^4\right)$

= (7) (3) (16) = 48 (7) = 336

Hence, the correct option is (A).

13. Prahaas can select the questions in the following combinations

(i) 3, 3, 2 (ii) 3, 2, 3

(iii) 2, 3, 3 (iv) 4, 2, 2

(v) 2, 4, 2 (vi) 2, 2, 4

The number of selections in each of the cases (i), (ii) and (iii) is ${}^6C_3 \times {}^6C_3 \times {}^6C_2$ while

The number of selections in each of the cases (iv), (v), (vi) is ${}^6C_4 \times {}^6C_2 \times {}^6C_2$

Hence total selections are

$3 \times {}^6C_3 \times {}^6C_3 \times {}^6C_2 + 3 \times {}^6C_4 \times {}^6C_2 \times {}^6C_2$

$= 3\,[6000 + 3375] = 28125$

Hence, the correct option is (C).

14. The word has 7 letters I, I, N, N, K, L, G. The following are the possibilities while selecting 4 letters.

Case (i): All 4 are distinct.

Case (ii): Two are alike and two are distinct.

Case (iii): Two are alike of one kind and two alike of the other.

Now the number of selections and arrangements in each case is given below.

	Combinations	**Permutations**
Case (i)	${}^5C_4 = 5$	$5 \times 4! = 120$
Case (ii)	${}^2C_1 \times {}^4C_2 = 12$	$12 \times \frac{4!}{2!} = 144$
Case (iii)	${}^2C_2 = 1$	$1 \times \frac{4!}{2!2!} = 6$

Hence total Combinations are 18.

Hence, the correct option is (D).

Solutions for questions 15 to 25:

15. The number of ways that the man can invite at least $n + 1$ friends for a dinner is

${}^{(2n+1)}C_{n+1} + \ldots + {}^{2n+1}C_{2n+1} = 4096$ (1)

$\therefore\ {}^{2n+1}C_0 + {}^{2n+1}C_1 + \ldots + {}^{2n+1}C_n = 4096$ (2)

(1) + (2) $\Rightarrow$

${}^{2n+1}C_0 + {}^{2n+1}C_1 + \ldots + {}^{2n+1}C_{2n-1} = 8192$

$\Rightarrow 2^{2n+1} = 8192$ ($\because\ {}^nC_0 + {}^nC_1 + \ldots + {}^nC_n = 2^n$)

$\Rightarrow 2^{2n+1} = 2^{13}$

$\Rightarrow 2n + 1 = 13$

$\therefore$ the number of friends = 13

Hence, the correct option is (D).

16. The digits in the thousands place and the possible number of digits in the hundreds, tens and units places (enclosed in brackets) are tabulated below.

Th	H	T	U	
3	(5)	(7)	(7)	245
4	(7)	(7)	(7)	343
5	(7)	(7)	(7)	343
7	(3)	(7)	(7)	147
				1078

These 1078 numbers include 3200, but not 7300.

$\therefore$ The number of numbers between 3200 and 7300 is 1077.

Hence, the correct option is (D).

17. The six-digit number may start with 15 or 26 and also it is an even number. We can have the following possible cases.

Case 1:

1 5 – – – 6

If the last digit is 6, then in the remaining 3 places, one place can be filled by 6 and the other two places can be filled in 9(9) ways.

$\therefore$ Hence, the number of trials = 9(9) (3) = 243

Case 2:

1 5 – – – –

If last position is filled by one of the digits 0, 2, 4, 8, then in the remaining 3 places, two of the place can be filled by 6 and third place can be filled by 9 ways.

$\therefore$ Hence, required number of trials = 4(9) (3) = 108

Case 3:

2 6 – – – 6

If the last position is filled by 6, then the remaining 3 positions can be filled in 9(9) (9) ways.

$\therefore$ Hence, required number of trials = $9 \times 9 \times 9 = 729$

Case 4:

0, 2, 4, 6, 8

2 6 6 $\downarrow^9$ $\downarrow$9 $\downarrow$

If the last position is filled with 0, 2, 4, 8, then in remaining 3 positions one position has to be filled by 6 and the other two positions can be filled in 9(9) ways.

$\therefore$ Hence, the required number of trials = 9(9) (3) (4) = 972

$\therefore$ At the most, Raju has to make 243 + 108 + 729 + 972 or 2052 trials to succeed.

Hence, the correct option is (B).

18. A 4×3 matrix has 12 elements. Each element can be 0 or 1 or 2.

The total number of matrices 0, 1, 2 as the elements is 3^{12}

Hence, the correct option is (C).

19. One postcard can be dropped into 8 letter boxes, in 8 ways, 4 postcards can be dropped in 8^4 ways.

Hence, the correct option is (C).

20. 12 pens can be distributed among 3 children and each one gets 4 pens.

$\therefore$ Hence, required number. of ways $= {}^{12}C_4 \times {}^{8}C_4 \times {}^{4}C_4$

$$= \frac{12!}{8!4!} \times \frac{8!}{4!4!} = \frac{12!}{(4!)^3}$$

Hence, the correct option is (D).

21. The initial digits and the number of numbers are tabulated below.

Initial Digits	Number of numbers
3	24
4	24
6	24
8	24

The 96^{th} number is 89643

The 95^{th} number is 89634

Hence, the correct option is (C).

22. We need to divide a group of 50 into groups of 25, 15 and 10 and this can be done in $\frac{50!}{25!15!10!}$.

Hence, the correct option is (B).

Note: A group of $(p + q + r)$ items can be divided into groups of p, q, r items in

$$\frac{(p+q+r)!}{p!q!r!}$$

23. In an 8×8 chess board there are 8 rows and 8 columns. In every row and column there are four white squares each. Number of ways of selecting two white squares which are in same row or column $= 8 \times {}^{4}C_2 + 8 \times {}^{4}C_2 = 8\,[6 + 6] = 96$

Hence, the correct option is (C).

24. $\therefore$ Required number of ways = (total 4 digited telephone numbers) – (The number of 4 digited numbers without repetition)

$= (9)\,(10^3) - 9\,({}^{9}P_3)$

$= 9\,(1000 - (9)\,(8)\,(7))$

$= 9\,(1000 - 504)$

$= 9\,(496) = 4464$

Hence, the correct option is (B).

25. We know that, if p things are alike of one kind, q things are alike of second kind and r things are alike of third kind, then one or more things can be selected in $(p + 1)\,(q + 1)\,(r + 1) - 1$ ways.

$\therefore$ Required number of ways $= (4 + 1)\,(3 + 1)\,(2 + 1) - 1$

$= (5)\,(4)\,(3) - 1 = 59$

Hence, the correct option is (B).

Practice Problems 2

1. A palindrome is a word which when read from left to right or right to left, remains the same.

In a palindrome, only in the first half the letters are different. The same letters that appear in the first half are repeated in the second. i.e. in a five letter palindrome word, first three letters will be different.

First place can be filled in 7 ways,

Similarly second and 3^{rd} places can be filled in 7 ways

$\therefore$ required number of palindrome words formed is

$= 7 \times 7 \times 7 = 343$

Hence, the correct option is (C).

2. We have to fill four blanks. Each blank can be filled by any of the six digits.

6	6	6	6

$\therefore$ Required number of ways $= 6^4$

Hence, the correct option is (D).

3. A number is even if its units place is divisible by 2

$\therefore$ units place can be filled by either 2 or 4 i.e. in 2 ways.

The remaining 4 places can be filled by the remaining 4 digits in 4! ways

Total number of even numbers that can be formed

$= 2\,(4!) = 48$

Hence, the correct option is (B).

4. Assume that the three students have to sit together as one unit. Now there are 8 units (7 students + 1 unit of three students) and they can be arranged at a circular table in 7! ways. Again the three students can be arranged among themselves in 3! ways.

$\therefore$ The total number of ways = 7! 3!.

Hence, the correct option is (B).

5. Each letter can be posted in 5 ways.

Four letters can be posted in 5(5)(5)(5)(5) or 625 ways.

Hence, the correct option is (C).

6. First arrange the 7 boys.

$\times B_1 \times B_2 \times B_3 \times B_4 \times B_5 \times B_6 \times B_7 \times$

Now there are 8 gaps between boys, marked by $\times$.

7 boys can be arranged in 7 places in 7! ways

The 6 girls can be arranged in these gaps in ${}^{8}P_6$ ways

$\therefore$ Required number of ways $= {}^{8}P_6 \times 7!$

Hence, the correct option is (B).

7. The number of ways (or combinations) of selecting atleast one of n different things is

${}^nC_1 + {}^nC_2 + \ldots + {}^nC_n = 2^n - 1$

$\therefore$ The number of ways that the man can invite at least one of his friends for dinner

$= 2^7 - 1 = 128 - 1 = 127$

Hence, the correct option is (C).

8. Number of squares (S) that can be formed in an $n \times n$ chess board

$$= \sum_1^n i^2 = \frac{n(n+1)\,(2n+1)}{6}$$

As $n = 8$, $S = \dfrac{8(9)\,(17)}{6} = 204$

Hence, the correct option is (A).

9. We know that,

$\therefore$ The number of positive integral solutions of

$x_1\, x_2 + \ldots + x_r = n$ is ${}^{n-1}C_{r-1}$

$\therefore$ Required answer is ${}^{19}C_2 = \dfrac{19\times18}{2} = 171$

Hence, the correct option is (D).

10. In all 10 squares in which 7 letters to be arranged and it can be done is ${}^{10}P_7$.

Top row vacant, this can be done in ${}^8P_7 = 8!$ ways.

Middle row vacant, this can be done in ${}^8P_7 = 8!$ ways.

$\therefore$ Required arrangements $= {}^{10}P_7 - 8! \times 2$

$$= \frac{10!}{6} - 8!\,2! = 8!\left(\frac{90}{6} - 2\right) = 13 \times 8!$$

Hence, the correct option is (D).

11. In the given word, there are 4 vowels and 2 consonants.

$\therefore$ It is not possible to arrange the letters as required.

Hence, the correct option is (D).

12. The first digit (thousands) can be selected in 4 ways.

The other 3 places can be filled in ${}^4P_3$ ways.

We can form a total of 4 ${}^4P_3$ four-digited numbers by using all the even digits. We have to add all these numbers. Let us look at the contribution of each of the digit.

3 (3!) numbers contain 2 in the units place

3 (3!) numbers contain 2 in the tens place

3 (3!) numbers contain 2 in the hundreds place

Similarly, we can work out the value contributed by the other digits. The digits and the total contribution of the digits is tabulated below.

Digit	Contribution
2	24 (2000) + 18 (222)
4	24 (4000) + 18 (444)
6	24 (6000) + 18 (666)
8	24 (8000) + 18 (888)

The total sum is 24 (20000) + 18 (20) (111)

$= 480000 + 39960 = 519960$

Hence, the correct option is (A).

13. The possible digits in the different places and the corresponding number of numbers are tabulated below. The units digit or the tens digit cannot be 3.

	Possible Digits				No. of Number	
A.	3	6	__	odd	9 (4) =	36
B.	3	__	6	odd	10 (4) =	40
C.	__	3	6	odd	9 (4) =	36
D	__	__	__	odd	8 (9) (9) (4) =	2592
						2704

The numbers 3361, 3365, 3367 and 3369 have been counted in B as well as C.

The numbers 3661, 3665, 3667, 3669 has been counted in A as well as B.

$\therefore$ The required number of numbers is 2696.

Hence, the correct option is (C).

14. Case 1:

The number of 2-digit and 3-digit numbers having only one 5 is 3 (9) (9) = 243

Case 2:

$\underline{5}\ \underline{5}\ \overset{\uparrow}{}_9$

The number of 2-digit and 3-digit numbers having exactly two 5's is 3 (9) = 27

Case 3:

$\underline{5}\ \underline{5}\ \underline{5}$

The number of numbers having exactly three 5*s* is 1.

$\therefore$ The total number of times 5 occurs, in all possible natural numbers less than 1000 is 243 + 2 (27) + 3(1) = 300

The number of times 5 occurs in between 9 to 1000 is

$300 - 1 = 299$

Hence, the correct option is (D).

15. The number of black and green balls and the number of ways they can be arranged in the 5 bowls, so that no two adjacent bowls have green balls are tabulated below.

b (6)	*g* (5)	No. of arrangements	Position of g balls
5	0	1	
4	1	5	1, 2, 3, 4, 5
3	2	6	1, 3 or 1,4 or 1,5 or 2,4 or 2, 5 or 3, 5
2	3	1	1, 3, 5

$\therefore$ The total number of arrangements is 13.

Hence, the correct option is (C).

16. $\underset{\downarrow}{}^{9}\ \_\ \_\ \underset{\downarrow}{}^{10}$

As there is no restriction on the units place, this place can be filled by any of the 10 digits. The, thousands place can be filled by any of the 9 digits. (all except 0)

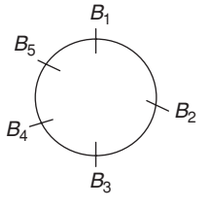
To fill the other two places, we have to select two distinct digits. This can be done in ${}^{10}C_2$ ways.

Required number of ways = ${}^{10}C_2$ (10) (9) = 4050

Hence, the correct option is (D).

17. The 5 boys can be seated around a table in 4! ways. In between them, there are 5 places.

The 3 girls can be placed in the 5 places in ${}^5P_3$ ways.



$\therefore$ Required number of ways = 4! ${}^5P_3 = 24 \times 60$

$= 1440$

Hence, the correct option is (D).

18. In the given word, there are 2 *M*'s, 2 *T*'s, 2 *A*'s and 5 single letters.

Taking the 2 *M*'s as one unit and 2 *T*'s as one unit, with remaining 7 letters can be arranged $\frac{9!}{2!}$ ways. (There are 2 *A*'s)

$\therefore$ Required number. of ways $= \frac{9!}{2!}$

Hence, the correct option is (D).

19. The word is INSTITUTE

Letter	N	S	U	E	I	T
Number of times repeated	1	1	1	1	2	3

The distribution of the 5 letters, combination and permutations are tabulated below.

Distribution	Combinations	Permutations
1, 1, 1, 1, 1	${}^6C_5 = 6$	6 (5!) = 720
1, 1, 1, 2	2 (${}^5C_3$) = 20	$20\left(\frac{5!}{2!}\right) = 1200$
1, 1, 3	${}^5C_2 = 10$	$10\left(\frac{5!}{3!}\right) = 200$
1, 2, 2	${}^4C_1 = 4$	$4\left(\frac{5!}{2!2!}\right) = 120$
2, 3	1	$\frac{5!}{2!3!} = 10$
	41	2250

5 letters can be selected in 41 ways.

Hence, the correct option is (A).

20. Arranging the letters of the word 'AGAIN' in dictionary order is A, A, G, I, N.

The letters and the number of words are tabulated below.

Initial Letters	Number. of words
A	24
GAA	2
GAIAN	1
GAINA	1

$\therefore$ The 28th word GAINA

Hence, the correct option is (B).

21. 12 pens can be distributed into 3 parcels in

$$\frac{12!}{(4!)^3\,3!} \text{ ways}$$

Hence, the correct option is (D).

22.

Section (1) (4 Qns)	Section (2) 4 (Qns)	Required combinations
2	3	${}^4C_2\ {}^4C_3$
3	2	${}^4C_3\ {}^4C_2$

Required number of ways = 2 (${}^4C_3$) (${}^4C_2$)

$= 2\ (4)\ (6) = 48$

Hence, the correct option is (C).

23. We know that,

The number of triangles formed with '*n*' non collinear points is ${}^nC_3$.

$\therefore$ Here, number of triangles = ${}^{15}C_3 - {}^4C_3 - {}^5C_3 - {}^6C_3$

$= 455 - 4 - 10 - 20$

$= 455 - 34 = 421$

Hence, the correct option is (D).

24. For each book, 0 or 1 or 2 copies can be selected. Hence, the required number of ways = $3^8 - 1$

(At least 1 book has to be selected)

Hence, the correct option is (B).

25. Not younger player

$\_\ \_\ \_\ \_\ \_\ \_\ \ \underset{\uparrow}{}$

The last ball can be thrown by any of the remaining 6 players. The first 6 players can throw the ball in ${}^6P_6$ ways.

$\therefore$ The required number of ways = 6 (6!) = 4320

Hence, the correct option is (C).

Chapter 14 Data Interpretation

Exercises

Practice Problems I

Directions for question 1: Select the correct alternative from the given choices.

1. The table shows the total marks of four students P, Q, R, and S in all their subjects for the two years 2012 and 2013.

Students	2012	2013
P	997	1295
Q	664	876
R	585	732
S	480	689

How many students had a percentage Increase in their total marks of more than 35% from 2012 to 2013?

(A) 1 (B) 2
(C) 3 (D) 4

Directions for questions 2 to 4: These questions are based on the following data which give some details of new states joining the United States of America across time.

State	Capital	Joined the union	Union rank	Population	Number of representatives in the house of representatives
Washington	Olympia	Nov 11, 1889	42	62,87,759	9
Texas	Austin	Dec 29, 1845	28	2,28,59,968	32
Delaware	Dover	Dec 7, 1781	1	8,43,524	1
Virginia	Raleigh	Nov 21, 1789	12	86,83,242	13
Minnesota	St. Paul	May 11, 1832	32	51,32,799	8
Kansas	Topeka	Jan 29, 1861	34	27,44,687	4
Illinois	Springfield	Dec 3, 1818	21	1,27,63,371	19
New Hampshire	Concord	June 21, 1788	9	13,09,940	2
Arizona	Phoenix	Feb 14, 1912	48	59,39,292	8
Hawai	Honolulu	Aug 21, 1959	50	12,75,194	2
Indiana	Indianapolis	Dec 11, 1816	19	62,71,973	9
Vermont	Montpelier	March 14, 1791	14	6,23,050	1
Nebraska	Lincoln	March 1, 1867	37	17,58,787	3
Georgia	Atlanta	Jan 2, 1788	4	9,07,256	13

Union rank is the chronological order in which the states joined the Union.

2. How many states joined the Union from March 1, 1867, to Feb 14, 1912?

(A) 11 (B) 12
(C) 13 (D) 14

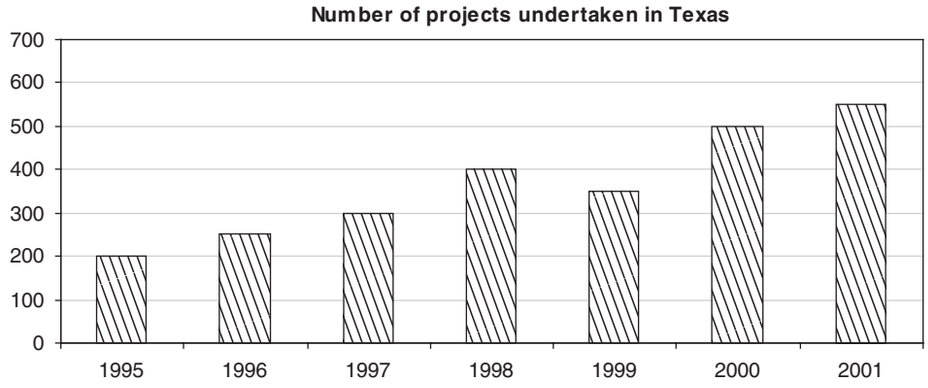
3. If it is known that the House of Representatives of USA has a strength of 535 members, then the number of representatives in the House of Representatives of the given states will form what approximate percentage of the total strength of the House of Representatives?

(A) 16 (B) 19
(C) 21 (D) 23

4. In how many of the given states is the population less than 15 million but the number of representatives is not less than six?

(A) 4 (B) 5
(C) 6 (D) 7

Directions questions 5 to 7: These questions are based on the following graphs.



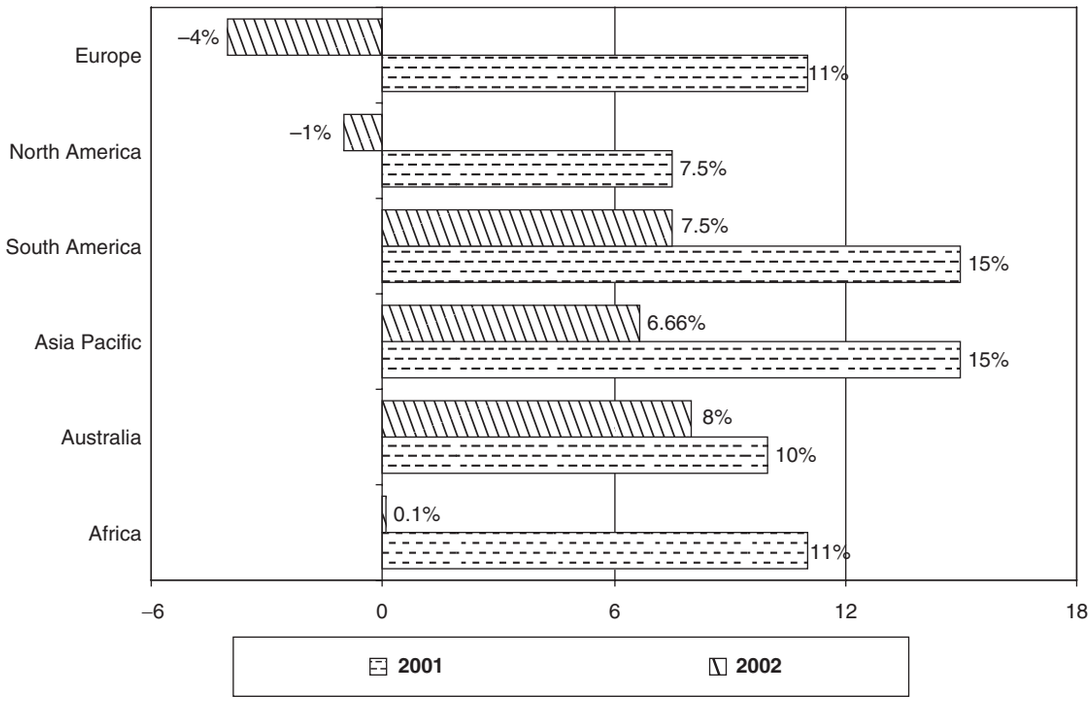
5. In how many years was the number of projects undertaken in Mexico greater than that in Texas?
(A) 3 (B) 6
(C) 5 (D) 4

6. How many projects were undertaken in the year 1998 in both places together?
(A) 1000 (B) 900
(C) 750 (D) 500

7. In which of the following years was the average (arithmetic mean) number of projects undertaken in both places the highest?
(A) 2000 (B) 1996 (C) 1998 (D) 1999

Directions for questions 8 to 10: These questions are based on the bar graph.

The graph given below shows the year over year (YOY) percent change in the air traffic in the different regions of the world. Air traffic is measured in revenue passenger kilometers. The number of revenue passenger kilometers of an aircraft is calculated by multiplying the number of revenue paying passengers aboard the aircraft by the distance travelled.

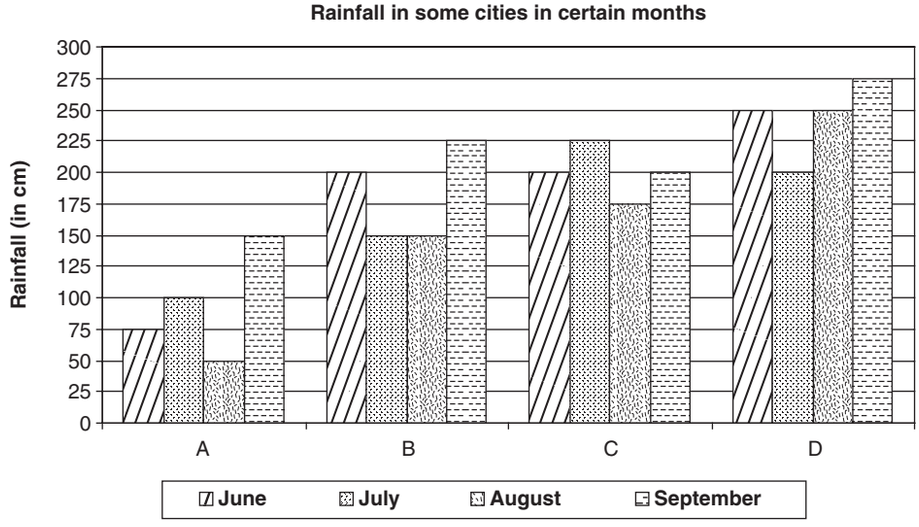


8. The air traffic in the year 2002 is the same for all the six regions of the world mentioned above. Which among the following regions had the highest air traffic (in revenue passenger kilometers) in the year 2000?
(A) Africa
(B) North America
(C) South America
(D) Asia Pacific

9. Which of the six regions has the highest growth in air traffic (in revenue passenger kilometers) from 2000 to 2002?
(A) Australia
(B) South America
(C) Europe
(D) Cannot be determined

10. If the air traffic in South America in the year 2000 was 20 million revenue passenger kilometers, what was the air traffic in South America in the year 2002?
(A) 247.25×10^7
(B) 24725×10^4
(C) 247.25×10^5
(D) 24725×10^2

Directions for questions 11 to 13: These questions are based on the graph given below.



11. When compared to the previous month, the percentage increase in rainfall is the highest in
(A) City A in September
(B) City B in September
(C) City C in September
(D) City D in August

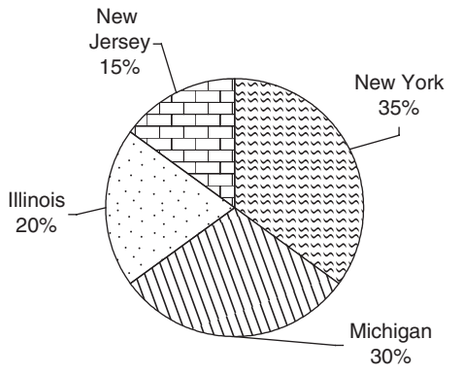
12. What is the average rainfall in the month of August in the four cities?
(A) 135.25 cm (B) 146.25 cm
(C) 156.25 cm (D) 168.75 cm

13. The average rainfall in cities *B* and *C* in August and September is what percentage less/more than the average rainfall in city *A* during July, August, and September?
(A) 87.5% less (B) 75% less
(C) 75% more (D) 87.5% more

Directions for questions 14 to 16: The table below shows the percentages of colleges offering the courses mentioned in medicine in four states, New York, New Jersey, Illinois, and Michigan, in a certain year.

Sl. No.	Course	New York	New Jersey	Illinois	Michigan
1.	Biochemistry	86	80	74	68
2.	Biophysics	74	92	88	64
3.	Biomechanics	59	82	84	68
4.	Biostatistics	56	84	86	70

The total number of colleges offering courses in medicine in the four states is 2000. The percentage-wise distribution of the number of colleges in the four states is as shown below.

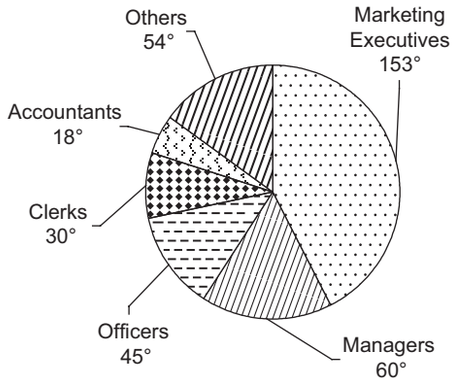


14. The number of colleges offering the Biochemistry course is more than 230 in
(A) All the four states.
(B) Exactly three states.
(C) Exactly two states.
(D) Exactly one state.

15. What percentage of the colleges in the four states do not offer Biophysics as well as Biochemistry?
(A) 41%
(B) 36%
(C) 34%
(D) Cannot the determined

16. What is the total number of colleges offering Biostatistics in all the four states?
(A) 1392 (B) 1408 (C) 1432 (D) 1476

Directions for questions 17 to 19: The pie chart given below indicates the software training imparted to 1440 employees of various categories in an organization. Study this chart and answer these questions.



17. In which of the following categories is twice the number of officers trained equal to thrice the number of employees imparted training?
(A) Accountants
(B) Marketing executives
(C) Clerks
(D) Managers

18. What percentage of the total employees trained are accountants?
(A) 15% (B) 18% (C) 5% (D) 10%

19. Among the number of employees trained, what is the ratio of the number of officers to the number of marketing executives?
(A) 5 : 17 (B) 5 : 16
(C) 15 : 23 (D) 15 : 31

Directions for questions 20 and 21: Study the given table and answer the questions that follow.

Percentage break-up of the workers working in six different factories *J, K, L, M, N*, and *P*.

Factory	Total no. of workers	Percentage		
		Men	Women	Boys
J	4800	50	37.5	12.5
K	8750	40	36	24
L	5250	24	56	20
M	12000	35	25	40
N	8500	38	30	32
P	2700	45	40	15

20. By what percent is the number of women working in factory *P* is more than the number of boys working in factory *J*?

(A) 20% (B) $44\frac{4}{9}\%$

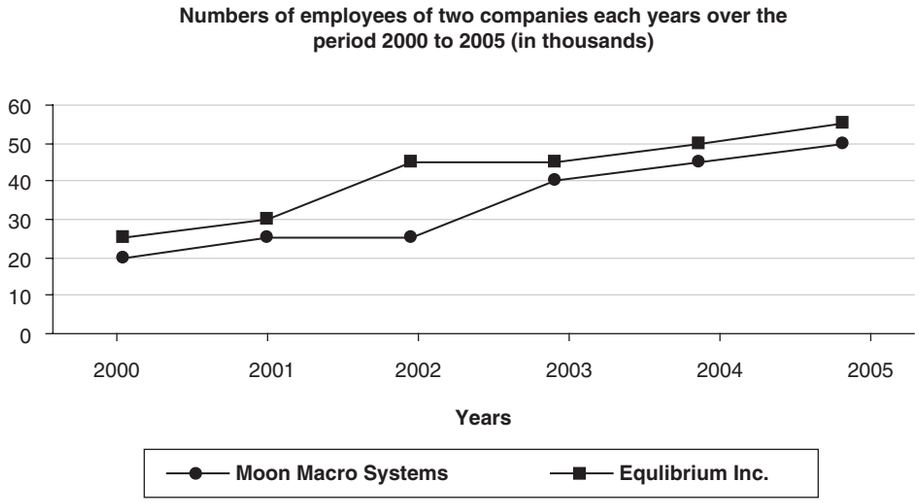
(C) 80% (D) 180%

21. What is the ratio of the number of men working in factory *M* to the number of women working in factory *L*?

(A) 7 : 10 (B) 10 : 7

(C) 7 : 5 (D) 5 : 7

Directions for questions 22 to 24: These questions are based on the following line graph.



22. What is the ratio of the average number of employees of company Moon Macro Systems for the period 2001, 2002 and 2003 and the average number of employees of company Equilibrium Inc. for the same period?

(A) 2 : 5 (B) 3 : 5 (C) 3 : 4 (D) 4 : 3

23. During which of the following years was the percentage increase in the number of employees of company Equilibrium Inc. over that in the previous year the highest?

(A) 2000 (B) 2001

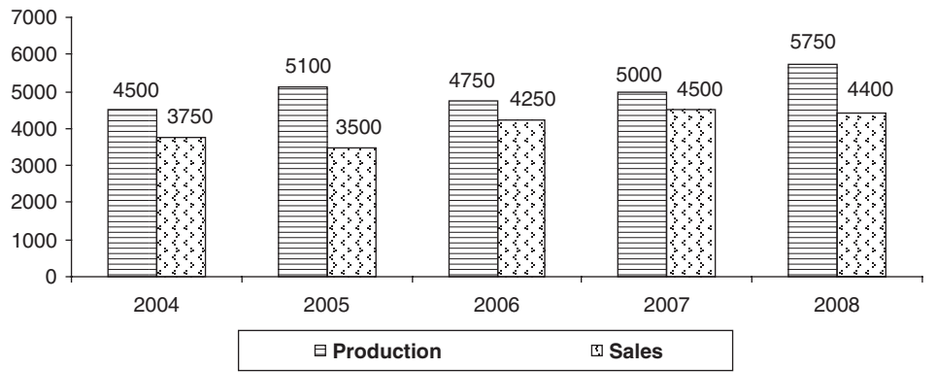
(C) 2002 (D) 2004

24. What is the approximate percentage increase in the total number of employees of the two companies from 2004 to 2005?

(A) 8.5% (B) 9%

(C) 9.5% (D) 10.5%

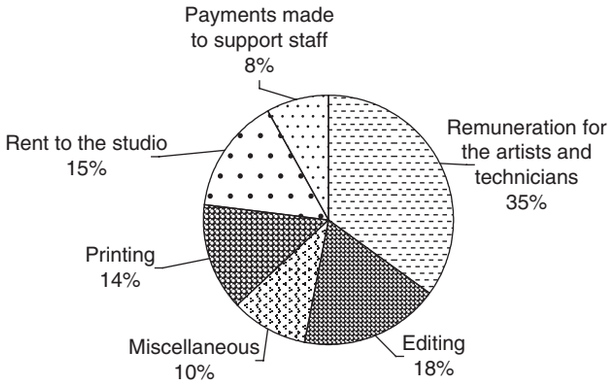
Directions for questions 25 and 26: These questions are based on the following bar graph that gives the production and sales of a company across five years from 2004 to 2008.



25. In the given period, what percentage was the average production more than the average sales?
(A) 20% (B) 23% (C) 25% (D) 28%

26. The percentage increase/decrease in the total sales of the company in a given year with respect to that in the previous year was highest in which of the following years?
(A) 2005 (B) 2006 (C) 2007 (D) 2008

Directions for questions 27 and 28: The following questions are based on the pie chart given that gives the projected total cost incurred in the production of a movie.

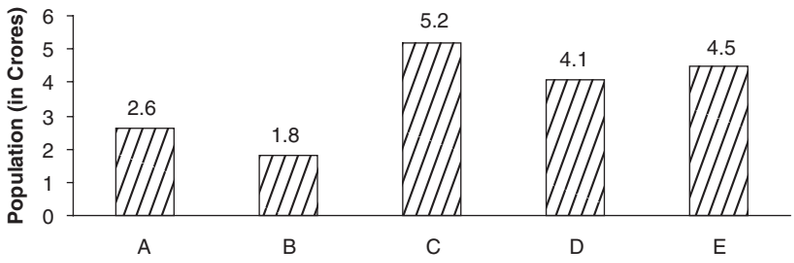


27. Approximately by what percentage is the total projected cost for printing less than that for editing?
(A) 15% (B) 22%
(C) 25% (D) 32%

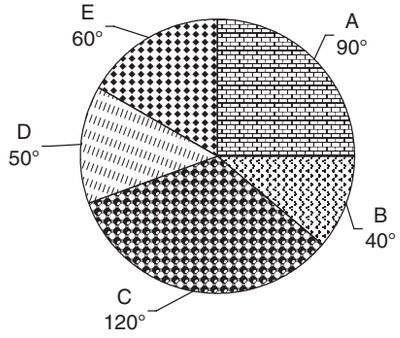
28. During the production of the movie, there was a 12.5% increase in the projected cost due to some unforeseen expenses which amounted to ₹7.2 lakhs. What was the projected remuneration to artists and technicians?
(A) ₹14,40,000 (B) ₹16,80,000
(C) ₹20,16,000 (D) ₹30,20,000

Directions for questions 29 to 32: Study the following data to answer these questions.

Population in five states in 2011



Share of female population in the given states in 2011 is shown in following pie chart.



Total female population = 8.1 crores

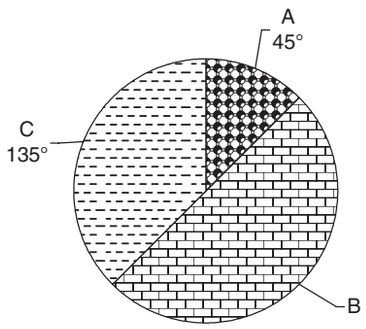
29. Which of the following states has the highest male population?
(A) E (B) C (C) A (D) D

30. How many states have more number of females than males?
(A) 3 (B) 4 (C) 1 (D) 2

31. What is the ratio of the total number of males in the states A and E together to that of the females in the same states?
(A) 3 : 1 (B) 13 : 7
(C) 149 : 135 (D) 44 : 27

32. What percentage does the number of females in the states C and D form of the number of males from the same states?
(A) 51.36% (B) 63.16%
(C) 69.86% (D) 75.86%

Directions for questions 33 to 35: These questions are based on the following pie chart and table. The pie chart shows the classification of number of companies according to male–female workers ratio in a Southern town of Hawaii.



Number of companies = 200

A The number of companies in which number of female workers is more than the number of male workers.

B The number of companies in which number of male workers is more than the number of female workers.

C The number of companies in which number of male workers is equal to the number of female workers.

The following table shows the classification of these 200 companies according to the number of working hours per day.

Number of working hours per day	*X*	*Y*	*Z*
4 hours	30	16	12
6 hours	45	24	15
8 hours	16	12	8
More than 8 hours	12	10	–

Here *X* shows the companies which work only during the day, *Y* shows the companies which work only during the night and *Z* shows the companies which work both day and night.

33. The maximum number of companies in any one of the four types of working hours and working only during the day is what percentage of the companies in which the number of female workers is less than the number of male workers?
(A) 45%
(B) 73%
(C) 87%
(D) 91%

34. The number of companies having equal number of male and female workers is what approximate percentage of the number of companies that work either only in the night or only in the day?
(A) 72.72%
(B) 63.63%
(C) 55.55%
(D) 45.45%

35. If all the companies where the number of male workers is less than the number of female workers continue/start to work only during the night for at least 8 hours, then how many companies will be changing their number of working hours?
(A) 0 (B) At least 1
(C) At least 2 (D) At least 3

Practice Problems 2

Directions for questions 1 to 3: These questions are based on the following table that shows the Btu content of common types of energy.

Type of energy	Btu content
1 barrel of crude oil (42 gallons)	5,838,000
1 gallon of gasoline	124,000
1 gallon of heating oil	139,000
1 barrel of residual fuel oil	6,287,000
1 cubic foot of natural gas	1,026
1 gallon of propane	91,000
1 short ton of coal	20,681,000
1 kilowatt hour of electricity	3,412

Note 1: British Thermal Unit (Btu) represents the amount of energy required to raise the temperature of 1 pound of water by 1°F. Also 1 calorie represents the amount of energy required to raise the temperature of 2.2 pounds (1 kg) of water by 1.8°F (1°C).

Note 2: 1 Joule = 0.2390 calories and

1 Btu = 1.06 kilojoules

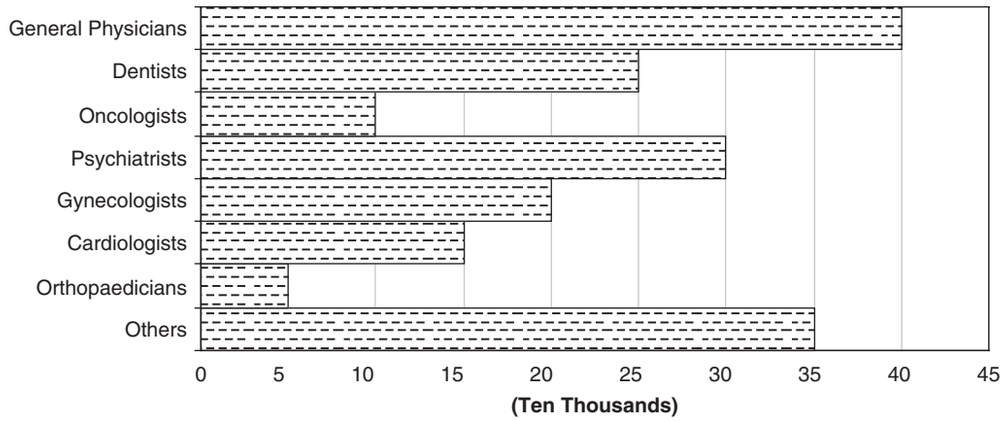
1. The Btu content in 42 gallons of heating oil is equal to the Btu content in which of the following?
(A) 60 gallons of propane
(B) 42 gallons of crude oil
(C) 48 gallons of gasoline
(D) 57 cubic feet of natural gas

2. The Btu content in 60 gallons of propane is less than that in one short ton of coal by
(A) 15,221,000 units (B) 16,290,000 units
(C) 17,820,000 units (D) 18,630,000 units

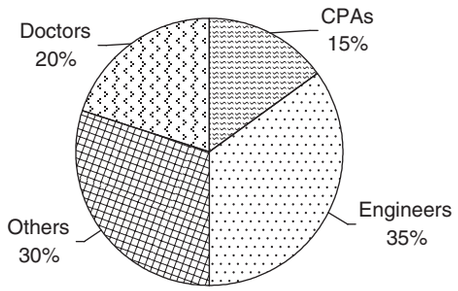
3. 1 Btu is equal to _____ calories
(A) 228.72 (B) 244.68
(C) 253.34 (D) 262.44

Directions for questions 4 to 6: These questions are based on the graph and pie chart given below.

Distribution of US doctors across specializations in a certain year



Distribution of professionals in USA that year

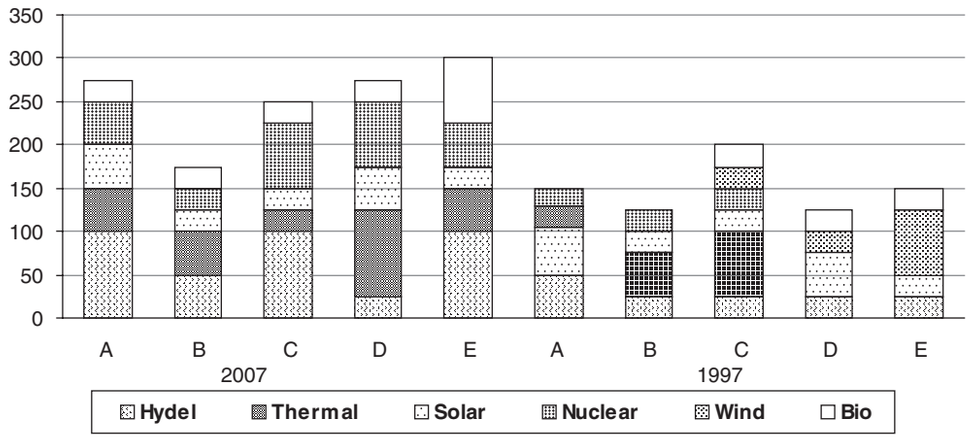


4. Gynaecologists constitute what percent of the total number of doctors?
(A) 11% (B) 25% (C) 20% (D) 15%

5. What is the ratio of the number of engineers to the number of general physicians?
(A) $\frac{63}{8}$ (B) $\frac{47}{8}$ (C) $\frac{33}{7}$ (D) $\frac{23}{5}$

6. If the population of USA was 360 millions that year, then how many doctors are available per 1000 people?
(A) 20 (B) 10 (C) 5 (D) 4

Directions for questions 7 to 9: The following bar graph shows the quantity (in million watts) of different types of power generated in five states in a certain country in the year 2007 and 1997.



(*Continued*)

7. For how many of the given states can we say that at least one particular type of power was not produced in 2007 while the same type of power was produced in 1997?
(A) 4 (B) 3 (C) 2 (D) 1

8. For how many types is the total production in 2007 more than that in 1997 for all the given states combined?
(A) 5 (B) 4 (C) 3 (D) 2

9. Of the total power generated in 2007 of the types wind and solar, only 85% and 90%, respectively, were useful for consumption. By how many million watts is the consumable power generated by state *D* in 2007 using solar energy less than that generated using wind energy?
(A) 20.25 (B) 19.50
(C) 18.75 (D) 17.50

Directions for questions 10 to 12: These questions are based on the following data that show the number of representatives of different political parties in the Parliament of UK, which has three wings—The Northern Ireland Assembly, The Scottish Parliament and the National Assembly of Wales.

Political parties	Northern ireland assembly		Political parties	Scottish parliament		Political parties	National assembly of wales	
	Males	Females		Males	Females		Males	Females
DUP	36	12	Labour	48	8	Labour	12	4
UUP	10	3	Democrats	16	4	SDLP	15	2
SDLP	24	2	Conservative	24	6	Conservative	6	5
Others	18	3	Republicans	14	9	Others	9	7

10. Only among parties with at least 25 members in any wing, the number of young parliamentarians is more than eight. What is the maximum number of parliamentarians in the two wings—Northern Ireland Assembly and Scottish Parliament who are not young?
(A) 201 (B) 144 (C) 165 (D) 176

11. If there are six Catholic parliamentarians for each party in the Scottish Parliament, then how many Scottish parliamentarians are not members of the Conservative party nor are Catholics?
(A) 27 (B) 42 (C) 60 (D) 81

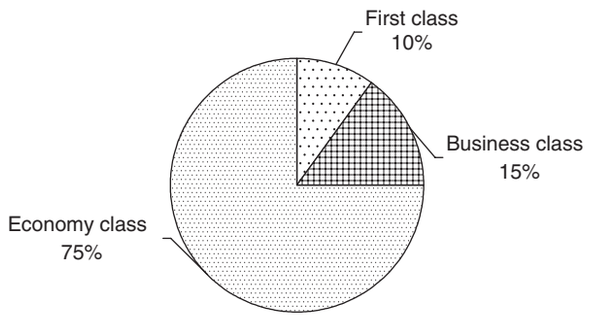
12. The number of parliamentarians who are graduates in the National Assembly of Wales is 45. How many parliamentarians in the National Assembly of Wales are either females or graduates?
(A) 27
(B) 30
(C) 32
(D) Cannot be determined

Directions for questions 13 to 15: These questions are based on the table and the pie chart given below.

Details of Airports in USA

Name of the Airport	Flights per day		Number of passengers per day (in hundreds)
	Arrivals	Departures	
Atlanta	1420	1380	560
JFK	1380	1420	700
Florida	1300	1500	360
Chicago	1440	1560	720
National	1200	1300	360
Los Angeles	920	880	468
Dallas	800	700	405
Detroit	690	710	392
Seattle	1100	1000	441

Classification of passengers in each of the flights

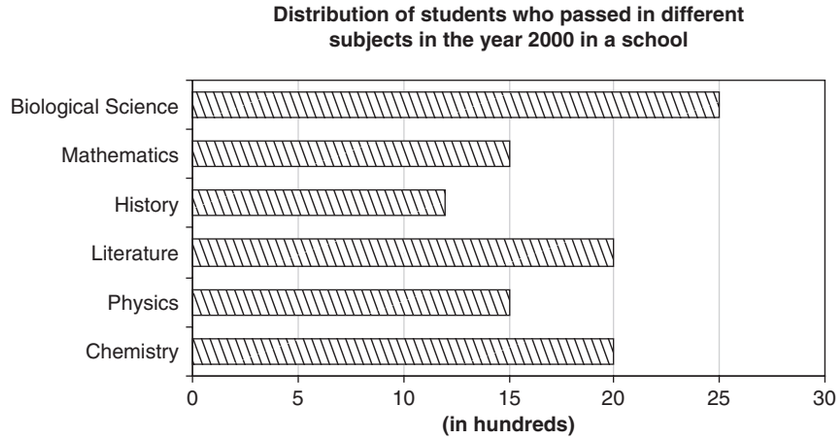


13. How many airports handle more than 1500 flights per day?
(A) 5 (B) 6 (C) 7 (D) 8

14. Which of the following airports carries the maximum number of passengers per flight per day?
(A) Chicago (B) JFK
(C) Detroit (D) Atlanta

15. How many passengers (in hundreds) travel in the Economy class at JFK Airport?
(A) 310 (B) 490 (C) 525 (D) 595

Directions for questions 16 to 18: These questions are based on the graph and the table given below.



Percentage change in the number of students who passed over the previous year in the school

	2000	2001
Biological Science	10%	20%
Mathematics	15%	20%
History	–25%	–10%
Literature	30%	–20%
Physics	20%	20%
Chemistry	15%	25%

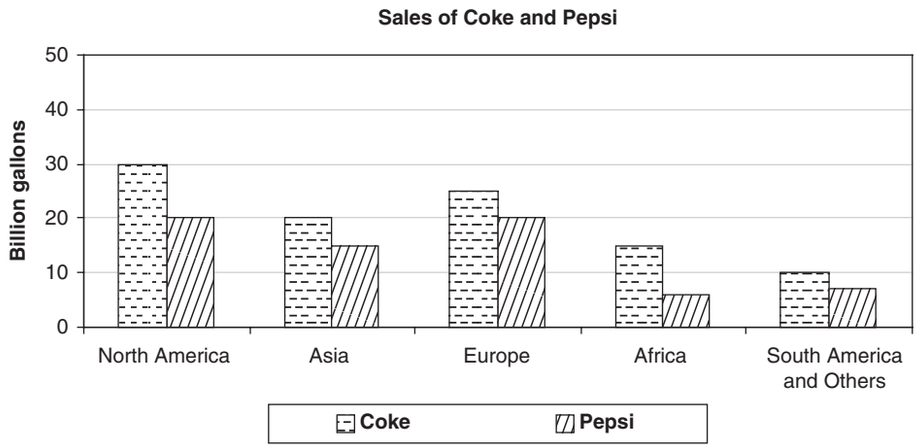
Note: The negative sign indicates a decrease.

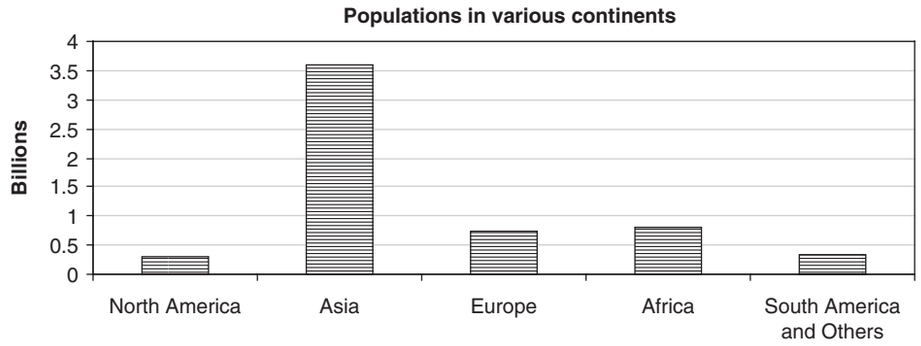
16. What is the ratio of the number of students who passed in History in 1999 to that in 2001?
(A) 9 : 4 (B) 8 : 5 (C) 40 : 27 (D) 6 : 5

17. By what percent did the number of students who passed in Chemistry increase from 1999 to 2001?
(A) 35% (B) 37.5%
(C) 40% (D) 43.75%

18. In which of the following pairs of subjects did an equal number of students pass in the year 2001?
(A) Mathematics and Literature
(B) Literature and Chemistry
(C) Mathematics and Physics
(D) History and Physics

Directions for questions 19 to 21: These questions are based on the graphs given below.





19. By what percent is the sales of Pepsi in Asia lower than that in North America?

(A) 40%
(B) 33%
(C) 25%
(D) 20%

20. Per capita consumption = $\frac{\text{Total consumption}}{\text{Total population}}$

Using the above definition what is the per capita consumption of Coke in Europe?

(A) $\frac{4}{3} \times 10$ gallons/person

(B) $\frac{100}{3}$ gallons/person

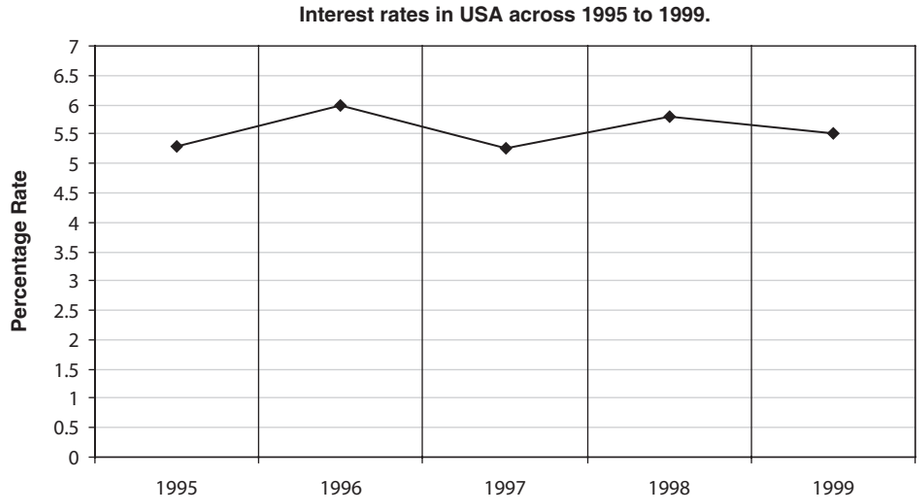
(C) $\frac{4}{3} \times 10^2$ gallons/person

(D) $\frac{1000}{7}$ gallons/person

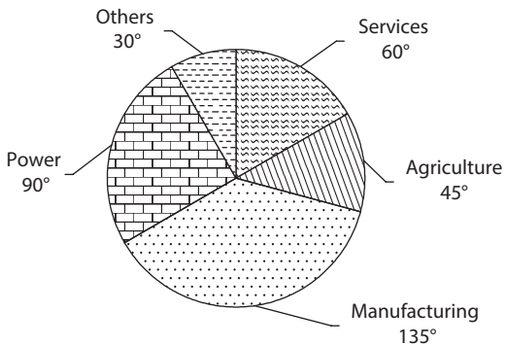
21. If the average (arithmetic mean) of the cost of Coke in all the continents is US$11, per gallon, then what is the total sales revenue on selling Coke in all the continents?

(A) $1835 billion
(B) $1675 billion
(C) $1535 billion
(D) $1100 billion

Directions for questions 22 to 24: These questions are based on the graph, pie chart, and the table given below.



Distribution of loans availed of by various sectors of the industry for each of the years from 1995 to 1999



Total loan amount availed of by the industry (In Billions of US$)	
1995	36
1996	48
1997	72
1998	108
1999	144

22. In the years 1998 and 1999 combined, the total interest paid by exactly one sector as a percentage of the total interest paid by all the sectors of the industry cannot be which of the following?

(A) $8\frac{1}{3}\%$ (B) $16\frac{2}{3}\%$

(C) 25% (D) 40%

23. ENRON Inc. has availed 25% of the loan pertaining to one of the sectors in 1997. What is the minimum possible amount of interest that they could have paid in 1997 in millions of dollars? [Consider others to be a single sector.]

(A) 60.75 (B) 66.75

(C) 72.75 (D) 78.75

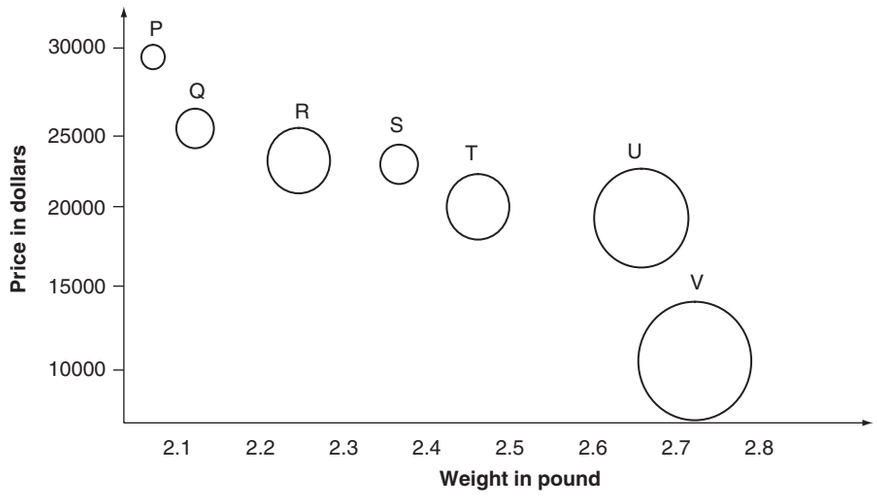
24. The total interest paid in 1995 and 1996 was more than $390 million for

(A) at least three sectors (B) exactly one sector

(C) exactly two sectors (D) exactly three sectors

Directions for questions 25 to 27: Answer the questions on the basis of the information given below.

In the bubble graph below, the horizontal axis shows the weight of the product in pounds and the vertical axis shows the price in dollars. The relative size of the circles indicates how many of the products were sold. The largest circle indicates 4000 products, and there is a decrease of 10% in the number of products for an incremental decrease in the size of the circles.



25. A shop sells one unit each of all products having a weight of 2.2 to 2.5 pound. The sales generated in dollars is _____
(A) 42500 (B) 70000
(C) 47500 (D) 72500

26. The number of the heaviest product sold was _____ that of any product shown and was the _____
(A) greater than; most expensive
(B) greater than; least expensive
(C) lesser than; most expensive
(D) lesser than; most expensive

27. The total numbers of products of type *U* and *T* sold were _____.
(A) 5400 (B) 6000
(C) 6840 (D) 8000

Directions for questions 28 to 30: These questions are based on the information given further.

Six activities—*A, B, C, D, E* and *F*—need to be performed. Time required for each activity is given in the table, along with the list of activities that need to be completed before that activity is started.

Activity	Time to complete (in min.)	Activities to be performed before
A	50	–
B	60	*A, C*
C	45	–
D	40	*B, C*
E	30	*D*
F	20	*B*

Assume that two or more activities can be performed simultaneously.

28. Find the minimum time in which all the activities can be completed.
(A) 180 minutes (B) 175 minutes
(C) 195 minutes (D) 225 minutes

29. If the work is to be completed in the least possible time, then what is the maximum possible time gap between starting the activities *D* and *F*?
(A) 20 minutes (B) 40 minutes
(C) 50 minutes (D) 80 minutes

30. Now, two new activities—*Y*, taking 10 minutes, and *Z*, taking 30 minutes—are included. If *Y* and *Z* are to be completed before *A* and *D*, respectively, then what is the least possible time in which the entire work is completed?
(A) 205 minutes (B) 210 minutes
(C) 180 minutes (D) 190 minutes

Directions for questions 31 and 32: Answer the questions on the basis of the information given below.

The table below gives the sector wise sales of *XYZ* Ltd., which is a group of companies, for the financial year 2000–2001. Study the table and answer the following questions.

Sl. No	Sector	Sales (₹Million)	Percentage share
1.	Electronics (Es)	52,920	11.2
2.	Financial Services (FS)	79,380	16.8
3.	Consumer Goods (CG)	41,580	8.8
4.	Information Systems (IS)	122,850	26
5.	Pharmaceuticals (Ph)	40,635	8.6
6.	Construction (Con)	100,170	21.2
7.	Fertilizers (Fz)	34,965	7.4
	Total	472,500	100.0

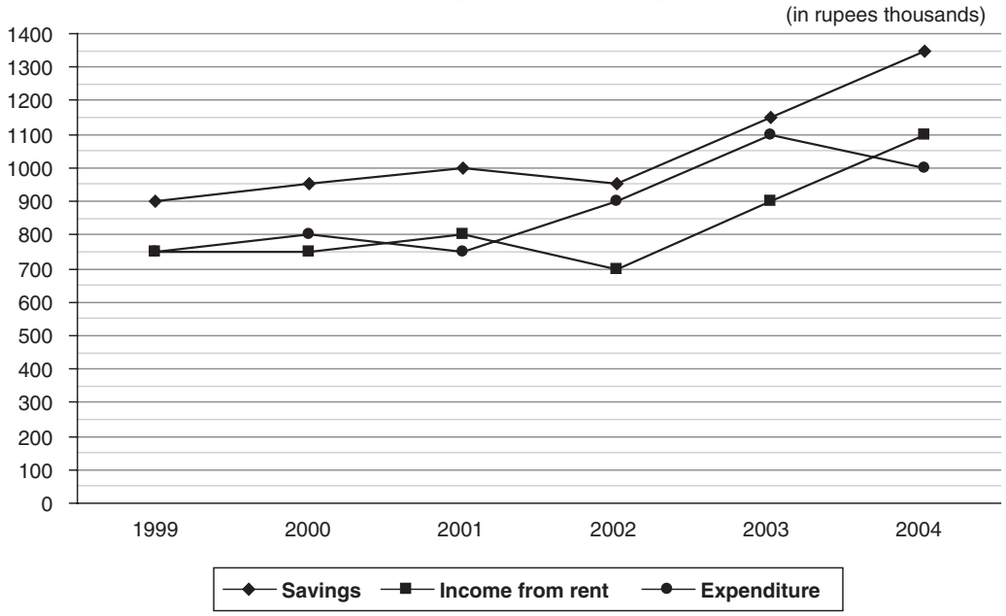
Note:
(A) Sales turnover at US $10.5 billion (₹47,250 crore) represents a year-on-year growth of 28 percent in dollar terms and 30.2 percent in rupee terms.
(B) Profit percentage = Profit as a percentage of sales.

31. The group has decided to focus its attention on the minimum number of sectors contributing together at least 75% of the sales of the group. Identify the correct set of sectors.
(A) IS, FS, Con, and Fz
(B) IS, FS, Con, and Es
(C) IS, Con, CG, Fz, and Ph
(D) IS, FS, Es, and Fz

32. The average profit percentage across the sectors is 10, that for consumer goods is 15, and that for electronics is 10. What is the average profit percentage across the remaining sectors?
(A) 9.25 (B) 9.45
(C) 9.65 (D) 9.85

Directions for questions 33 to 35: These questions are based on the following information.

The graph given below shows the savings, income from rent and the expenditure of a family for each of the six years.



Savings = Income from business + Income from rent – Expenditure

33. What is the approximate percentage growth in the average annual savings of the last three years compared to that of the first three years?
(A) 6% (B) 31% (C) 15% (D) 21%

34. If the income from business is plotted on the graph, then which of the following is true about the graph?
(A) Increase followed by decrease in the alternate years
(B) Decrease followed by increase in the alternate years
(C) Linear curve
(D) None of these

35. Which of the following is the difference between the average expenditure per annum and the average income from rent per annum?
(A) ₹50000
(B) ₹51660
(C) ₹41660
(D) ₹45000

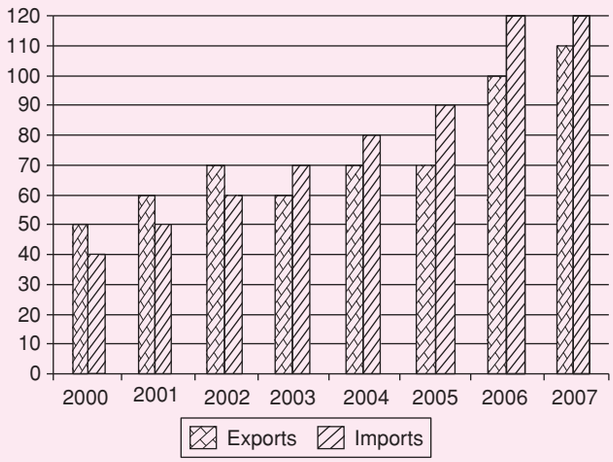
Previous Years' Questions

1. The statistics of runs scored in a series by four batsmen are provided in the following table. Who is the most consistent batsman of these four? **[2014]**

Batsman	Average	Standard deviation
K	31.2	5.21
L	46.0	6.35
M	54.4	6.22
N	17.9	5.90

(A) K (B) L
(C) M (D) N

2. A train that is 280 meters long, travelling at a uniform speed, crosses a platform in 60 seconds and passes a man standing on the platform in 20 seconds. What is the length of the platform in meters? **[2014]**



3. The exports and imports (in crores of ₹) of a country from 2000 to 2007 are given in the following bar chart. If the trade deficit is defined as excess of imports over exports, in which year is the trade deficit 1/5th of the exports? **[2014]**

(A) 2005 (B) 2004
(C) 2007 (D) 2006

4. You are given three coins: one has heads on both faces, the second has tails on both faces, and the third has a head on one face and a tail on the other. You choose a coin at random and toss it, and it comes up heads. The probability that the other face is tails is **[2014]**

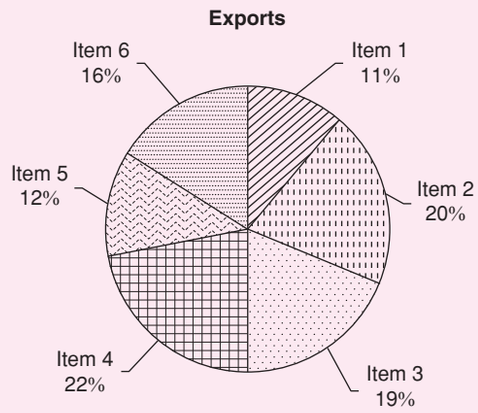
(A) $\frac{1}{4}$ (B) $\frac{1}{3}$
(C) $\frac{1}{2}$ (D) $\frac{2}{3}$

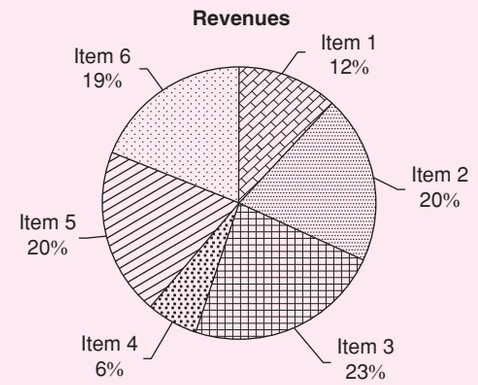
5. A regular die has six sides with numbers 1 to 6 marked on its sides. If a very large number of throws show the following frequencies of occurrence: $1 \rightarrow 0.167$; $2 \rightarrow 0.167$; $3 \rightarrow 0.152$; $4 \rightarrow 0.166$; $5 \rightarrow 0.168$; $6 \rightarrow 0.180$. We call this die **[2014]**

(A) irregular (B) biased
(C) Gaussian (D) insufficient

6. The sum of eight consecutive odd numbers is 656. The average of four consecutive even numbers is 87. What is the sum of the smallest odd number and second largest even number? **[2014]**

7. The total exports and revenues from the exports of a country are given in the two charts shown below. The pie chart for exports shows the quantity of each item exported as a percentage of the total quantity of exports. The pie chart for the revenues shows the percentage of the total revenue generated through export of each item. The total quantity of exports of all the items is 500 thousand tonnes and the total revenues are 250 crore rupees. Which item among the following has generated the maximum revenue per kg? **[2014]**





(A) Item 2 (B) Item 3
(C) Item 6 (D) Item 5

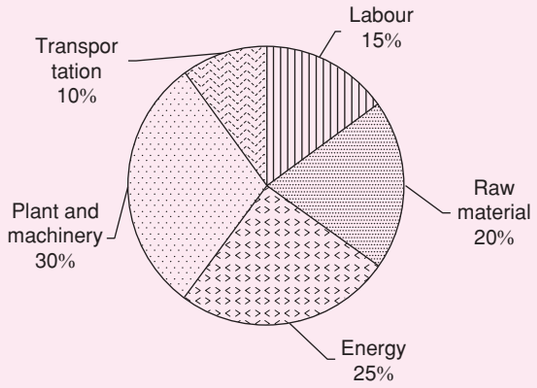
8. If takes 30 minutes to empty a half-full tank by draining it at a constant rate. It is decided to simultaneously pump water into the half-full tank while draining it. What is the rate at which water has to be pumped in so that it gets fully filled in 10 minutes? **[2014]**
(A) 4 times the draining rate
(B) 3 times the draining rate
(C) 2.5 times the draining rate
(D) 2 times the draining rate

9. The multi-level hierarchical pie chart shows the population of animals in a reserve forest. The correct conclusions from this information are: **[2014]**

(i) Butterflies are birds
(ii) There are more tigers in this forest than red ants.
(iii) All reptiles in this forest are other snakes or crocodiles.
(iv) Elephants are the largest mammals in this forest.
(A) (i) and (ii) only
(B) (i), (ii), (iii), and (iv)
(C) (i), (iii), and (iv) only
(D) (i), (ii), and (iii) only

10. A man can row at 8 km/h in still water. If it takes him thrice as long to row upstream, as to row downstream, then find the stream velocity in km/h. **[2014]**

11. A firm producing air purifiers sold 200 units in 2012. The following pie chart presents the share of raw material, labor, energy, plant and machinery, and transportation costs in the total manufacturing cost of the firm in 2012. The expenditure on labour in 2012 in ₹4,50,000. In 2013, the raw material expenses increased by 30% and all other expenses increased by 20%. If the company registered a profit of ₹10 lakhs in 2012, at what prices (in ₹) was each air purifier sold? **[2014]**



12. A batch of one hundred bulbs is inspected by testing our randomly chosen bulbs. The batch is rejected if even one of the bulbs is defective. A batch typically has five defective bulbs. The probability that the current batch is accepted is _____. **[2014]**

13. Let $f(x, y) = x^n y^m = P$. If x is doubled and y is halved, the new value of f is **[2014]**
(A) $2^{n-m} P$ (B) $2^{m-n} P$
(C) $2(n-m)P$ (D) $2(m-n)P$

14. In a sequence of 12 consecutive odd numbers, the sum of the first 5 numbers is 425. What is the sum of the last 5 numbers in the sequence? **[2014]**

15. Industrial consumption of power doubled from 200–2001 to 2010–2011. Find the annual rate of increase in percent assuming it to be uniform over the years. **[2014]**
(A) 5.6 (B) 7.2
(C) 10.0 (D) 12.2

16. A five digit number is formed using the digits 1, 3, 5, 7, and 9 without repeating any of them. What is the sum of all such possible five digit numbers? **[2014]**
(A) 6666660 (B) 6666600
(C) 6666666 (D) 6666606

17. In the summer of 2012, in New Delhi, the mean temperature of Monday to Wednesday was 41°C and of Tuesday to Thursday was 43°C. If the temperature on Thursday was 15% higher than that of Monday, then the temperature in °C on Thursday was **[2013]**
(A) 40 (B) 43
(C) 46 (D) 49

18. A car travels 8 km in the first quarter of an hour, 6 km in the second quarter and 16 km in the third quarter. The average speed of the car in km/h over the entire journey is **[2013]**
(A) 30 (B) 36
(C) 40 (D) 24

19. Find the sum to n terms of the series $10 + 84 + 734 +$ **[2013]**

(A) $\frac{9(9^n+1)}{10}+1$ (B) $\frac{9(9^n-1)}{8}+1$

(C) $\frac{9(9^n-1)}{8}+n$ (D) $\frac{9(9^n-1)}{8}+n^2$

20. The set of values p for which the roots of equation $3x^2 + 2x + p(p-1) = 0$ are opposite sign is **[2013]**

(A) $(-\infty, 0)$ (B) $(0, 1)$

(C) $(1, \infty)$ (D) $(0, \infty)$

21. What is the chance that a leap year, selected at random, will contain 53 Saturdays? **[2013]**

(A) $\frac{2}{7}$ (B) $\frac{3}{7}$

(C) $\frac{1}{7}$ (D) $\frac{5}{7}$

22. If $(1.001)^{1259} = 3.52$ and $(1.001)^{2062} = 7.85$, then $(1.001)^{3321} =$ **[2012]**

(A) 2.23 (B) 4.33

(C) 11.37 (D) 27.64

23. The data given in the following table summarize the monthly budget of an average household **[2012]**

Category	Amount (₹)
Food	4000
Clothing	1200
Rent	2000
Savings	1500
Other expenses	1800

The approximate percentage of the monthly budget **NOT** spent on savings is

(A) 10% (B) 14%

(C) 81% (D) 86%

24. A and B are friends. They decide to meet between 1 pm and 2 pm on a given day. There is a condition that whoever arrives first will not wait for the other for more than 15 minutes. The probability that they will meet on that day is **[2012]**

(A) $\frac{1}{4}$ (B) $\frac{1}{16}$

(C) $\frac{7}{16}$ (D) $\frac{9}{16}$

25. Raju has 14 cureccy notes in his pocket consisting of only ₹20 notes and ₹10 notes. the total money value of the notes is ₹230. the number of ₹10 notes that Raju has is **[2012]**

(A) 5 (B) 6

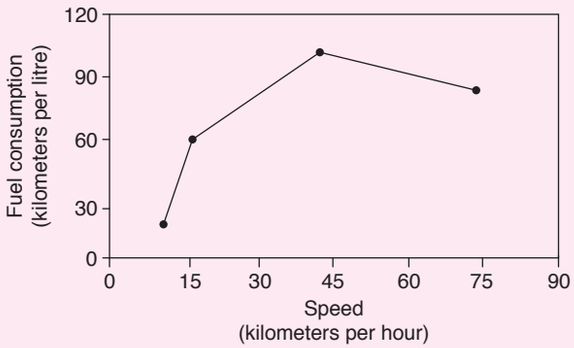
(C) 9 (D) 10

26. There are two candidates P and Q in an election. During the campaign, 40% of the voters promised to vote for P, and rest for Q. However, on the day of election, 15% of the voters went back on their promise to vote for P and instead voted for Q. 25% of the voters went back on their promise to vote for Q and instead voted for P. Suppose, P lost by 2 votes, then what was the total number of voters? **[2011]**

(A) 100 (B) 110

(C) 90 (D) 95

27. The fuel consumed by a motorcycle during a journey while travelling at various speeds is indicated in the graph below. **[2011]**



The distance covered during four laps of the journey are listed in the table.

Lap	Distance (Kilometers)	Average speed (Kilometers per hour)
P	15	15
Q	75	45
R	40	75
S	10	10

Four the given data, we can conclude that the fuel consumed per kilometer was least during the lap

(A) P (B) Q

(C) R (D) S

28. Three friends, R, S, and T, shared toffee from a bowl. R took 1/3rd of the toffees, but returned four to the bowl. S took 1/4th of what was left but returned three toffees to the bowl. T took half of the remainder but returned two back into the bowl. If the bowl had 17 toffees left, how many toffees were originally there in the bowl? **[2011]**

(A) 38 (B) 31

(C) 48 (D) 41

29. Given that $f(y) = \frac{|y|}{y}$, and q is any non-zero real number, the value of $|f(q) - f(-q)|$ is **[2011]**

(A) 0 (B) −1

(C) 1 (D) 2

30. The sum of n terms of the series $4 + 44 + 444 + \cdots$. Is **[2011]**

(A) $\left(\frac{4}{81}\right)[10^{n+1} - 9n - 1]$

(B) $\left(\frac{4}{81}\right)[10^{n-1} - 9n - 1]$

(C) $\left(\frac{4}{81}\right)[10^{n+1} - 9n - 10]$

(D) $\left(\frac{4}{81}\right)[10^{n} - 9n - 10]$

31. If $137 + 276 = 435$, how much is $731 + 672$? **[2010]**

(A) 534 (B) 1403

(C) 1623 (D) 1513

32. 5 skilled workers can build a wall in 20 days, 8 semi-skilled workers can build the wall in 25 days, and 10 unskilled workers can build the wall in 30 days. If a team has 2 skilled, 6 semi-skilled, and 5 unskilled workers, how long will it take to build the wall? **[2010]**

(A) 20 (B) 18

(C) 16 (D) 15

33. From the digits 2, 2, 3, 3, 3, 4, 4, 4, 4, how many distinct 4-digit numbers greater than 3000 can be formed? **[2010]**

(A) 50 (B) 51

(C) 52 (D) 54

Test
Quantitative Ability

Time: 30 min.

Directions for questions 1 to 30: Select the correct alternative from the given choices.

1. P, Q, R, and S have a total amount of ₹220 with them. P has ₹30 more than Q. S has half the amount with Q. R has ₹10 more than S. Find the amount with S (in ₹).
(A) 20 (B) 40
(C) 30 (D) 50

2. In a room, there are some girls and some benches. If 5 girls sit on each bench, three girls will have no bench to sit on. If there is one bench less, 6 girls can sit on each bench. Find the number of benches.
(A) 6 (B) 8
(C) 7 (D) 9

3. A test has 60 questions. Each correct answer fetches 1 mark. For each wrong answer and each unanswered question 1 mark is deducted. A candidate who wrote this test scored 20 marks. Find the number of questions he correctly answered.
(A) 50 (B) 45
(C) 35 (D) 40

4. If $\frac{a+b-c}{x}=\frac{a-b+c}{y}=\frac{c+b-a}{z}$ then $x(b-a)+y(a-c)+z(c-b)=$
(A) 0 (B) 2
(C) 3 (D) 1

5. The value of a diamond varies directly with the cube of its weight. It broke into two pieces whose weights are in the ratio 3 : 4. The loss due to breakage is ₹504000. Find its initial value (in ₹).
(A) 1029000 (B) 686000
(C) 1372000 (D) 1715000

6. The average of 25 observations is 120. By mistake one of the observations, 144, is taken as 169. Find the average of the 25 observations, after the mistake is corrected.
(A) 120 (B) 119
(C) 125 (D) 132

7. The average of 13 observations is 50. The average of first seven observations is 45 and the average of last seven observations is 52. Find the value of seventh observation.
A) 41 (B) 30
(C) 29 (D) 62

8. A container contains 100 litres of milk. 10 litres of milk is replaced by 10 litres of water. From the solution formed, 10 litres of solution is replaced by 10 litres of water, and this process is repeated one more time. Find the percentage of water in the resulting solution.
(A) 33.3% (B) 67%
(C) 36.5% (D) 27.1%

9. In a 729 ml of solution, the ratio of acid to water is 7 : 2. How much more water should be mixed so that the resulting mixture contains acid and water in the ratio 7 : 3 (in ml)?
(A) 100 (B) 40
(C) 37 (D) 81

10. A merchant buys sulphuric acid at a certain rate per gallon and after mixing it with water, sells it at the same rate. If the merchant makes a profit of 20%, how many gallons of water are there per gallon of acid?
(A) 0.2 (B) 0.5
(C) 0.7 (D) 0.25

11. If A travelled a certain distance at 6 km/h, he would have reached his destination 10 minutes early. If he travelled it at 4 km/h, he would have reached his destination 10 minutes late. Find the speed at which he must travel to reach his destination on time (in km/h).
(A) 5 (B) 5.4
(C) 4.8 (D) 4.5

12. A car travelled the first hour of its journey at 30 km/h, the next 5 hours of its journey at 50 km/h, and the remaining 4 hours of its journey at 75 km/h. Find its average speed for its journey (in km/h).
(A) 56 (B) 60
(C) 58 (D) 62

13. Without stoppages, a train can cover 54 km in an hour. With stoppages it can cover 36 km in an hour. Find its stoppage time per hour in a journey it covers with stoppages (in minutes).
(A) 15 (B) 18
(C) 20 (D) 12

14. A and B can complete a job in 40 days. B and C can complete it in 30 days. A and C can complete it in 20 days. Find the time taken by A to complete it (in days).
(A) $\frac{180}{7}$ (B) 30
(C) 48 (D) $\frac{240}{7}$

15. 3 men and 4 women can complete a job in 10 days. 24 men and 2 women can complete it in 2 days. Find the time taken by 5 men and 10 women to complete it (in days).
(A) 4 (B) 5
(C) 3 (D) 6

16. Abhilash spends 25% of his income towards rent, 20% of the remaining income towards food, 8% of the

remaining towards medical expenses, and 25% of the remaining towards miscellaneous expenses. If he saves ₹82,800, what is his income?
(A) ₹2,00,000 (B) ₹2,25,000
(C) ₹2,40,000 (D) ₹2,50,000

17. By selling 30 articles, a shopkeeper gained the selling price of 10 articles. Find the profit percent.
(A) 20% (B) 30%
(C) 50% (D) 40%

18. When 1036 is divided by N, the remainder is 12 and when 1545 is divided by N, the remainder is 9. Find the greatest possible value of N.
(A) 128 (B) 512
(C) 250 (D) 64

19. Five bells toll at regular intervals of 10, 15, 20, 25, and 30 seconds, respectively. If they toll together at 8:00 am, then at what time will they toll together for the first time after 8:00 am?
(A) 8:04 am (B) 8:06 am
(C) 8:05 am (D) 8:07 am

20. Find the value of

$$\frac{3}{\sqrt{2}+\sqrt{11}}+\frac{3}{\sqrt{5}+\sqrt{8}}+\frac{3}{\sqrt{8}+\sqrt{11}}+\cdots+\frac{3}{\sqrt{26}+\sqrt{29}}.$$

(A) $\sqrt{29}-\sqrt{2}$ (B) $\sqrt{26}+\sqrt{5}$
(C) $\sqrt{29}+\sqrt{26}$ (D) $\sqrt{26}-\sqrt{8}$

21. If $a+b+c=0$, find the value of $(3^{a^8})^{\frac{1}{a^6bc}}(3^{b^8})^{\frac{1}{ab^6c}}(3c^8)^{\frac{1}{abc^6}}$.
(A) 2 (B) 6
(C) 27 (D) 81

22. If $3^{x+3}-3^{x+2}=486$, then find x.
(A) 3 (B) 5
(C) 6 (D) 2

23. $\log_p q=\frac{5}{4}, \log_r q=\frac{5}{6}$ and $\log_r p=3x$, then find x.
(A) $\frac{1}{9}$ (B) $\frac{2}{3}$
(C) $\frac{1}{18}$ (D) $\frac{2}{9}$

24. $\frac{5}{1+\log_p qr}+\frac{5}{1+\log_q pr}+\frac{5}{1+\log_r pq}=$
(A) 0 (B) 1
(C) 5 (D) 10

25. If $\log_{10}2=0.3010$ then find the number of digits in 2^{55}.
(A) 17 (B) 11
(C) 18 (D) 16

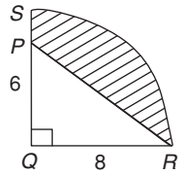
26. The maximum sum of the arithmetic progression 45, 41, 37, ... is
(A) 256 (B) 274
(C) 276 (D) 264

27. The greatest value of n such that $1+3+3^2+3^3+\cdots+3^n$, which is less than 3000 will be:
(A) 6 (B) 8
(C) 7 (D) 9

28. $3+33+333+\cdots+$ upto n terms =
(A) $\frac{(10^n-1)}{3}-n$ (B) $\frac{10(10^n-1)}{27}-n$
(C) $\frac{(10^n-1)}{3}-\frac{n}{9}$ (D) $\frac{10(10^n-1)}{27}-\frac{n}{3}$

29. What is the minimum value of the function $f(x)=x^2-15x+9$?
(A) $\frac{289}{4}$ (B) $\frac{-200}{9}$
(C) $\frac{295}{2}$ (D) $\frac{-189}{4}$

30. Find the area of the shaded region, where PQR is a triangle and QRS is a quadrant. $PQ=6$ cm and $QR=8$ cm.



(A) $4\pi-8$ sq. cm. (B) $2\pi-3$ sq. cm.
(C) $8\pi-16$ sq. cm. (D) $16\pi-24$ sq. cm.

HINTS/SOLUTIONS

Practice Problems I

Solutions for question 1:

1. Percentage increase for the student

$$P = \left(\frac{1295-997}{997}\right)\times 100 = 29.8\%$$

Percentage increase for the student

$$Q = \left(\frac{876-664}{664}\right)\times 100 = 31.9\%$$

Percentage increase for the student

$$R = \left(\frac{732-585}{585}\right)\times 100 = 25.12\%$$

Percentage increase for the student

$$S = \left(\frac{689-480}{480}\right)\times 100 = 43.5\%$$

$\therefore$ Clearly only student 'S' has percentage increase more than 35%.

Hence, the correct option is (A).

Solutions for questions 2 to 4:

2. On March 1, 1867 – 37th state joined the Union and on Feb 14, 1912 – 48th state joined the Union. Including them a total of 48 – 37 +1 i.e., 12 states have joined.

Hence, the correct option is (B).

3. Required percentage $= \frac{124}{535}\times 100 = 23\%$.

Hence, the correct option is (D).

4. Number of representatives is not less than six for Washington, Texas, Virginia, Minnesota, Illinois, Arizona, Indiana and Georgia. Of these, only Texas has a population of more than 15 million. Hence, the remaining seven states will satisfy both the given conditions.

Hence, the correct option is (D).

Solutions for questions 5 to 7:

5. The number of projects in Mexico and Texas

Year	Mexico		Texas
1995	250	>	200
1996	300	>	250
1997	250	<	300
1998	350	<	400
1999	400	>	350
2000	350	<	500
2001	400	<	550

During the three years, the number of projects undertaken in Mexico are greater.

Hence, the correct option is (A).

6. The number of projects undertaken in 1998

$= 350 + 400 = 750$.

Hence, the correct option is (C).

7. The average number of projects undertaken in Mexico and Texas

in 1995 $= \frac{250+200}{2} = 225$.

in 1996 $= \frac{300+250}{2} = 275$.

in 1998 $= \frac{350+400}{2} = 375$.

in 1999 $= \frac{400+350}{2} = 375$

in 2000 $= \frac{350+500}{2} = 425$.

Hence, the correct option is (A).

Solutions for questions 8 to 10:

8. From the graph in order to have the highest air traffic in 2000, the overall growth rate from 2000 to 2002 must be the least. In the least growth rate was for North America. In 2002, only Europe and North America had a negative growth rate with Europe having a higher negative growth rater, $\therefore$ one of them must have the highest air traffic in 2000.

In the choices as Europe is not given, answer should be North America.

Hence, the correct option is (B).

9. As the air traffic of all regions in the year 2000 is not known, the region showing the highest growth cannot be found.

Hence, the correct option is (D).

10. Air traffic of South America in revenue passenger kilometers

$$= 20\times 10^6 \times \frac{107.5}{100}\times\frac{115}{100} = 24725\times 10^3$$

(revenue passenger kilometer.) $= 247.25\times 10^5$.

Hence, the correct option is (C).

Solutions for questions 11 to 13:

11. The maximum increase in the rainfall quantity over the previous month occurred for city A in September. Also the minimum rainfall quantity occurred for city A in August. $\therefore$ Maximum percentage increase occurred from August to September for city A.

Hence, the correct option is (A).

12. Average rainfall in the month of August

$$= \frac{50+150+175+250}{4} = 156.25 \text{ cm}.$$

Hence, the correct option is (C).

13. Average rainfall in cities B and C during August and September

$$= \frac{150+225+175+200}{4} = 750/4$$

Average rainfall in city A during July, August and September

$$= \frac{100+50+150}{3} = 100.$$

$$\therefore \text{ Required percentage} = \frac{\frac{750}{4}-100}{100} \times 100$$
$$= 87.5\%.$$

Hence, the correct option is (D).

Solutions for questions 14 to 16:

14. The least percentage of colleges offering courses in medicine is for New Jersey and the least percentage of colleges offering Biochemistry is for Michigan.

$\therefore$ The least number of colleges offering Biochemistry in the four states

$$= \text{Min}\left(\frac{15}{100}.\frac{80}{100}.2000, \frac{30}{100}.68.2000\right)$$

$$= \frac{15}{100}.\frac{80}{100}2000 = 240$$

$\therefore$ Each state has atleast 240 colleges offering Biochemistry i.e., more than 230 colleges offering Biochemistry.

Hence, the correct option is (A).

15. We do not know whether each college offering Biophysics in any of the four states offers Biochemistry or not. $\therefore$ we cannot answer the question.

Hence, the correct option is (D).

16. Total number of colleges offering Biostatistics in all the four states =

$$\left(\frac{56}{100}.\frac{35}{100}+\frac{84}{100}.\frac{15}{100}+\frac{86}{100}.\frac{20}{100}+\frac{70}{100}.\frac{30}{100}\right).2000$$

$$= 56.\frac{7}{20}.20 + 84.\frac{3}{20}.20 + 86.\frac{1}{5}.20 + 70.\frac{3}{10}.20$$

$$= 392 + 252 + 344 + 420 = 1408.$$

Hence, the correct option is (B).

Solutions for questions 17 to 19:

17. Twice the number of officers trained = $2 \times 45° = 90°$

As this is equal to thrice the other category, angle representing that category is = $(90° \div 3) = 30°$

As 30° angle is represented by clerks, required answer is clerks.

Hence, the correct option is (C).

18. Accountants who were imparted training as a percentage of total number of employees trained

$$= \frac{18}{360} \times 100 = 5\%.$$

Hence, the correct option is (C).

19. Ratio of the number of employees trained in the officers category to those who are trained in the category of marketing executives = 45° : 153° = 5 : 17.

Hence, the correct option is (A).

Solutions for questions 20 and 21:

According to the given data, the distribution of the workers is as follows:

Factory	Total	Men	Women	Boys
J	4800	2400	1800	600
K	6750	2700	2430	1620
L	5250	1260	2940	1050
M	12000	4200	3000	4800
N	8500	3230	2550	2720
P	2700	1215	1080	405

20. Required percentage

$$= \frac{1080-600}{600} \times 100 = 80\%.$$

Hence, the correct option is (C).

21. The required ratio = 4200 : 2940

$$= 10 : 7.$$

Hence, the correct option is (B).

Solutions for questions 22 to 24:

22. The required ratio $= \frac{25+25+40}{30+45+45} = \frac{90}{120} = 3 : 4.$

Hence, the correct option is (C).

23. Of $\frac{30-25}{25}, \frac{45-30}{30}, \frac{50-45}{45}$ and $\frac{55-50}{50}, \frac{45-30}{30}$ is the highest which is for the year 2002.

Hence, the correct option is (C).

24. The required percentage increase

$$= \frac{(50+55)-(45+50)\times 100}{(45+50)} \simeq 10.5\%.$$

Hence, the correct option is (D).

Solutions for questions 25 and 26:

25. The average sales over the given period (in 000's of rupees)

$$= \frac{3750+3500+4250+4500+4400}{5} = 4080$$

The average production over the given period (in 000's of rupees)

$$= \frac{4500+5100+4750+5000+5750}{5} = 5020$$

Therefore the average production over the given period was more than the average sales by

$$= \frac{5020-4080}{4080} \times 100 = 23\% .$$

Hence, the correct option is (B).

26. Percentage change in sales from 2004 to 2005

$$= \frac{3750-3500}{3750} \times 100 = 6.66$$

Percentage change in sales from 2005 to 2006

$$= \frac{4250-3500}{3500} \times 100 = 21.43\%$$

Percentage change in sales from 2006 to 2007

$$= \frac{4500-4250}{4250} \times 100 = 5.88$$

Percentage change in sales from 2007 to 2008

$$= \frac{4500-4400}{4500} \times 100 = 2.22\%$$

Therefore the percentage change was the highest in the year 2006.

Hence, the correct option is (B).

Solutions for questions 27 and 28:

27. The total cost incurred in printing was less than that incurred in editing by

$$\frac{18-14}{18} \times 100 = 22\frac{2}{9}\%.$$

Hence, the correct option is (B).

28. It is given that, 12.5% of the planned total production cost was 7.2 lakhs.

Let the total production cost (planned) be ₹T.

12.5% (T) = 7.2 lakh. $\Rightarrow T = 57.6$ lakhs.

Therefore the expenditure due to payments made to artists and technicians = 35% (57.6) lakhs.

= 20.16 lakhs. = ₹20,16,000.

Hence, the correct option is (C).

Solutions for questions 29 to 32:

29.

(in crore)

State	Males	Females
A	0.575	2.025
B	0.9	0.9
C	2.5	2.7
D	2.975	1.125
E	3.15	1.35

Clearly from the above table, we can say that state E has highest male population

Hence, the correct option is (A).

30. From the above table we can say that only state *A* and *C* has more number of females than males.

Hence, the correct option is (D).

31. Total number of males in the states *A* and *E* = 0.575 + 3.15 = 3.725 crores

Total number of females in the states *A* and *E*

= 2.025 + 1.35 = 3.375 crores.

$\therefore$ The required ratio = 3.725 crores : 3.375 crores.

= 149 : 135.

Hence, the correct option is (C).

32. The percentage of number of females in the states *C* and *D* w.r.t the number of the males from the same states

$$= \frac{(2.7+1.125)}{(2.5+2.975)} \times 100\%$$

$$= \frac{3.825}{5.475} \times 100\% = 69.86\%.$$

Hence, the correct option is (C).

Solutions for questions 33 to 35:

33. The number of companies in which number of male workers is more than that of female workers is

$$\frac{180^\circ}{360} \times 200 = 100.$$

Out of them, maximum 45 companies can work for 6 hours. (Highest number of companies are working during day time work for 6 hours).

$\therefore$ Required percentage $= \frac{45}{100} \times 100 = 45.$

Hence, the correct option is (A).

34. The number of companies which work either only in the night or only in the day = 103 + 62 = 165

The number of companies in which the number of male and female workers is same = 75

$\therefore$ Required percentage $= \frac{75}{165} \times 100 = 45.45\%.$

Hence, the correct option is (D).

35. The number of companies where the number of male workers is less than the number of female workers is 25.

The number of companies which work only during night for at least 8 hrs = 22.

$\therefore$ At least 3 companies should change their position to satisfy this condition. ($\because$ If all 22 companies are those where there are less males and females, 3 companies should change their number of working hours. Otherwise more than 3 companies should change).

Hence, the correct option is (D).

Practice Problems 2

Solutions for questions 1 to 3:

1. Btu content in 42 gallons of heating oil = 42 × 139,000 = 5838000 i.e., the same in 1 barrel of Crude oil or 42 gallons of crude oil.

Hence, the correct option is (B).

2. Btu content in 60 gallons of propane = 60 × 91,000

= 54,60,000 Btu content in one short ton of coal

= 20,681,000

∴ The required difference = 15,221,000 units.

Hence, the correct option is (A).

3. 1 Btu = 1.06 Kilo joules i.e., 1060 joules

But 1 joule = 0.2390 calories

∴ 1 Btu = 1060 × 0.2390 calories i.e., 253.34 calories.

Hence, the correct option is (C).

Solutions for questions 4 to 6:

4. Gynaecologists = $\frac{20}{180} \times 100 = \frac{1}{9} \times 100 = 11\%$.

Hence, the correct option is (A).

5. The number of engineers as a fraction of total number of general physicians

$$= \frac{35 \times 180}{40 \times 20} = \frac{63}{8}.$$

Hence, the correct option is (A).

6. (Number of doctors ÷ Total population) × 1000

$$= \frac{180 \times 10^4}{360 \times 10^6} \times 1000$$

= 5 per thousand.

Hence, the correct option is (C).

Solutions for questions 7 to 9:

7. For state *A*, we observe that wind power was not produced in 2007 and was produced in 1997. Similarly for *C* and *E* also, atleast one type of power was produced in 1997, but not in 2007.

Hence, the correct option is (B).

8. Observing the given stacked bar, we can say that the production of power from Hydel, Thermal, Nuclear, Wind as well as Bio is more in 2007 when compared to that in 1997.

Hence, the correct option is (A).

9. Consumable power from solar energy in state *D* in 2007 = 90% of 50 = 45 MW. That from wind energy = 85% of 75 = 63.75 MW.

∴ Required difference = 18.75 million watts.

Hence, the correct option is (C).

Solutions for questions 10 to 12:

10. In Northern Ireland Assembly, the maximum number of parliamentarians who are young = (48 – 9) + 13 + (26 – 9) + 21 = 90. The same for Scottish parliament = (56 – 9) + (20) + (30 – 9) + 23 = 111.

∴ 90 + 111 i.e. 201 is the required answers.

Hence, the correct option is (A).

11. The number of parliamentarians who are neither Catholic nor members of the Conservative party = (48 + 8 – 6) + (16 + 4 – 6) + (14 + 9 – 6) = 81.

Hence, the correct option is (D).

12. The number of parliamentarians in the National Assembly of Wales who are either females or graduates cannot be determined because there is no information on the number of females who are not graduates.

Hence, the correct option is (D).

Solutions for questions 13 to 15:

13. There are 7 Airports handling more than 1500 flights per day.

Hence, the correct option is (C).

14. Number of passengers per flight per day carried by the given airports are as follows:

(A) Chicago = $\frac{720}{3000} = \frac{6}{25} = 0.24$

(B) JFK = $\frac{700}{2800} = \frac{1}{4} = 0.25$

(C) Detroit = $\frac{392}{1400} = \frac{7}{25} = 0.28$

(D) Atlanta = $\frac{560}{2800} = \frac{1}{5} = 0.2$

As thousand is common in all the fractions, eliminating that Detroit has the maximum number of passengers per flight.

Hence, the correct option is (C).

15. Number of passengers travelling in Economy class at JFK Airport (in hundreds) = 75% of 700

$$= \frac{3}{4} \times 700 = 525.$$

Hence, the correct option is (C).

Solutions for questions 16 to 18:

16. Let the number of students who passed in History in 1999 be 100 K.

∴ The number of students who passed in 2000 will be 75 K.

Also, the number of students in 2001 will be

$$75\text{ K} \times \frac{90}{100} \text{ i.e., } \frac{135}{2}\text{ K}$$

∴ The required ratio will be 40 : 27.

Hence, the correct option is (C).

17. Let the number of students who passed in Chemistry in 1999 be 100 K.

The number of students who passed in 2000 = 115 K

The number of students in 2001 = $\frac{125}{100} \times 115$ K

$= \frac{125}{100} \times 115 \text{ K} = 143.75$ K

The increase in the percentage of students who passed from 1999 to 2001

$= 143.75 - 100\text{K} \times 100 = 43.75\%$.

Hence, the correct option is (D).

18. The number of students of Mathematics who passed in 2001 = 120% of 15 = 18.

The number of students of Literature who passed in 2001= 80% of 20 = 16.

The number of students who passed in Chemistry 2001=125%of 20 = 25.

The number of students who passed in Physics 2001 = 120% of 15 = 18.

The number of students of History who passed in 2001 = 90% of 12 = 10.8.

The number of students of Biological sciences who passed in 2001 = 120% of 25 = 30.

The number of Maths students and the number of students of Physics is equal in 2001.

Hence, the correct option is (C).

Solutions for questions 19 to 21:

19. Sales or Pepsi in Asia and in North America are 15 billion gallons and 20 billion gallons respectively.

Therefore the required ratio = $\frac{20-15}{20} \times 100 = 25\%$.

Hence, the correct option is (C).

20. Per capita consumption of coke in Europe

$= \frac{25 \times 10^9}{0.75 \times 10^9}$ gallons/person

$= \frac{100}{3}$ gallons/person.

Hence, the correct option is (B).

21. The total volume of Coke in all the continents

= 100 billion gallons

Revenue = 100 × 11 i.e. \$1100 billions.

Hence, the correct option is (D).

Solutions for questions 22 to 24:

22. Interest paid by manufacturing sector in 1998

$= 108 \times \frac{135}{360} \times 5.75\%$

Interest paid by manufacturing sector in 1999

$= 144 \times \frac{135}{360} \times 5.5\%$

Total interest paid by all sectors of the industry in 1998 and 1999

= 5.75% of 108 + 5.5% of 144

∴ Interest paid by manufacturing sector forms

$108 \times \frac{135}{360} \times 5.75 + 144 \times \frac{135}{360} \times 5.5 \ \frac{135}{360}\ 100\%$

$108 \times 5.75 + 144 \times 5.5$. 100% of the total interest paid by all the sectors of the industry = 37.5%

From the above, we can see that interest paid by any sector forms $\frac{\theta}{360}.100\%$ of the total interest paid by all the sectors of the industry where θ = central angle representing loan the amount which was availed by that sector.

Maximum θ was for the manufacturing sector.

∴ Maximum percentage also was for the manufacturing sector.

∴ The required percentage cannot be 40%.

Hence, the correct option is (D).

23. Interest paid by ENRON Inc

$= \frac{5.25}{100}\left(\frac{25}{100}\left(72\left(\frac{30^\circ}{360^\circ} \text{ or } \frac{60^\circ}{360^\circ} \text{ or } \frac{135^\circ}{360^\circ} \text{ or } \frac{90^\circ}{360^\circ}\right)\right)\right)$

∴ Minimum possible interest that ENRON In*c* could have paid

$= \frac{5.25}{100}\left(\frac{25}{100}\left(72\frac{30^\circ}{360^\circ}\right)\right) = \frac{5.25}{100}\left(\frac{1}{4}.6\right)$

$= \frac{1}{4}\left(\frac{31.50}{100}\right) = \$0.07875$ billion = \$78.75 million.

Hence, the correct option is (D).

24. The total interest amount paid by 'others' in 1995 and 1996

$= \frac{5.25}{100}\left(\frac{30}{360}\right)(36) + \frac{6}{100}\left(\frac{30}{360}\right)(48)$

$= \frac{5.25}{100}\left(\frac{1}{12}.36\right) + \frac{6}{100}\left(\frac{1}{12}.48\right) = \frac{15.75+24}{100}$

= \$0.3975 billion = \$397.5 million.

The total interest for the service sector, the power sector, the agriculture sector, the agricultural sector and the manufacturing sector will be more than \$397.5 million and hence more than \$390 million. (∴ These sectors have a greater central angle than 'others'). Others consist of one or more sectors. We do not know the breakup of the total interest amount paid by 'others'. We can only say that at least 4 sectors paid more than \$390 million as interest.

Hence, the correct option is (A).

Solutions for questions 25 to 27:

25. R, S and T corresponds to products whose weights lie in the range 2.2 to 2.5 pounds

$\therefore$ \$24000 + \$24000 + \$22000 = \$70000.

Hence, the correct option is (B).

26. The heaviest product sold corresponds to circle V. The size of bubble V is the largest in the graph

$\therefore$ It was the least expensive (\$12500).

Hence, the correct option is (B).

27. Product $V = 4000$ numbers

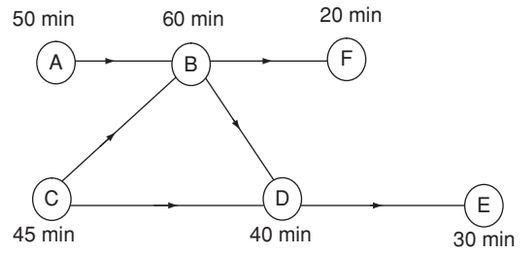
Product $U = \frac{90}{100} \times 400 = 3600$ [10% decrease for a corresponding decrease in size of all bubble]

Product $T = \frac{90}{100} \times 3600 = 3240$

$\therefore$ Total number of products U and $T = 3600 + 3240 = 6840$.

Hence, the correct option is (C).

Solutions for questions 28 to 30:



28. The minimum time that is required to complete all the tasks is 50 + 60 + 40 + 30 i.e., 180 minutes.

Hence, the correct option is (A).

29. The maximum possible time gap between starting of activities D and F is 50 minutes. Since F can be started even after 50 minutes of starting of D.

Hence, the correct option is (C).

30. If Y and Z are added then the minimum time required increased by only 10 minutes, since Z can be completed before D begins (along with A and C). Therefore, 180 + 10 = 190 minutes.

Hence, the correct option is (D).

Solutions for questions 31 and 32:

31. IS + FS + Con + Es = $26 + 16 \times 8 + 21 \times 2 + 11 \times 2 = 75.2$.

Hence, the correct option is (B).

32. Average profit across sectors = $\frac{10}{100} \times 100 = 10$

Profit from electronics = $\frac{10}{100} \times 11.2 = 1.12$.

Profit from consumer goods = $\frac{15}{100} \times 8.8 = 1.32$

$\therefore$ Profit from other sectors = 7.56

Sales from other sectors = 80.

$\therefore$ % Profit from other sectors = $\frac{7.56}{80} \times 100 = 9.45$.

Hence, the correct option is (B).

Solutions for questions 33 to 35:

33. Average annual savings in first 3 years

$$= \frac{900 + 950 + 1000}{3} = 950$$

Annual savings in the last 3 years

$$= \frac{950 + 1150 + 1350}{4} = 1152$$

% growth = $\frac{200}{950} \times 100 = \frac{4}{19} \times 100 \cong 21\%$.

Hence, the correct option is (D).

34. The income from business for various years is

1999: 900 + 750 – 750 = 900

2000: 950 + 800 – 750 = 1000

2001: 1000 + 750 – 800 = 950

2002: 950 + 900 – 700 = 1150

2003: 1150 + 1100 – 900 = 1350

2004: 1350 + 1000 – 1100 = 1250.

Hence, the correct option is (D).

35. Avg. expenditure per annum = $\frac{5300}{6} \times 1000$

Avg. income from rent per annum = $\frac{5000}{6} \times 1000$

Required difference = $\frac{30}{6} \times 1000 = ₹50000$.

Hence, the correct option is (A).

Previous Years' Questions

1. The most consistent batsman is the batsman for whom the standard deviation of the scorers is the least. ∴ K is the most consistent batsman.
Hence, the correct option is (A).

2. Let the speed of the train be S. Let the length of the platform be L

$\frac{280+L}{S}=60$ and $\frac{280}{S}=20$

$280+L=60S$ and $S=14$

$L=60(14)-280=560$.

Alternate Method

The train crosses its own length in passing a man standing on the platform. ∴ Speed of the train $=\frac{280 \text{ m/sec}}{20}=14$ m/sec.

∴ In 60 seconds, the train will cover 840 m. Of this 280 m is the length of the train. ∴ The remaining 560 m is the length of the platform.

Hence, the correct answer is 560.

3. Trade deficit $=\frac{1}{5}$(Exports)

Imports – Exports $=\frac{1}{5}$(Exports)

Imports $=\frac{6}{5}$(Exports)

This relation is satisfied in 2006.
Hence, the correct option is (D).

4. The faces of one coin are H and H.

The faces of the other coin are H and T.

A coin is chosen randomly and tossed. It comes up H.

The other face can be H, H or T.

Probability (Other face is T) $=\frac{1}{3}$

Hence, the correct option is (B).

5. For an unbiased die thrown a very large number of times, the frequency of occurrence of each of the numbers must be the same.

As the given frequencies of occurrence are not the same, the die must be biased.
Hence, the correct option is (B).

6. Let the eight consecutive odd numbers be

$x, x+2, x+4, x+6, x+8, x+10, x+12, x+14$

$x+(x+2)+(x+4)+(x+6)+(x+8)+(x+10)+(x+12)+(x+14)=656 \Rightarrow 8x=600 \Rightarrow x=75$

Let the four consecutive even numbers be $y, y+2, y+4, y+6$

$y+(y+2)+(y+4)+(y+6)=87(4) \Rightarrow 4y=336$

$y=84$

Smallest odd number $=x=75$

Second largest even number $=y+4=88$

Sum = 163

Hence, the correct answer is 163.

7. We can straight away reject the items for which the revenue figure is less than or equal to the export figure. We have to consider items 1, 3, 5 and 6 only. As all the percentage figures have to be multiplied by the same factors, to get the actual revenue and actual exports, we can ignore these factors. Of $\frac{12}{11}, \frac{23}{19}, \frac{20}{12}, \frac{19}{16}$ the greatest is $\frac{20}{12}$. ∴ Item 5 has generated the maximum revenue per kg

Hence, the correct option is (D).

8. Let the rate at which water is pumped in, be f

Half the tank has to be filled in 10 minutes.

∴ Effective rate of filling $=\frac{\frac{1}{2}}{10}=\frac{1}{20}$

$f-\frac{\frac{1}{2}}{30}=\frac{1}{20}$ i.e., $f-\frac{1}{60}=\frac{1}{20}$

$f=\frac{1}{15}=4\left(\frac{1}{60}\right)=4$ (Draining rate)

Hence, the correct option is (A).

9. According to the given chart,

Hawk, Butterfly, Drongo, Bulbul are birds.

∴ Butterflies are birds

(i) is a correct conclusion

(iii) is also a correct conclusion

'Tiger' occupies a greater proportion of the 'pie' chart when compared to 'Red ant'.

∴ There are more tigers than red ants in the forest.

(ii) is correct.

Elephants are present in the largest number they need not be the largest mammals.

No information is given to support (iv)

∴ (iv) is not necessarily correct

(i), (ii), (iii) are correct.
Hence, the correct option is (D).

10. Let the stream velocity be V Km/h.

Upstream travel time = 3(Downstream travel time)

$\frac{d}{8-V}=3\left(\frac{d}{8+V}\right) \Rightarrow V=4$

Hence, the correct answer is 4.

11. Expenditure on Labour = 4, 50, 000

Total expenditure $= 4,50,00\left(\frac{100}{15}\right) = 3$ lakhs

Profit = 10 Lakhs

Total selling price = 40 Lakhs

Selling price of each purifier $= \frac{40 \text{ Lakhs}}{200} = 20{,}000$

Hence, the correct answer is 20,000.

12. Probability that the current batch is accepted = Probability of none of the four bulbs tested being defective $= \left(1-\frac{5}{100}\right)^4 = 0.8145$

($\therefore$ Probability of any bulb being non-defective $= 1-\frac{5}{100}$).

Hence, the correct answer is 0.8145.

13. New value of $f = (2x)^n\left(\frac{y}{2}\right)^m$

$= 2^{n-m}x^n y^m = 2^{n-m}P$

Hence, the correct option is (A).

14. Let the 12 consecutive odd numbers be a, $a+2$, $a+4$, $a+6$, $a+8$, $a+10$, $a+12$, $a+14$, $a+16$, $a+18$, $a+20$, $a+22$

Sum of the first 5 numbers = $5a+20$

$5a+20 = 425 \Rightarrow a = 81$

Sum of the last 5 numbers = $a+22+a+20+a+18+a+16+a+14 = 103+101+99+97+95 = 495$

Hence, the correct answer is 495.

15. Let the industrial consumption of power in 2000–2001 be P

Industrial consumption of power in 2010–2011 = $2P$

Let the rate be r%

$$2P = P\left(1+\frac{r}{100}\right)^n \quad 2 = \left(1+\frac{r}{100}\right)^{10}$$

$$1+\frac{r}{100} = 2^{\frac{1}{10}} \approx 1.072 \qquad (1)$$

$\Rightarrow r = 7.2$

Hence, the correct option is (B).

16. Let the total cost in 2012 be x

Raw material expenses in 2012 = $0.2x$

Other expenses in 2012 = $0.8x$

Raw material expenses in 2013 = $(0.2x)(1.3) = 0.26x$

Other expenses in 2013 = $(0.8x)(1.2) = 0.96x$

Total cost in 2013 = $1.22x$

Percentage increase in total cost $= \frac{1.22x-x}{x}\times 100\% = 22\%$

Hence, the correct answer is 22%.

Note: Labour cost in 2012 is redundant data

17. Number of five digit numbers which can be formed using the digits 1, 3, 5, 7, 9 without repeating any of them = 5! = 120

Any five digit number formed is equally likely to begin with each of the five digits 1, 3, 5, 7, 9. It is also equally likely to have each of the five digits in any of its places from 2nd to 5th.

Sum of all possible five digit numbers $= \frac{120}{5}(1(10^4+10^3+10^2+10+1) + 3(10^4+10^3+10^2+10+1)+5(10^4+10^3+10^2+10+1)+7(10^4+10^3+10^2+10+1)+9(10^4+10^3+10^2+10+1))$

$= (24)(1+3+5+7+9)(11111) = 6666600$

Hence, the correct option is (B).

18. Total of the temperatures

Monday to Wednesday = $41\times 3 = 123°C$

Tuesday to Thursday = $43\times 3 = 129°C$

The temperature on Thursday–the temperature on Monday

$= 129°C - 123°C = 6°C$

Let the temperature on Monday be x°C

Temperature on Thursday =

$$x\left(1+\frac{15}{100}\right) = \frac{23}{20}x°C.$$

Given, $\frac{23}{20}x - x = 6 \quad \Rightarrow \quad x = 40$

$\therefore \quad \frac{23}{20}x = 46$

Hence, the correct option is (C).

19. Total travel time of the journey = 3 quarters of an hour $= \frac{3}{4}$ hrs

Average speed

$$= \frac{\text{Total distance travelled}}{\text{Total travel time}} = \frac{8+6+16}{\frac{3}{4}}$$

= 40 km/h

Hence, the correct option is (C).

20. Required sum = Sum to n terms of the series 1 + 9, 3 + 81, 5 + 729, $\cdots$ = (1 + 3 + 5 + $\cdots$ n terms) + (9 + 81 + 729 + $\cdots$ + n terms) = n^2 + (Sum of the terms of a G.P whose first term as well as common ratio is 9)

$$= n^2 + \frac{9(9^n-1)}{9-1} = n^2 + \frac{9}{8}(9^n-1)$$

Note: When $n = 2$, the sum evaluates to 94. Only Choice (D) evaluates to 94 when $n = 2$.

Hence, the correct option is (D).

21. The product of the roots will be negative when the roots are of opposite sign.

$\therefore \quad \frac{p(p-1)}{3} < 0.$

$\therefore p(p-1) < 0.$

$\therefore 0 < p < 1.$

Hence, the correct option is (B).

22. Any leap year has 52 weeks and 2 odd days.

$\therefore$ A leap year will have 53 Saturdays if any one of the 2 odd days are Saturdays.

$\therefore$ Probability of 53 Saturdays $= \frac{2}{7}$

Hence, the correct option is (A).

23. $(1.001)^{3321} = (1.001)^{1259} \times (1.001)^{2062}$

$= 3.52 \times 7.85$ ($\because (1.001)^{1259}$

$= 3.52$, and $(1.001)^{2062} = 7.85$)

$= 27.64$ (approx)

Hence, the correct option is (D).

24. From the table total income = ₹10,500

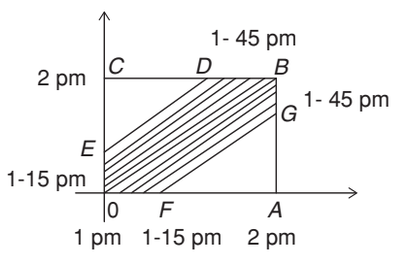
Total expenditure (other than savings) = ₹9000

$\therefore$ the required percentage

$$= \frac{9000}{10500} \times 100 = \frac{1800}{21} = 86 \quad \text{(approx)}$$

Hence, the correct option is (D).

25. We can plot A's arrival along the x – axis and B's arrival along the y axis. They will meet when $|x - y| \le 15$ minutes. When A arrives before B, they will meet if $x - y \le 15$ or else when B arrives first, they will meet when $y - x \le 15$. The sample space is the entire square, whereas the favourable region is the shaded one. [In the figure FG corresponds to $x - y = 15$ and DE corresponds to $y - x = 15$].



The required probability

$$= \frac{\text{shaded area}}{\text{Area of the square}}$$

$$= \frac{\text{Area of the square} - \text{unshaded area}}{\text{Area of the square}}$$

$$= \frac{(60)^2 - 2\frac{1}{2}(45 \times 45)}{60^2} = \frac{7}{16}$$

$\therefore$ The probability that the meeting takes place is $\frac{7}{16}$.

Hence, the correct option is (C).

26. Let the number of ₹20 notes be x and ₹10 notes be y

$\therefore x + y = 14$ (1)

and value of them is

$20x + 10y = 230.$ (2)

Solving (1) and (2) we get $x = 9$ and $y = 5$

$\therefore$ Number of ₹10 notes are 5.

Hence, the correct option is (A).

27. Let the total number of voters be x.

Number of voters who promised to vote for P and Q are $0.4x$ and $0.6x$ respectively.

Number of voters who actually voted or P = $0.4x - 0.15(0.4x) + 0.25(0.6x) = 0.49x$

Number of voters who actually voted for Q = $0.6x + 0.15(0.4x) - 0.25(0.6x) = 0.51x$

Given that $0.51x - 0.49x = 2$

$\Rightarrow x = 100.$

Hence, the correct option is (A).

28. Using the given graph and the table we can easily summarize the following

Lap	Speed Km/h	Mileage Km/litre	Fuel consumed per Km (litres/Km)
P	15	60	$\frac{1}{60}$
Q	45	90	$\frac{1}{90}$
R	75	75	$\frac{1}{72}$
S	10	30	$\frac{1}{30}$

From the above table, it can be observed that the fuel consumed per km is least for the lap Q.

Hence, the correct option is (B).

29. Let the number of toffees in the bowl originally be n.

Number of toffees left after R took his share

$$= \frac{2n}{3} + 4$$

Number of toffees left after S took his share

$$= \frac{3}{4}\left(\frac{2n}{3} + 4\right) + 3 = \frac{n}{2} + 6$$

Number of toffees left after T took his share

$$= \frac{1}{2}\left(\frac{n}{2} + 6\right) + 2 = \frac{n}{4} + 5$$

Given that $\frac{n}{4} + 5 = 17$

$\therefore n = 48$

Hence, the correct option is (C).

30.

	$y > 0$,	$y < 0$
$f(y)$	$\frac{y}{y} = 1$	$\frac{-y}{y} = -1$
$f(-y)$	$\frac{y}{-y} = -1$	$\frac{-y}{-y} = -1$

For $q > 0$, $|f(q) - f(-q)| = |1 - (-1)| = 2$

For $q < 0$, $|f(q) - f(-q)| = |-1 - 1)| = 2$

$\therefore |f(q) - f(-q)| = 2$

Hence, the correct option is (D).

31. Let $S = 4 + 44 + 444 + \cdots$ upto n terms

$\frac{9S}{4} = 9 + 99 + 999 + \cdots$ upto n terms

$= (10 - 1) + (10^2 - 1) + (10^3 - 1) + \cdots + (10^n - 1)$

$= (10 + 10^2 + 10^3 + \cdots + 10^n) - n$

$= \frac{10(10^n - 1)}{9} - n = \frac{10^{n+1} - 9n - 10}{9}$

$\therefore S = \frac{4}{81}(10^{n+1} - 9n - 10)$

Hence, the correct option is (C).

32. Let the base considered for the additions be b.

$137 + 276 = 435$

$(137)_b + (276)_b = (435)_b$

$b^2 + 3b + 7 + 2b^2 + 7b + 6 = 4b^2 + 3b + 5$

$b^2 - 7b - 8 = 0$

$(b - 8)(b + 1) = 0$

$b = 8$ ($\because$ b must be positive)

$$\begin{array}{r} 7\ 3\ 1 \\ +\ 6\ 7\ 2 \\ \hline 1\ 6\ 2\ 3 \end{array}$$

We perform the addition on base 8.

$1 + 2 = 3 = 3_8$

$3 + 7 = 10 = 12_8$

$1 + 7 + 6 = 14 = 16_8$

Hence, the correct option is (C).

33. Let the rate of work of each unskilled worker be 1 unit/day.

10 unskilled workers can build the wall in 30 days.

$\therefore$ Work = $1(10)(30) = 300$ units.

$\therefore$ Rates of work of each skilled worker and each semi-skilled worker are $\frac{300}{5 \times 20}$ units/day and $\frac{300}{8 \times 25}$ units/day respectively

i.e., 3 units/day and $\frac{3}{2}$ units/day respectively.

Combined rate of work of 2 skilled, 6 semi-skilled and 5 unskilled workers

$= 2(3) + 6\left(\frac{3}{2}\right) + 5(1) = 20$ units/day.

$\therefore$ Time taken by them to complete the work

$= \frac{300}{20}$ days $= 15$ days

Hence, the correct option is (D).

34. If there is no restriction on the number of times each digit is used, we can form (3) (3) (3) numbers between 3000 and 4000 as well as between 4000 and 5000.

$\therefore$ A total of 54 numbers could have been formed as there are only 2 twos and 3 threes.

$\therefore$ We cannot form 3222 and 3333. Also, we cannot form 4222.

A total of 51 numbers can be formed.

Hence, the correct option is (B).

TEST

HINTS/SOLUTIONS

1. Let the amounts with P, Q, R and S be ₹p, ₹q, ₹r and ₹s respectively

$p + q + r + s = 220$

$p = q + 30$

$s = \frac{q}{2}$ i.e., $q = 25 \quad \therefore \quad p = 25 + 30$

$r = s + 10$

$2s + 30 + 2s + s + 10 + s = 220$

$6s = 180$

$s = 30$

Hence, the correct option is (C).

2. Let the number of the girls and the benches be g and b respectively. If 5 girls sit on each bench, a total of $5b$ girls can sit.

$\therefore g - 3 = 5b$, i.e., $g = 5b + 3$

If there is one bench less, on $b - 1$ benches 6 $(b - 1)$ girls can sit.

$\therefore g = 6\,(b - 1)$

$g = 5b + 3 = 6\,(b - 1)$

$b = 9.$

Hence, the correct option is (D).

3. Let the number of questions correctly answered, wrongly answered and unanswered by the candidate be C, W and U respectively

$C + W + U = 60$ (1)

Candidate's mark $= C - W - U = 20$ (2)

(1) + (2) $\Rightarrow 2C = 80$, i.e., $C = 40$.

Hence, the correct option is (D).

4. $\frac{a+b+c}{x} = \frac{a-b+c}{y} = \frac{c+b-a}{z}$ (1)

if $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$ then, $\frac{a+c}{b+d} = \frac{c+e}{d+f} = \frac{a+e}{b+f}$

$\therefore$ (1) $\Rightarrow \quad \frac{a+b-c+a-b+c}{x+y} = \frac{a-b+c+c+b-a}{y+z}$

$= \frac{a+b-c+c+b-a}{x+y}$

$\therefore \quad \frac{2a}{x+y} = \frac{2c}{y+z} = \frac{2b}{x+z}$

Let each of these be k.

Then $2a = k\,(x + y)$, $2c = k\,(y + z)$ and $2b = k\,(x + z)$

$x(b - a) + y\,(a - c) + z\,(c - b)$

$\frac{xk}{2}(z - y) + \frac{yk}{2}(x - z) + \frac{zk}{2}(y - x)$

$= \frac{k}{2}\{xz - xy + xy - yz + yz - xz\}$

$= \frac{k}{2}\{0\} = 0$

Hence, the correct option is (A).

5. Let the weight of the diamond be W. Let the weights of the pieces be $3x$ and $4x$. Initial value of the diamond equals kW^3, where K is a constant.

$W = 7x$

$\Rightarrow$ Initial value of the diamond (in ₹)

$= k(7x)^3 = 343\ kx^3$

Total value of the pieces (in ₹) $= k(3x)^3 + k(4x)^3 = kx^3\,(3^3 + 4^3) = 91kx^3$

Loss (in ₹) $= 343\ kx^3 - 91\ kx^3 = 252\ kx^3 = 504000$

$\therefore\ kx^3 = 2000$

$\therefore$ Initial value $= 343kx^3 = 343 \times 2000 =$ ₹686000.

Hence, the correct option is (B).

6. Number of observations = 25

Average = 120

Correction = –(25)

Corrected average $= 120 - \frac{25}{25}$

$= 120 - 1 = 119$

Hence, the correct option is (B).

7. The average of 13 observations = 50

$\therefore\ x_1 + x_2 + x_3 + \cdots + x_{13} = 650$

Average of the first seven observations

$x_1 + x_2 + \cdots + x_7 = 45 \times 7 = 315$ (2)

Average of last seven observations

$x_7 + x_8 + + \cdots + x_{13} = 7 \times 52 = 364$ (3)

By adding (2) and (3)

$x_7 + (x_1 + x_2 + \cdots + x_{13}) = 679$

From (1)

$x_7 + 650 = 679 \Rightarrow x_7 = 29$

Hence, the correct option is (C).

8. The concentration of the milk after 3 replacements

$= \left(\frac{100 - 10}{100}\right)^2 = 0.729$

$\therefore$ Concentration of water = 0.271

Required % = 27.1%

Hence, the correct option is (D).

9. Total volume of the solution = 729 ml

Quantity of acid $= \frac{7}{6} \times 729 = 7 \times 81 = 567$ ml

Quantity of water = 162 ml

Let us assume n ml of water is to be added to make the ratio of acid to water as 7 : 3

$$\frac{162+x}{729+x}=\frac{3}{10} \Rightarrow 1620+10x=2187+3x$$

$7x = 2187 - 1620 = 567$

$x = 81$

$\therefore$ 81 litres of water needs to be mixed.

Hence, the correct option is (D).

10. Let the price of acid per gallon be ₹100

Also the quantity of water mixed per gallon of acid be x gallons.

Cost price of $(x + 1)$ gallon = ₹100

Cost price of 1 gallon solution $= \frac{100}{x+1}$

Selling price of 1 gallon solution = ₹100.

$$\therefore \quad \frac{100}{x+1}\times 1.2 = 100$$

$$\frac{120}{100} = x+1$$

$1.2 = x + 1$

0.2 gallon $= x$

Volume of water mixed per gallon of acid = 0.2 gallon

Hence, the correct option is (A).

11. Let the usual time taken be t hours.

$$\text{Distance} = 6\left(t-\frac{10}{60}\right) = 4\left(t+\frac{10}{60}\right) t = \frac{5}{6}$$

$$\text{Required speed} = \frac{\text{Distance}}{t}$$

$$= \frac{6\left(\frac{5}{6}-\frac{10}{60}\right)}{\frac{5}{6}} = 4.8 \text{ km/h.}$$

Hence, the correct option is (C).

12. Total distance (in Km) = (30)(1) + (50)(5) + (75)(4) = 580

Total time = 10 hours

Average speed (in km/h) $= \frac{580}{10} = 58.$

Hence, the correct option is (C).

13. In an hour, the distance it can cover without stoppages = 54 Km.

In this time, the distance it can cover with stoppages = 36 Km

It does not cover 18 Km due to its stoppages.

Stoppage time/h = Time taken to cover 18 Km

$= \frac{18}{54}$ hrs = 20 minutes.

Hence, the correct option is (C).

14. Let the times in which A, B and C can complete the job be a days, b days and c days respectively.

$$\frac{1}{a}+\frac{1}{b}=\frac{1}{40} \quad (1)$$

$$\frac{1}{b}+\frac{1}{c}=\frac{1}{30} \quad (2)$$

$$\frac{1}{a}+\frac{1}{c}=\frac{1}{20} \quad (3)$$

Subtracting (1) from (2),

$$\frac{1}{c}-\frac{1}{a}=\frac{1}{120}$$

$$\Rightarrow \quad \frac{1}{c}=\frac{1}{120}+\frac{1}{a} \text{ substituting this in Eq. (3),}$$

$$\frac{1}{a}+\left(\frac{1}{120}+\frac{1}{a}\right)=\frac{1}{20} \text{ or } \frac{2}{a}=\frac{5}{120}$$

or $a = 48$

Hence, the correct option is (C).

15. Let the times in which each man and each woman can complete the job be m days and w days respectively.

Part of the job which can be completed by a man in a day $= \frac{1}{m}$.

Par of it which can be completed by a woman in a day $= \frac{1}{w}$.

$$\text{Job} = \left[3\left(\frac{1}{m}+4\left(\frac{1}{w}\right)\right)\right](10)$$

$$= \left[24\left(\frac{1}{m}+2\left(\frac{1}{w}\right)\right)\right](2)$$

$$\frac{30}{m}+\frac{40}{w}=\frac{48}{m}+\frac{4}{w}$$

$$\frac{36}{w}=\frac{18}{m}$$

$$\frac{2}{w}=\frac{1}{m}$$

$$\therefore \quad \text{job} = \frac{30}{m}+\frac{40}{w} = 30\left(\frac{2}{w}\right)+\frac{40}{w}=\frac{100}{w}$$

Part of the job which can be completed by 5 men and 10 women in a day $= 5\left(\frac{1}{m}\right)+10\frac{1}{w}$

$$= 5\left(\frac{2}{w}\right)+10\frac{1}{w}=\frac{20}{w}$$

Required time (in days) $= \dfrac{\dfrac{100}{w}}{\dfrac{20}{w}} = 5$

Hence, the correct option is (B).

16. Let initial income be $100x$.
Rent = $25x$.
Food = $15x$.
Medical = $4.8x$
Miscellaneous = $13.8x$
Total expenses = $58.6x$
Total savings = $41.4x = 82800$
$x = 2000$.
Income = 200000
Hence, the correct option is (A).

17. $P(30) = S(10)$

$3P = S$

$\dfrac{P}{S} = \dfrac{1}{3}$

$\dfrac{P}{C} = \dfrac{1}{2}$

$P\% = 50$

Hence, the correct option is (C).

18. $1036 - 12 = 1024$

$1545 - 9 = 1536$

The HCF of 1024 and 1536 = 512

Hence, the correct option is (B).

19. Time interval for the bells to toll together is the LCM of 10, 15, 20, 25 and 30 = 300 sec = 5 min.

The next time the bells toll together is at 8:05 am.

Hence, the correct option is (C).

20. $\dfrac{3}{\sqrt{2}+\sqrt{5}} + \dfrac{3}{\sqrt{5}+\sqrt{8}} + \dfrac{3}{\sqrt{8}+\sqrt{11}} + \cdots + \dfrac{3}{\sqrt{25}+\sqrt{29}}$

$= \dfrac{2}{\sqrt{5}+\sqrt{2}} \times \dfrac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}} + \dfrac{3}{\sqrt{8}-\sqrt{3}} \times \dfrac{\sqrt{8}-\sqrt{3}}{\sqrt{8}-\sqrt{5}}$

$+ \dfrac{3}{\sqrt{11}+\sqrt{2}} \times \dfrac{\sqrt{11}-\sqrt{8}}{\sqrt{11}-\sqrt{8}} + \cdots + \dfrac{3}{\sqrt{29}+\sqrt{26}} \times \dfrac{\sqrt{29}-\sqrt{26}}{\sqrt{29}-\sqrt{26}}$

$= \left[\begin{array}{l} \dfrac{3(\sqrt{5}-\sqrt{2})}{3} + \dfrac{3(\sqrt{8}-\sqrt{5})}{3} + \dfrac{3(\sqrt{11}-\sqrt{8})}{3} \\ + \cdots + \dfrac{3(\sqrt{29}-\sqrt{26})}{3} \end{array}\right]$

$= \left\lfloor \sqrt{5}-\sqrt{2}+\sqrt{8}-\sqrt{5}+\sqrt{11}-\sqrt{8}+\cdots+\sqrt{29}-2\sqrt{6} \right\rfloor$

$= -\sqrt{2}+\sqrt{29} = \sqrt{29}-\sqrt{2}.$

Hence, the correct option is (A).

21. Given, $a + b + c = 0$

If $a + b + c = 0$ then $a^3 + b^3 + c^3 = 3abc$ (1)

$(3^{a^8})^{\frac{1}{a^6bc}} \cdot (3^{b^8})^{\frac{1}{ab^6c}} \cdot \dfrac{c^8}{3^{abc^6}}$

$\Rightarrow \quad 3^{\frac{a^2}{bc}} \cdot 3^{\frac{b^2}{ac}} \cdot 3^{\frac{c^8}{abc^6}} \quad \therefore \quad (a^m)^n = a^{mn}$

$= 3^{\frac{a^2}{bc}+\frac{b^2}{ac}+\frac{c^2}{ab}} \quad (\therefore \quad a^m a^n = a^{m+n})$

$= 3^{\frac{a^3+b^3+c^3}{abc}} = 3^{\frac{3abc}{abc}} \quad (\therefore \text{ from (1)})$

$= 3^3 = 27$

Hence, the correct option is (C).

22. Given, $3^{x+3} - 3^{x+2} = 486$

$\Rightarrow 3^x \cdot 3^3 - 3^x \cdot 3^2 = 486$

$\Rightarrow 27\,(3^x) - 9(3^x) = 486$

$\Rightarrow 18\,(3^x) = 486$

$\Rightarrow \quad 3^x = \dfrac{486}{18} = 27$

$\Rightarrow 3^x = 3^3 \Rightarrow x = 3.$

Hence, the correct option is (A).

23. $\log_p q$ and $\log_r q$ are given. We can invert the first expression and get $\log_q p$. Then we get $\log_r p$. We can find x.

$\log_p q = \dfrac{5}{4} \quad \Rightarrow \quad \log_q p = \dfrac{4}{5}$

$\log_r q = \dfrac{5}{6}$

$\therefore \quad \log_r p = \left(\dfrac{4}{5}\right)\left(\dfrac{5}{6}\right) = \dfrac{2}{3} = 3x$

$\therefore \quad x = \dfrac{2}{9}$

Hence, the correct option is (D).

24. $\dfrac{5}{1+\log_p qr} + \dfrac{5}{1+\log_q pr} + \dfrac{5}{\log_r pq + 1}$

$= \dfrac{5}{\log_p p + \log_p qr} + \dfrac{5}{\log_q q + \log_q pr} + \dfrac{5}{\log_r pq + \log_r r}$

$= \dfrac{5}{\log_p pqr} + \dfrac{5}{\log_q pqr} + \dfrac{5}{\log_r pqr}$

$\Rightarrow \quad (\log_{pqr} pqr + \log_{pqr} q + \log_{pqr} r)$

$\left(\because \quad \log_b a = \dfrac{1}{\log_a b}\right)$

$\Rightarrow 5\log_{pqr} pqr \Rightarrow 5 \times 1 = 5.$

Hence, the correct option is (C).

25. Given, $\log_{10} 2 = 0.3010$

2^5 let $P = 2^{55}$

$\log_{10} p = \log_{10} 2^{55} = 55\log 2$

$= 55(0.3010) = 16.555$

$\therefore$ The number of digits = 16 + 1 = 17.

Hence, the correct option is (A).

26. Given series is 45, 41, 37, ...

to get the maximum value of sum, the nth term is positive.

$t_n > 0$

$45 + (n-1)(-4) > 0$

$-4n + 49 > 0$

$4n < 49$

$n < \frac{49}{4} \quad \Rightarrow \quad n < 12.25$

$n = 12$

$S_{12} = \frac{12}{2}[2 + (11)4] = 276$

Hence, the correct option is (C).

27. Let $1 + 3 + 3^2 + 3^3 + \cdots$ to n terms < 3000

$= 1 \cdot \frac{3^n - 1}{3 - 1} < 3000$

$= 3^n - 1 < 6000$

$3^n \le 3^7 < 6001$

$\therefore$ The greatest value of n is 7.

Hence, the correct option is (C).

28. $S = 3 + 33 + 333 + \cdots +$ upto n terms

$\therefore 3S = 9 + 99 + 999 + \cdots +$ upto n terms

$3S = (10 - 1) + (100 - 1) + (1000 - 1) + \cdots + (10^n - 1)$

$= 10 + 100 + \cdots + 10^n - (1 + 1 + \cdots$ upto n times)

$\therefore \quad 3S = \frac{10(10^n - 1)}{27} - n$

$\Rightarrow \quad S = \frac{10(10^n - 1)}{27} - \frac{n}{3}$

Hence, the correct option is (D).

29. Given, $f(x) = x^2 - 15x + 9 = 0$

Here $a = 1 > 0$

$\therefore f(x)$ has a minimum value.

The minimum value $= \frac{4ac - b^2}{4a}$

$= \frac{4(1)(9) - (-15)^2}{4} = \frac{36 - 225}{4} = \frac{-189}{4}$

Hence, the correct option is (D).

30. Area of shaded region = Area of quadrant–Area of triangle $= \left(\frac{1}{4}\pi(64) - \frac{1}{2}(8)(6)\right) \text{cm}^2$

$= (16\pi - 24)\ \text{cm}^2$

Hence, the correct option is (D).